MASTER THESIS

Applying Mathematics in the Natural Sciences

An Unreasonably Effective Method

Lisanne Coenen

Master Philosophy of Natural Sciences
July 2015

Leiden University
Faculty of Humanities
Institute for Philosophy

Supervisor:
Victor Gijsbers

Abstract

In this thesis the unreasonable effectiveness of mathematics in the natural sciences is discussed. I will show that this is a deep philosophical problem for which no easy solution is available. A historical analysis of the role of mathematics in science shows that basic mathematics, an abstraction from empirical observation, evolved into complex mathematics, a human invention completely detached from its empirical roots. The conclusion of this analysis is that the applicability of mathematics cannot be explained by adhering to the empirical roots of mathematics. This poses a philosophical problem: how can something that is anthropocentric describe and predict the intricate workings of natural phenomena so accurately? This question is my main research question and is also thoroughly discussed by Mark Steiner (1998). He places emphasis on the predictive power of mathematics in the natural sciences and I will show that Steiner's main argument, that anthropocentric elements in mathematics play a crucial, and unreasonable effective, role in the discovery of new physical theories is a valid observation in need of an explanation. The mapping accounts of Pincock (2004) and Bueno and Colyvan (2011) are discussed, who attempt to render the anthropocentric elements in mathematics intelligible. They both turn out to be incomplete and therefore, I have provided an improved inferential mapping account that is able to render parts of the anthropocentric influences in mathematics intelligible. However the successful use of tractability assumptions cannot be explained by this mapping account. This leads to the conclusion that the world looks 'user-friendly', because our anthropocentric assumptions result in correct knowledge about the natural world. Therefore, one cannot refrain from a metaphysical discussion about the relation between mathematics, mind and world. I discuss several metaphysical accounts, of which the most reasonable is the simple explanation that we just 'see what we look for'. A price needs to be paid however; complete knowledge about the world around us will never be possible. Moreover, it remains mysterious that we are able to control natural phenomena in such a detailed way, whilst only having knowledge of a small part of it. The final chapter mentions the changing role of mathematics in science in the last 30 years, where advancements in theoretical physics increased the importance of mathematical methods, whereas advancements in computer science decreased this role. I conclude that now more than ever, it is important to reflect on the role of mathematics in the scientific method.

Contents

Chapter 1

Introduction

Science is an extremely powerful tool; both in its ability to describe the world and as the starting point for many innovations and novel technologies. The scientific method relies heavily on mathematics which is used to quantify the phenomena in the world. Old mathematical structures are re-invented and new mathematical structures are put in place to quantify observations and theories and every time it became apparent that the mathematical toolbox perfectly fitted onto the physical description of the world. This raised suspicion about the true status of mathematics and Eugene Wigner, in his famous article 'the unreasonable effectiveness of mathematics in the natural sciences' described this suspicion (Wigner, 1960). How can it be, he asked,

> that the mathematical formulation of the physicist's often crude experience leads in an uncanny number of cases to an amazingly accurate description of a large class of phenomena. This shows that the mathematical language has more to commend it than being the only language which we can speak; it shows that it is, in a very real sense, the correct language. (Wigner, 1960, pp. 5-6)

What Wigner makes clear is that the view that mathematics is merely a tool for scientists cannot be the whole story. He asked *why* the language of mathematics is able to describe and predict natural phenomena - and why it does that so accurately. A result that exemplifies this incredible accuracy of general mathematical methods is the determination of the theoretical value of the gyromagnetic ratio g, a constant that was important for determining the magnetic moment of an electron. The magnetic moment of an electron follows the equation $\mu = g(eh/2mc)S$, where g was determined by the by then accepted Dirac equation and should equal 2 according to that equation. However, experiments showed a deviation from this value, which was strange since the Dirac equation predicted results with an accuracy way better than this deviation. The attempts to solve this anomaly resulted in the new field of relativistic quantum electrodynamics, in which state of the art mathematics was used to describe the behavior of small systems. After long, devious calculations and very precise new measurements during the 70s and 80s, atomic physicists found the following values for g (Gross, 1988, p. 8372):

$$g_{theory} = 2 \cdot (1,000159652459 \pm 0,000000000123)$$
$$g_{experiment} = 2 \cdot (1,000159652459 \pm 0,000000000004)$$

This result has even been improved upon since the first experiments, which led to a relative standard uncertainty of $7,6 \cdot 10^{-13}$ in 2006 (Odom, Hanneke, D'Urso, & Gabrielse, 2006). You cannot but wonder how this impressive result came about and what the apparently strong relation is between the pure mathematical structure underlying quantum electrodynamics and the natural world. It doesn't seem like an approximation anymore, when the value is accurate up to thirteen decimals.

It is cases like these that led Wigner to state that the applicability of mathematics is a *"wonderful gift which we neither understand nor deserve"* (p. 9). With his article he articulated more clear and more pressing than ever the mysterious applicability of mathematics in the natural sciences. However as Bochner (1966) and Colyvan (2001) both state, this philosophical problem has not received and is not receiving enough attention. The topic was never discussed in depth in philosophical and scientific circles and the few authors that do discuss it conclude with phrases like 'a suggestion is made', 'it remains an open question' and 'more work needs to be done'.

Wigner's question will be the main research question in this thesis. Is the use of mathematics in the natural sciences truly unreasonably effective or can it be rendered intelligible? I will approach the problem from three different perspectives: A historical perspective that shows the development of mathematics over time and its connection to natural science, a methodological perspective that shows *how* mathematics enters the scientific practice and a metaphysical perspective that questions our definition of mathematics. The structure of my thesis follows largely these three perspectives.

Chapter 2 focuses on the historical approach and discusses how mathematics and science got intertwined. It focuses mainly on the period during the scientific revolution, wherein the merge of mechanics and mathematics initiated the close collaboration between mathematics and science in general. Fresnel's theory of total reflection is thoroughly discussed, since it is believed that this is the first time in science that 'more came out of mathematics than was put in by it'. Two conclusions follow from this section: mathematics as we know it today is detached from its empirical origins and history has shown that mathematics developed without an application in sight could nevertheless be useful for describing natural phenomena.

Chapter 3 discusses the scope, relevance and validity of the main research question, because although many philosophers have recognized that there is something strange here, very few have actually taken up the task of defining and solving 'Wigner's puzzle'. This disinterestedness is not strange, since mathematics is such a normal part of our lives that its usefulness seems unproblematic and not worth of philosophical attention. I will take some time, therefore, to discuss Wigner's article in detail and to show that the relation between mathematics, science and the natural world is not so unproblematic as it looks, along the way rejecting some of the 'easy way out' solutions to Wigner's puzzle.

Chapter 4 is the start of the methodological approach and is concerned with one of the most important responses to Wigner's article: Mark Steiner's book *The Applicability of Mathematics as a Philosophical Problem*. I will discuss Steiner's anthropocentric argument and discuss his two most important examples that defend this argument: the quantization procedure and the prediction of the positron by Dirac. His methodological approach leads to the insight that the anthropocentric elements present in mathematics, such as the beauty of equations, play a crucial role in the development of new physical theories and moreover, that this role is unaccounted for and the mathematics therefore unreasonably effective. Although I grant that there are anthropocentric elements present in the mathematical methods, I question his conclusion that these anthropocentric elements are unintelligible in the scientific method. This question, whether the anthropocentric elements in mathematics can be rendered intelligible is therefore the main question of Chapter 5. Here I investigate several mapping accounts that show how mathematics is used in the natural sciences. I reject Pincock's mapping account and accept part of Bueno & Colyvan's inferential mapping account in which inferential relations between experiments and mathematical conclusions play an important role. However, I will show that their mapping account is not complete since it does not take into account the first and most important step in the process: the use of tractability assumptions to make the empirical situation mathematically tractable. I argue that tractability assumptions are used to handle the empirical situation mathematically, and that these assumptions are anthropocentric and cannot be made intelligible by invoking inferential relations. I provide an improved mapping account, in which these influences are displayed and where the role of experiments become more clear. It is concluded that part of Wigner's and Steiner's problems can be solved by adopting this improved mapping account though the role of tractability assumptions in the scientific method is still unaccounted for.

This leads to the realization that the world looks 'user-friendly' and that an answer has to be found to the question what mathematics really is and where it comes from. These are metaphysical questions and Chapter 6 will therefore be concerned with a metaphysical approach. Here I provide metaphysical solutions to the problem of the applicability of mathematics in the natural sciences without pretending to be fully exhaustive. Platonism is reviewed, a solution from cognitive science discussed, the simple solution that we just 'see what we look for' proposed and an insight from theoretical physics given. All these solutions question the way I have defined mathematics and its relation to the human mind and the natural world.

Chapter 2

The rise of complex mathematics

It is not immediately apparent that there is a problem with the effectiveness of mathematics. Many scientists never considered the relation between mathematics and science as problematic and have taken its effectiveness for granted. The ones that did puzzle over the close connection between mathematics and science all agree that there is something strange about the relationship - among them Albert Einstein:

> At this point an enigma presents itself which in all ages has agitated inquiring minds. How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality? Is human reason, then, without experience, merely by taking thought, able to fathom the properties of real things. (Einstein, 1922, p. 15)

Indeed when we look at the successes of science, it almost seems miraculous how well the mathematical predictions match the outcome of experiments. One explanation of the applicability of mathematics could be that our most profound and complex mathematical concepts can be led back to simple abstractions from Nature and that this is the reason that mathematics as we know it is so useful. Mac Lane (1990) defends this stance, claiming that this is the solution to Wigner's puzzle. I don't agree with him, or with any other defender of this claim and I will show that mathematics is more than an abstraction from Nature by looking at its historical narrative. What is the origin of mathematics and how did it get intertwined with science? This chapter has the aim of showing that complex mathematics, the mathematics as we know it today, is influenced by more than the structure of Nature.

2.1 The empirical origins of mathematics: basic vs. complex mathematics

First, a distinction has to be made between two types of mathematics, what I will call basic and complex mathematics.

Basic mathematics includes geometry and arithmetic. As the oldest branches of mathematics they have their origins in ancient Greece and still play a major role in mathematics and science today. Geometry and arithmetic have empirical origins and are an abstraction from experience. The need to count the number of sheep or estimate the area of a triangle-shaped land necessitated this abstraction from empirical observation.

Complex mathematics is a more recent development and has its origins in the 16th century. As mathematics evolved, mathematical structures and objects did not clearly resemble structures and objects in the natural world anymore. Numerous examples can be given, such as the development of calculus by Newton and Leibniz or more recently, the development of group theory. Moreover, anthropocentric elements like beauty and simplicity influenced the development of new mathematical theories which removed complex mathematics further from its empirical origins, as also noticed by Peat (1990):

> "Mathematics is not really concerned with specific cases but with the abstract relationships of thought that spring from these particular instances. Indeed, mathematics takes a further step of abstraction by investigating the relations between these relationships. In this fashion, the whole field moves away from its historical origins, towards greater abstraction and increasing beauty." (Peat, 1990, p. 156)

In this thesis and in the debate about the applicability of mathematics in general, the focus is on these more complex forms of mathematics. In basic mathematics, its applicability is not surprising since it has a direct connection to the natural world. In complex mathematics however, the question arises whether it still has its roots in experience or that it is completely detached from it. Wigner uses a conversation between two friends of which one is a statistician, to exemplify the way complex mathematics has become detached from its empirical origins. The statistician explains all the symbols that feature in the Gaussian distribution, to which the other friends asks in bewilderment what on earth the ratio of the circumference of the circle to its diameter, π, has to do with a statistical distribution. Indeed, statistics is a case of complex mathematics, in which all its elements can not be immediately led back to its meaning in the physical world, but that does however result in consistent predictions about that physical world.

The question in the debate about the applicability of mathematics now becomes the following: are these more complex forms of mathematics still *"evolutionarily tethered to those empirical origins"* (Oldershaw, 1990, p. 142) or have they become cut off from their roots? In arguing the former the applicability of mathematics becomes less of a mystery because it has all evolved from experience about the natural world. But in arguing the latter, it remains a mystery to be solved.

The aim of the following sections is to discuss the development of mathematics and science and to discover where they began to cross paths. From this it becomes clear when and how basic mathematics became complex mathematics. The example of Fresnel in section 2.4 shows how complex mathematics for the first time became 'unreasonably effective' in describing natural phenomena. I will make a distinction between three periods in science: ancient, classical and modern science (following Dijksterhuis (1961)). In this chapter, I put some emphasis on the transition from ancient to classical science

2.2 Mathematics in ancient Greece

Greek mathematics has its origin in the mathematical methods developed in Egypt and Babylon which is now called pre-Greek mathematics. Yet it was not until the Greek period that mathematical concepts and names for various areas of mathematics were introduced. The word 'mathematics' is therefore also a Greek word, and means something in the spirit of 'acquired knowledge' or 'knowledge acquirable by learning' (Bochner, 1966). Originally mathematics therefore had a more general scope than the mathematics we know today. It was not until Aristotle that mathematics had converged into what we now would describe as mathematics.

The mathematics in ancient Greece mainly consisted of two fields: arithmetic and geometry. It is remarkable that in Greek mathematics no mention is made of symbolic algebra. Algebraic methods were known and used by the Babylonians before them, but somehow Greek mathematicians made the choice to adopt geometry and arithmetic but to declare algebra superfluous.

Greek mathematics culminated in the 3rd century B.C. with Euclid's Elements. The greatest achievement of Euclid is his axiomatization of the geometry and arithmetic known at that time. This method of axiomatization has since then never left the field of mathematics; all mathematics is based on the use of axioms. Though a great achievement, Euclid's Elements is still what I call 'basic mathematics'. With the only branches being arithmetic and geometry, its mathematical structures were a direct abstraction from empirical observations. But although their mathematics has a clear empirical origin, the Greeks were not willing to apply their mathematics to problems outside the mathematical realm. In a Platonic spirit, they believed their mathematics was about the forms of the Ideal World, not about the maximization of a corn field. It would take until the 17th century before the realization dawned that an application outside mathematics was possible.

2.3 From ancient to modern science; the birth of complex mathematics

What the Greeks did not do, namely develop an algebraic system, was accomplished in the Renaissance in Italy.[1] An important role was played by the Italian mathematicians Tartaglia and Cardano: Tartaglia solved for the first time a cubic equation, whereas Cardano introduced negative numbers and negative roots to algebra (Burton, 2011). Other important developments until and during the scientific revolution were the re-introduction of symbolic algebra, last used in Pre-Greek mathematics by the Babylonians and the invention of logarithms in 1614. Algebra was not the only domain though in which new mathematics was developed in that period . Number theory was further developed by Fermat, Euler and Gauss, probability theory invented by Pascal, and Descartes and Fermat founded analytic geometry, combining algebra and geometry (Katz, 2009). We can safely say that these mathematical methods were no longer basic mathematics: difficult proofs and new mathematical structures and relations were put forward that had not much to do with the abstraction of an empirical observation. Hamming (1980) endorses this and furthermore claims that much of the development of mathematics in this period is influenced by aesthetics:

> Mathematics has been made by man and therefore is apt to be altered rather continuously by him. Perhaps the original sources of mathematics were forced on us, but as in he example I have used [how

[1] I leave out the role of Chinese, Islamic and Indian mathematics. This does not mean that no great advancements were done here. Many of the mathematics developed in the Renaissance in Europe is thought to be influenced by these mathematical cultures. See Katz (2009).

> number theory was extended with the number zero, the complex numbers, transcendental numbers, etc.] we see that in the development of so simple a concept as number we have made choices for the extensions that were only partly controlled by necessity and often, it seems to me, more by aesthetics. (Hamming, 1980, p. 87)

I will show in the next section that especially the introduction of complex functions is an example in mathematics of the moving away from its empirical origins.

The fast development of complex mathematics was one of the reasons that in the 16th century Galileo was able to make an explicit connection between mathematics and science and claim that the book of Nature was written in mathematical terms. When he formulated his law of falling bodies he made extensive use of mathematical methods. Galileo therefore marked the beginning of the mathematization of science in which the two disciplines influenced each other heavily. New mechanics made the introduction of new mathematical concepts necessary and new mathematics influenced the formulation of new mechanical theories. The birth of classical science was a fact and there was an essential difference with ancient and medieval times, as Dijksterhuis points out:

> Classical mechanics is mathematical not only in the sense that it makes use of mathematical terms and methods for abbreviating arguments which might, if necessary, also be expressed in the language of everyday speech; it is so also in the much more stringent sense that its basic concepts are mathematical concepts, that mechanics itself is a mathematics. (Dijksterhuis, 1961, p. 499)

Here, Dijksterhuis states that mathematics was not just a language for Galileo, Newton and others. Their mathematical formulae could not be translated in a different language or explained in a different way: the mathematical relations were all they had. The best way to exemplify this is by explaining how Newton formulated the law of gravitation.

When Galileo put forward his law of falling bodies he never intended it to be applicable beyond the realm of physical objects on earth. The law was also not very accurate, not in the least because measuring techniques were not yet well developed. Nevertheless, Newton used the law of free falling bodies to describe the motion of the planets. He used the insight that the trajectory of a rock thrown into the sky is much like the trajectory of a planet moving through space and used the numerical coincidence he found between the two phenomena to formulate his universal law of gravitation. The way

Newton arrived at his law was therefore not by renowned scientific methods or by experiments and deduction. According to Wigner, *philosophically, the law of gravitation as formulated by Newton was repugnant to his time and to himself. Empirically, it was based on very scanty observations"* (Wigner, 1960, p. 6). The only thing Newton knew for sure was that his mathematics was consistent and that there was a numerical coincidence between the trajectory of a rock and that of a planet. The law of gravitation, as formulated by Newton in 1687, is now known to be accurate to less than a ten thousandth of a per cent.

The other strange thing about the law of gravitation is that it cannot be articulated in any other way than in the mathematical form. Newton not only formulated his law of gravitation in mathematical terms, it was the only way in which he could account for the phenomena - by adhering to the inverse square law. When you think about it, you should be able to explain such a basic law in terms of physical phenomena. For example, we are able to reformulate Boyle's law that relates the volume and pressure of an ideal gas to a theoretical description of particles in a closed system moving faster or slower and bumping into each other. In the case of the law of gravitation this is not possible, as Richard Feynman also points out:

> [...] up to today, from the time of Newton, no one has invented another theoretical description of the mathematical machinery behind this law which does not either say the same thing over again, or make the mathematics harder, or predict some wrong phenomena. So there is no model of the theory of gravitation today, other than the mathematical form. (Feynman, 1967, p. 42)

The law of gravitation then shows two things. First, it shows that mathematics has indeed more to commend it than being just *a* language. Second, it proves the beginning of a new era, in which not the physical cause of a phenomenon was central but merely its description in mathematical terms, as also noticed by Kline:

> Mathematical deduction from the quantitative law proved so effective that this procedure has been accepted as an integral part of physical science. What science has done then, is to sacrifice physical intelligibility for the sake of mathematical description and mathematical prediction. (Kline, 1985, p. 122)

At the end of the 18th and the beginning of the 19th century, a massive amount of mathematics was created for mechanics which resulted in the realization that mathematics had lost a part of its 'pure' character. Since then a distinction is made between applied mathematics and pure mathematics.[2] Applied mathematics was a new branch in which the mathematics developed had the only purpose of describing natural phenomena such as motion and force. Pure mathematics was then only concerned with problems from the mathematical realm, problems for which it was believed that they were of no use in the natural sciences.

So far, I have made two distinctions: the distinction between basic and complex mathematics and the distinction between applied and pure mathematics. Greek mathematics was a case of basic pure mathematics. The mathematics used by Newton and Galileo was complex applied mathematics since the mathematics was invented just for this purpose. However, we have also seen that 'pure' mathematics was also developed greatly during the scientific revolution, in which it changed from basic to complex mathematics. It is this mathematics that I will turn to now, by showing that complex pure mathematics was found to be applicable in the natural sciences as well. The example that I use concerns the development of complex number theory and its application to optics.[3]

2.4 The complexification of mathematics

The need for complex numbers arose out of the need to solve cubic and quadratic equations for which the solution had no real roots. Cardano and Bombelli first used it in the 16th century, after which Descartes, Newton and Leibniz developed the theory of complex numbers further, however not seeing any application in science. Newton believed that complex roots showed the insolubility of a problem. Leibniz was more optimistic about the use of complex numbers which he called a 'hermaphrodite between existence and non-existence' (Remmert, 1991, p. 58).

It was Euler that eventually postulated a more or less complete theory of complex numbers that related the field to other mathematical disciplines. In 1728 he stated his famous formula that still surprises undergraduates today:

$$e^{ix} = cosx + isinx$$

and in particular, when x denotes an arc length of π:

[2] Applied mathematics has a changed meaning nowadays. Here I merely mean the mathematics that was invented for scientific purposes.

[3] Though complex number theory is a subbranch of what I call complex mathematics, the adjective 'complex' here refers to the square root of a negative number, not to an 'advanced' type of number theory.

$$e^{i\pi} = -1$$

The surprising thing is that it connects three symbols used extensively in mathematics: e, π and i. What Euler led to the introduction of the complex number i, which equals $\sqrt{-1}$, is strangely still not known: it appears out of nowhere and he made no attempt to prove it. Nonetheless, it quickly became clear that the formulas were extremely useful for almost all other fields in mathematics. In the years that followed complex numbers spread out in every corner of mathematics known by then but it wasn't until 1823, almost 100 years later, that the leap of faith was made into the domain of physics. It is believed by Bochner (1966) that with this step pure complex mathematics was used for the first time in the description of a natural phenomenon:

> We think that this was the first time that complex numbers or any other mathematical objects which are "nothing-but-symbols" were put into the center of an interpretative context of "reality", and it is an extraordinary fact that this interpretation, although the first of its kind, stood up so well to verification by experiment [...]. (Bochner, 1966, p. 242)

Bochner is talking about Fresnel's theory of total reflection, in which Fresnel showed that for certain angles, the incident light is completely reflected at the transition between two materials. Fresnel found out that the propagation of light in adjoining materials was dependent on three parameters; the angle of incidence α, the angle of refraction β, and μ, the ratio between the refractive indices of the two materials. He found the following geometric relation between these three parameters,

$$sin\alpha = \mu sin\beta,$$

but the question quickly arose what the angle of refraction would be if $sin\alpha > \mu$. In this situation, $sin\beta$ would be larger than 1, meaning that β would become a complex number. This was impossible, since β was supposed to represent a physical quantity. Because the situation that $sin\alpha > \mu$ was perfectly conceivable, Fresnel was not able to use Newton's strategy and declare the solution not real. Moreover, Fresnel already established in earlier work that the ratio between the amplitudes of reflected and incident light is

$$-\frac{sin(\alpha-\beta)}{sin(\alpha+\beta)}.$$

When he now calculated the absolute value of the ratio in the case that β was complex, he found the value 1 in all cases. These two facts, that β was a complex value and that in that case the ratio between the amplitude of reflected and incident light is 1, let him to conclude that the ray must be completely reflected and that somehow, the complex value tells us something about the natural phenomenon of total reflection. Fresnel (1831) states that he has no good explanation why this should be the case, apart from the argument that 'it seems the most natural explanation to him':

> Was die Formel (C) betrifft, welche ich auch daraus abgeleitet habe, und welche das Gesetz der durch die totale Reflexion eingeprägten Modificationen darstellt , so muss ich bekennen, dass sie sich nicht auf eine so notwendige Weise daraus ergiebt; allein sie scheint mir die natürlichste Auslegung zu seyn, wenn der Werth von v imaginär wird, und diese Auslegung, welche sich schon durch die Formeln selbst bewährt, wird uberdiess durch die fünf hier erwähnten Versuche, wie durch meine älteren Beobachtungen bestätigt. (Fresnel, 1831, p. 124)

Indeed, experiments confirmed Fresnel's gut feeling; the light rays were completely reflected when β became complex. It remains strange that Fresnel attached a meaning to a solution of complex values, instead of just claiming in a Newtonian way that for these solutions there was no counterpart in the natural phenomenon (Remmert, 1991) (Bochner, 1966).

Fresnel's theory of total reflection was the starting point for a wide variety of applications of complex numbers and functions in theoretical physics. Nowadays, complex numbers are used in all physical theories and they are even a major part of the most important equations in quantum mechanics - both Heisenberg's uncertainty principle and the Schrödinger equation are formulated as complex functions.

In conclusion, three important things can be concluded from this historical narrative. First, that complex mathematics (applied or pure) has deviated from its empirical origins and is influenced by more than just the structure of Nature. This means that the simple solution to Wigner's puzzle, that mathematics is useful because it mirrors the structure of Nature, is no longer available to us. Mathematics is an invention of the human mind and not just a reflection of Nature's harmony. The second conclusion is that not even applied mathematics is free from controversy, as was exemplified by Newton's law of gravitation. Here, the most general question of all can be asked: why does mathematics work *at all* in the description of natural phenomena and why does it do that so accurately? It is a mistake to think that it is only pure mathematics that is unreasonable - the law of gravitation shows that also the usefulness of applied mathematics is in need of an explanation. The third conclusion, following from the case of complex numbers, is that although complex pure mathematics was developed within the mathematical realm, it turned out to be applicable in the physical realm. These last two conclusions puts us in the same struggle Wigner finds himself in: how can it be that mathematics, an invention of the human mind with no direct ties to the empirical world, is so appropriate to describe the empirical world? A detailed discussion of this question and what it means to claim that mathematics is unreasonably effective, is the topic of the next chapter.

Chapter 3

What is unreasonable about the effectiveness of mathematics?

In this chapter I will explain what is truly unreasonable about the effectiveness of mathematics by giving a few definitions of mathematics in the 20th century and by discussing Wigner's article and Steiner's elaboration on that. Finally, I will discuss four 'easy way out' solutions to Wigner's problem, for which I will show that they are either wrong or incomplete solutions to the problem of the applicability of mathematics.

3.1 Mathematics in the 20th century: the 'big three' and Wigner's puzzle

Many philosophers, physicists and mathematicians have their own definition of mathematics, and in past centuries, the consensus on what mathematics is and what falls in the domain of mathematics has changed quite a bit. In the beginning of the 20th century, three ideas about the nature of mathematics dominated the philosophy of mathematics; logicism, formalism and intuitionism (Shapiro, 2000). Logicism is the stance that all mathematics can be led back to logic and consequently to logical necessary truths. Intuitionism claims that all mathematical statements are constructs. Even natural numbers are mental constructions and more complex mathematics is just a more complex construction of the human mind. Finally, formalism is the position attributed to David Hilbert, that mathematics is a formal game that follows simple rules. Formalism is closely related to a linguistic view of mathematics and states that for instance natural numbers are merely symbols that we can manipulate. Complex mathematics is then a formal game that has no direct interpretation in the physical world.

All three schools give definitions of mathematics in which mathematics is detached from the physical realm. This resulted from the desire to secure the truth-value of pure mathematics by letting it reside in a realm of its own. From these three stances it becomes clear that although they differ in many ways they all agree on one thing: mathematics is a human activity with no immediate ties to the physical world.

Because of these schools it is less surprising that Eugene Wigner began to wonder about the applicability of mathematics. In the wake of the 'big three', Wigner himself takes a similar stance on mathematics:

> "I would say that mathematics is the science of skillful operations with concepts and rules invented just for this purpose. The principal emphasis is on the invention of concepts." (Wigner, 1960, p. 2)

Here, Wigner states that mathematics is an invention of the human mind. We construct and devise mathematical concepts and they turn out to be useful in natural science. My claim, that complex mathematics is detached from its empirical origins, is also defended by early 20th century philosophers of mathematics and Wigner. However, in claiming that mathematics is a human invention the trouble starts. When mathematics is nothing more than a manipulation of meaningless symbols, mere deductions from axioms and when it has nothing to do with knowledge or truth in the physical world, this leaves the practical applicability of mathematics inexplicable. We have seen an example of this in Chapter 2, where the development of complex numbers took place in a mathematical environment. Indeed, complex numbers are not suggested by our experience, on the contrary: it is a concept invented for the consistency of mathematical theorems and the solvability of negative roots in equations. Surprisingly, it turned out to be highly useful for Fresnel's theory of total reflection, and for many more descriptions of natural phenomena after that.

Wigner then concludes that it is difficult to avoid the impression that something strange is going on here. How can it be that the mathematics invented, influenced for example by convenience for the physicist or the sense of beauty of the mathematician, maps so well onto the description of natural phenomena?

One of the most important reactions to Wigner's question comes from Mark Steiner who published *The applicability of mathematics as a philosophical problem* 38 years after the publishing of Wigner's article. In the meantime no real progress was made in solving the mystery. Steiner also does not provide a solution but he does provide a framework from which we can work towards a solution. I will discuss his arguments and examples extensively in Chapter 4, but in short, Steiner places an emphasis on the unreasonableness that becomes apparent in the discovery of new natural phenomena as opposed to the description of natural phenomena. He claims that scientists have adopted anthropocentric methods by using mathematical analogies instead of physical analogies to state new physical laws. The verification of these laws by experiment confirmed the strange relation of mathematics and science. From Steiner, but also from all examples already mentioned and available in the literature, two general patterns can be posited how new physical laws are discovered, that show the unreasonable effectiveness of mathematics:

1. Scientists start from an already known physical phenomenon. They map the physical concepts with the help of our mathematical language onto mathematical concepts. Then, they let the mathematics speak for itself. The mathematical results are mapped back onto the physical universe, predicting a new physical concept. Often, it is only years later that the physical concept is indeed confirmed in experiment. An example of this pattern is the discovery of the positron by Dirac.
2. Scientists are stuck with a certain theoretical hypothesis and do not know how to formulate their new theory. It then turns out that there is a whole mathematical framework already developed by mathematicians in their 'ivory tower' that is a perfect fit with the physical hypotheses. Using this mathematical framework, all calculations are more simple and elegant and new predictions follow from the mathematically consistent theory. Experiments confirm the predicted physical phenomena. An example is the use of a new mathematical structure by Einstein in his theory of relativity.

I wanted to mention the second scheme separately, since it speaks to the imagination and it immediately becomes clear that there is something strange about the relation between mathematics and science. However, I consider it to be a subcategory of the first scheme. The first scheme, also called a mapping account, deals with the more general question why mathematics works *at all* to discover new physical phenomena. In the second scheme the emphasis is on the applicability of pure mathematics in the verification a hypothetical physical theory, the first scheme asks the more general question why any mathematics works in the description of physical phenomena. Although the second scheme is the most extreme case of the unreasonable effectiveness of mathematics, it is the first scheme which is the most general and that I will use in this thesis.

In the first scheme, the unreasonableness consists in the mapping itself but also in the manipulation of the mapped mathematical concepts. It is not at all clear why the manipulation and mapping that consist of following the rules of a game *we* invented, containing anthropocentric elements, result in the prediction of a new physical phenomenon that is later verified by experiment.

3.2 Four common explanations of Wigner's puzzle

Many simple solutions that are proposed to Wigner's puzzle are flawed: they are wrong or incomplete explanations of the applicability of mathematics that instinctively seem correct but do not solve the puzzle. Four of these solutions are listed below with my counterarguments.

First, Mac Lane (1990), among others, states that the usefulness of mathematics can be explained by adhering to its empirical origins. According to this solution, all mathematics ultimately follows from the empirical world, which makes it no mystery that mathematics is equipped to describe that same natural world. I showed in Chapter 2 that although it is the case that complex mathematics may have its origin in basic mathematics and the empirical world, it is influenced by far more than only its empirical origins. Pragmatic and contextual considerations have played a major role in the development of mathematics and many mathematicians acknowledge that for instance beauty plays an immensely important role in the development of mathematical theories, among them the mathematician G.H. Hardy:

> The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas like the colours or the words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics. (Hardy, 1940, p. 14)

Mathematics is influenced by more than only the structure of the empirical world which discredits the solution given above.

The second explanation concerns the claim that many more mathematics is invented than is used by physicists. The applicability of mathematics is then explained by the fact that the scientist just picks out the piece of mathematics that is useful to him and that contains useful structures, leaving aside all mathematics that he cannot find an application for. This is an incomplete explanation of the applicability of mathematics since it does not explain why mathematics in general (instead of some other human capacity) maps so well onto the natural world. It only explains how scientists choose between mathematical structures already developed by the mathematician - it does not explain why any one of those structures is apt to describe the natural world.

The third common explanation can be formulated in relation to the merge of mechanics and mathematics during the scientific revolution (Section 2.3). It is stated that much mathematics was invented just for the purpose of describing Nature. Physicists invented new mathematics to be able to describe certain structures found in their experiments. The claim is that the applicability of mathematics is reasonable because the mathematics used in the natural sciences is invented by the physicists themselves. Lützen (2011, p. 242), for example, states that *"the development of geometry and analysis has been shaped by physics from the beginning and all the way up till the twentieth century. [...] this fact makes the applicability of mathematics seem rather reasonable."* Indeed, in the case of mechanics, this is correct: much mathematics was shaped by the need to describe systems with trajectories through space and time with forces acting on them. However, I have already discussed one example in which this is not the case. The development of complex numbers was done in a purely mathematical environment. As already pointed out, Newton did not believe there was a physical application for complex numbers and Euler developed the theory of complex numbers without having in mind any application. It seems that this explanation, that mathematics is shaped by physics, is only valid for the specific case of the relation between mathematics and mechanics during the scientific revolution. In modern physics, much mathematics is used that was developed in a mathematical environment, of which the use of Hilbert spaces and complex functions in the quantum formalism are the most telling examples. Referring again to Hardy (p. 49), he is convinced that *"I have never done anything 'useful'. No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world."*. Besides the fact that he was wrong,[1] it seems that the explanation that the mathematics used in science was developed for the development of science is in general not true.

The fourth answer to Wigner's puzzle is of a metaphysical nature and concerns the Galilean idea that the world is laid out in mathematical terms. This idea is not new and can be led back to Pythagoras. The Pythagorean ideal is that mathematics just *is* the structure of reality, in this way merging metaphysics and physics (Hacking, 2011, p. 12). Wilson (2000) translated this to

[1] The Hardy-Ramanujan asymptotic formula is widely applied in atomic physics and the Hardy-Weinberg theorem has become standard in population genetics.

a modern conception of mathematics and declares the belief in this ideal ’lazy mathematical optimism’:

> [...] somewhere deep within mathematics’ big bag must lie a mathematical assemblage that is structurally isomorphic to that of the physical world before us, even if it turns out that we will never be able get our hands on that structure completely. [...] To believe this is to accept what I call "lazy mathematical optimism". (Wilson, 2000, p. 297)

He claims that this kind of optimism, that every structure in the natural world possesses a representative in the mathematical realm, is not based on fact but on desire. It is an ideal that can explain the applicability of mathematics, but that is also a mere speculation, bordering on theology. There is no need for Nature to exhibit the regularities of our mathematical structures. Scientists assume that a natural phenomenon can be mapped to a - often more simple - structure which is a representative of a mathematical structure. The fact that this works in many cases is however not a solution to the problem - it is constitutive to the problem. I will come back to this in Chapter 4 in which I will discuss Steiner who denotes this as the ’apparent user-friendliness’ of the universe.

In conclusion, I have shown in this chapter that the applicability of mathematics in the natural sciences is a deep philosophical problem, that should be approached with care. A scheme has been put forward that shows how scientists use mathematical concepts and we have seen how mathematics can be unreasonably effective. Steiner uses this scheme as well and projects it onto the developments in physics in the beginning of the 20th century. He claims that in the shift from classical to modern science an even greater change has taken place in the scientific method. Mathematics has been given an even more important role, which led Bochner (p. 47) to remark that mathematics has changed in the beginning of the 20th century from the "handmaiden" to the "dictatorial mistress" of science. These developments and Steiner’s arguments are central to the following chapter.

Chapter 4

Steiner's anthropocentric argument

In response to Wigner's puzzle, Steiner investigated the major developments in physics in the 20th century. He concludes that Wigner's premise that the usefulness of mathematics is unreasonable holds true as the anthropocentric elements in mathematics cannot be made intelligible. In this chapter I provide a discussion of Steiner's claims. His examples and arguments form an excellent starting point for a discussion about epistemological and metaphysical questions that arise when the applicability of mathematics is considered.

As opposed to Wigner, who asked both epistemological and metaphysical questions, Steiner makes a strict distinction and states that he is only interested in the epistemological questions. How mathematics is used in the methodologies of science and whether that is reasonable or not is of interest to him - what mathematics really is and what its relation is to the human mind and the natural world is not. As a metaphysical default position, he assumes the same position as Wigner, namely that mathematics is a human invention without immediate ties to the natural world.

4.1 Steiner's anthropocentric claim

Steiner places emphasis on the discovery of new physical laws, as opposed to the description of natural phenomena. He asks himself the question:

> How did physicists discover successful theories concerning objects remote from perception and from processes which could have participated in Natural Selection? (Steiner, 1998, pp. 52-53)

and immediately answers:

> My answer: by analogy. Having no choice, physicists attempted to frame theories "similar" to the ones they were supposed to replace. (Steiner, 1998, p. 52-53)

The first type of analogy tried was a physical analogy. Did the new phenomenon resemble a phenomenon already present in Nature? It became clear that physical analogies were no help in discovering new laws. This is shown in for example atomic physics, where it was just not the case that the behavior of an atom was analogous to the behavior of a macroscopic body. Physicists were therefore forced to rely on non-physical analogies, of which mathematical analogies turned out to be the most successful.

According to Steiner, two different types of mathematical analogies were used in the discoveries early in the 20th century: Pythagorean analogy and formalist analogy. A Pythagorean analogy is a mathematical analogy between physical laws, that cannot be translated to non mathematical language at some point in the analogy. These are therefore analogies between mathematical concepts that are not physically based. A formalist analogy is a subcategory of a Pythagorean analogy, and is merely concerned with the analogy in notation or language between physical theories. He makes the strong claims that 1) these mathematical analogies are anti-naturalist and 2) that modern physics would not be possible or would not have come this far without them.

These two claims are central to the rest of the book; where the first one is an implicit criticism on the philosophical ideology of naturalism, the second one is the main reason why mathematics is unreasonable effective. With regard to the first claim Steiner sees naturalism in opposition to anthropocentrism, which is the statement that human beings are central to things - privileged in a way. Naturalism entails the claim that the world around us is indifferent to the hopes and wishes of the human beings living on it, which makes it a value for science. Science is aimed at knowing the natural world without taking into account the subject, and naturalism fits perfectly into that ideal. But as Steiner himself points out, naturalism is not the central notion of his book:

> My topic is anthropocentrism, and my goal in this book is to show in what way scientists have - quite recently and quite successfully - adopted an anthropocentric point of view in applying mathematics. (Steiner, 1998, p. 55)

What does it mean though, to say that scientists have adopted an anthropocentric point of view in applying mathematics? It means that at the turn of the century, physical analogies turned out to be useless in discovering new laws of nature and physicists had no other choice then to adopt other strategies which were anthropocentric. The use of Pythagorean analogies is the most important example of this. So where Dijksterhuis showed that something crucially changed in the relation between mathematics and science when going from ancient to classical science, Steiner here claims that there was another major change when shifting from classical to modern science. The values of the scientific revolution were overthrown in a way, because anthropocentric methods were introduced in science as a replacement of naturalistic ideals. The use of mathematical analogies in which aesthetic considerations and convenience for the physicist play a role was the primary way in which anthropocentric strategies entered the scientific method. Steiner sums up his conclusions regarding both science and philosophy of science:

> In sum, on the basis of the evidence about to be presented, I would argue for a weak and a strong conclusion. The weak conclusion is that scientists have recently abandoned naturalist thinking in their desperate attempt to discover what looked like the undiscoverable. This is a conclusion about scientists, not about nature. The strong conclusion is about naturalism: the apparent success of Pythagorean and formalist methods is sufficiently impressive to create a significant challenge to naturalism itself. (Steiner, 1998, p. 75)

The challenge to naturalism is that it seems to be the case that nature looks 'user friendly' to human inquiry and that somehow using anthropocentric elements in scientific research is actually helping to find out what the natural world around us is really like.

The question that remains is whether it is really true that scientists adopted anthropocentric strategies such as Pythagorean analogies to discover new laws of nature and moreover, whether these methods were crucial in the discovery of those laws. Steiner is careful to note that it is not only mathematics that has led to these discoveries. Without valuable empirical data and prior modeling, these laws could never have been formulated. He argues, however, that it is the role of mathematics and more precisely, the role of anthropocentric elements in mathematics that was a crucial step in the development of new physical theories. Steiner gives evidence for his two claims by providing different examples. Two of those I will discuss in the following sections.

4.2 The mystery of quantization

As is widely known, the radical change from classical to quantum physics was initiated at the turn of the century with the realization that classical mechanics was not able to adequately describe atomic phenomena. Four breakthroughs in physics prepared the ground for this realization (Todorov, 2012). In 1900, Max Planck discovered the formula for the spectral density of black body radiation, in which he made use of quantized energy packets which he called the quantum of action. Four years later Albert Einstein discovered that those energy packets had a physical meaning and that light was quantized in those energy packets. In 1911 Ernest Rutherford proposed the planetary atomic model, describing the way electrons orbit the nucleus much like planets orbiting the sun. Last but not least, in 1923 Louis de Broglie predicted that not only light could be expressed as particles - it was also possible to describe particles as waves.

In light of these developments, a new conceptual system was needed that could deal with all those phenomena. A system was needed that could describe the state of an atomic particle at different times t. In other words, a differential equation similar to Newton's second law of motion was needed for particles whose energy was quantized and behaved nothing like classical particles. This equation would become Schrödinger's equation and is derived below to show how anthropocentric elements influenced the development of quantum theory.

In quantum mechanics, a system is described by a vector space. A physical state of that system is described as a unit vector in the vector space. Now the goal is to describe the movement of the unit vector through time and mathematically, this is equivalent to a unitary transformation U(t):

$$U(t) = e^{-\frac{i}{\hbar}Ht},$$

with initial condition U(0) = I and H the Hamiltonian that describes the system's energy. Combined with the initial condition of the particle $\Psi(0)$, a future state $\Psi(t)$ could be predicted:

$$\Psi(t) = e^{-\frac{i}{\hbar}Ht}\Psi(0),$$

which is a solution of the differential equation

$$i\hbar\frac{d\Psi}{dt} = -H\Psi(t). \tag{4.1}$$

So in order to find out what the state of the particle was, the only thing needed was to find out what H was. Schrödinger knew that energy should be quantized, so he relied on the classical equation of energy and 'quantized' the Hamiltonian. In classical mechanics, the energy of a particle is

Energy = Kinetic Energy + Potential Energy

with the kinetic energy $KE = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2)$ and the potential energy V(x) dependent on the environment of the particle. Schrödinger substituted the position, momentum and energy parameters in the classical equation for their quantized versions:

$$
\begin{aligned}
E &\rightarrow i\hbar\frac{\delta}{\delta t} \\
p_x &\rightarrow -i\hbar\frac{\delta}{\delta x} \\
p_y &\rightarrow -i\hbar\frac{\delta}{\delta y} \\
p_z &\rightarrow -i\hbar\frac{\delta}{\delta z}
\end{aligned}
$$

Because the energy is quantized, the position and momentum of the particle are also quantized and become operators. If we now substitute these back into the original equation and take for the Hamiltonian the quantized version of the classical Hamiltonian, we get the Schrödinger equation for one particle:

$$i\hbar\frac{\delta\Psi(x,y,z,t)}{\delta t} = [-\frac{\hbar^2}{2m}(\frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta y^2} + \frac{\delta^2}{\delta z^2} + V(x,y,z)]\Psi(x,y,z,t)$$

Schrödinger's equation was tested by predicting the energy levels of the hydrogen atom. The hydrogen atom was modeled classically, by assuming that the electron was a point particle rotating around the nucleus with the Coulomb attraction holding it in its orbit. The electron's potential was therefore modeled proportionate to $-\frac{e^2}{r}$ with r the distance from the nucleus. It turned out to work: the theoretical predictions matched the experimental data to a high degree of certainty. The next step was to try and find the energy levels of heavier atoms, starting with the helium atom. The same quantization procedure was used again, but now for a system of two electrons and here, the method worked as well.

Looking back, Schrödinger made three distinctively anthropocentric choices, based on a formal analogy, in discovering this equation. First, there is the decision to create the Schrödinger equation out of the classical equation. At the time, it was already known that position and momentum could never have a definite value at the same exact moment. Inserting in the quantum Hamiltonian an equation that had both position and momentum in it, as is the case when both kinetic and potential energy are present in the system, was physically speaking meaningless: it was a purely formal analogy that led Schrödinger to his equation.

The second decision that was strange was to represent the hydrogen atom as a particle with the electron orbiting the nucleus. Again, it had already become clear that this was probably not the right depiction of an atom, for instance because de Broglie showed that electrons were waves as well as particles. For lack of an alternative they tried it. The final decision that led to the discovery of the energy levels of the helium atom was to generalize the method for the energy levels of the hydrogen atom to the energy levels of the helium atom. The rationale for this step was that the success of quantization in the case of the hydrogen atom argues for the success in the case of the helium atom. However, the helium atom was a more complex structure, with two electrons instead of one. There was no reason to assume that quantization would work here. Steiner then sums up his findings as follows:

> My claim then, is: the lack of an algorithm to "quantize" classical systems makes the analogy between classical and quantum mechanics distinctly formalist. (Steiner, 1998, p. 153)

Steiner concludes that all three decisions are anthropocentric and formalist in character. However in all three decisions Schrödinger turned out to be right. As Wigner pointed out, they were crucial in the development and success of quantum mechanics:

> The mathematical formalism was too dear and unchangeable so that, had the miracle of helium not occurred, a true crisis would have arisen. (Wigner, 1960, p. 7)

This shows that by that time, scientists had so much trust in the formal rules of quantum mechanics that a disagreement with experiment would highly surprise everyone. This trust was based at least partially on the instinct that the rules of quantum mechanics were simple, its equations quite beautiful and the mathematics consistent; all anthropocentric arguments.

4.3 The prediction of the positron by Dirac

The second example from Steiner's account is that of the prediction of the positron by Dirac. It is another instance of the 'mystery of quantization' and a clear case of formal reasoning. Schrödinger's equation was an equation that was only capable of solving the trajectory and state of non-relativistic particles. However, as was proved by Einstein in 1905, particles behave differently when approaching the speed of light. Schrödinger therefore pursued the relativistic version of his equation and argued that the same procedure can be followed here as was done in the non-relativistic case. So where he used Hamilton's energy equation, $E = \frac{p^2}{2m}$ in the non-relativistic version, he quantized Einstein's energy-mass equation, $E^2 - p^2 = m^2$ in the relativistic case.[1] The result is known as the Klein-Gordon equation and is said to be derived by five different authors in the time-span of half a year (Steiner, 1998, footnote 27, p. 157): Starting from the general differential equation

$$i\hbar \frac{\delta \Psi}{\delta t} = E\Psi(t),$$

we substitute E with the mass-energy equation of Einstein. Since this is an equation that uses E^2 we square both sides of the equation:

$$-\hbar^2 \frac{\delta^2 \Psi}{\delta t^2} = E^2 \Psi(t),$$

$$-\hbar^2 \frac{\delta^2 \Psi}{\delta t^2} = (p^2 + m^2)\Psi(t),$$

After which the quantized version of the momentum was inserted:

$$\hbar^2 [\frac{\delta^2}{\delta t^2} - (\frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta y^2} + \frac{\delta^2}{\delta z^2}]\Psi(t) + m^2 \Psi(t) = 0,$$

However, it soon became clear that there was something wrong with it. For one, it had a second time derivative in it; which was physically strange because that means that more information than the initial state is needed to predict future states. Dirac then came into the picture and posited that there had to be an equation that was first order in both time and space (this follows from the introduction of a space-time continuum by Einstein in which space and time are symmetrical). But Dirac also believed in the quantization procedures that worked so well for the non-relativistic equations. So instead of searching for a different solution altogether, he proposed to factor the mass-energy relation in

[1] For the clarity of the derivation, the speed of light c is set to 1.

order to arrive at first-order solutions:

$$\hat{E}^2 - \hat{p}_x^2 - \hat{p}_y^2 - \hat{p}_z^2 - m^2 = 0$$
$$(\hat{E} + \alpha_1 \hat{p}_x + \alpha_2 \hat{p}_y + \alpha_3 \hat{p}_z + \alpha_4 m)(\hat{E} - \alpha_1 \hat{p}_x - \alpha_2 \hat{p}_y - \alpha_3 \hat{p}_z - \alpha_4 m) = 0$$

The only way that this factorization is possible, is when the following relations hold:

$$\alpha_1^2 = \alpha_2^2 = \alpha_3^2 = \alpha_4^2 = 1$$
$$\alpha_k \alpha_l = -\alpha_l \alpha_k \ \ (k \neq l)$$

As can be seen quickly, there are no numbers that satisfy these relations, so it seems that Dirac was stuck. However, still a firm believer in the formalism, he went ahead and posited four 4x4 matrices that did satisfy the equation. The solution Ψ of what is now called the Dirac equation therefore consisted of four components: a spinor with two positive energy solutions and two negative energy solutions. Dirac then went even one step further and posited that these must represent an electron with spin 'up' and spin 'down' and a new particle with a negative energy with its states spin 'up' and spin 'down'. He called these new particles positrons and stated that they must belong to a class of 'anti-matter'. 4 Years later in 1932, the existence of the positron was proven experimentally by Carl Anderson. The description of the spin states of the electron and the positron also turned out to be the right description. It was the start of the successful field of particle physics.

The discovery of the positron is a clear example of the use of formal analogies and anthropocentric decisions playing a crucial role in discovering Nature's workings. There is first the decision to have trust in Schrödinger's formal analogy that classical equations can be quantized. Then there is the decision that the relativistic equation should have the same general mathematical form as the non-relativistic equation. But the most astonishing part of Dirac's derivation comes from placing his belief in the formal analogy regarding the spinor. He posited that the spinor resulting from the factorization of the Klein-Gordon equation had to be a physically existing state and that all four elements describe existing states of particles. It was Dirac's belief in the equation and in formal reasoning that led to his discovery. Steiner sees this as no less than a miracle. There was no reason to believe that positrons should exist, no experimental data was ever found that particles with negative energy solutions could exist.

According to Steiner, this was also the start of the new meaning of the term 'prediction'. From then on, prediction meant the assumption that when something is mathematically possible, it was presumed to be physically real as well. This rule has been adopted by most physicists and mathematicians later on, and has turned out to be true in 'an uncanny number of cases'. Dirac's discovery of the positron is the most uncanny example of the predictive power of mathematics and it is at the heart of Wigner's trouble with mathematics.

4.4 Two criticisms of Steiner

Steiner's thesis rests on two assumptions. First, that mathematics has an anthropocentric character and second, that the anthropocentrism plays an essential role in the development of the physical theories from the 20th century. He renders this anthropocentrism unreasonably effective. Because he defined naturalism as the antonym of anthropocentrism, he declares that one cannot be a naturalist whilst believing in the scientific methods of the 20th century.

Two questions can be asked that could put Steiner's claim on shaky grounds: does mathematics indeed have an anthropocentric character and if yes, is the anthropocentric element in science truly unreasonably effective in the discovery of new physical phenomena? Since I feel Steiner's examples sufficiently show the anthropocentric character of the formal reasoning in complex mathematics, it is the second question that I will address below. The question then becomes, whether the anthropocentric element in mathematics is really unreasonably effective or can be made intelligible. Steiner claims that the anthropocentrism in mathematics alone is enough to conclude that the predictions done by science are unreasonably effective. I think that he jumps to that conclusion too soon, without any arguments. In my view, solving the mystery is explaining this anthropocentrism and showing that it is an intelligible part of science, not an unreasonable part. I will turn to this issue in Chapter 5, by discussing mapping accounts of scientific research to find out what the role is of the anthropocentric elements in mathematics.

There is another component of Steiner's argument that does not add up. From the beginning of his book, he is clear about the fact that he is concerned with the epistemological problem of the applicability of mathematics in the natural sciences, not with metaphysical questions. He concludes however, at the end of his book, that the universe looks 'user-friendly'. This is expressed by Bangu (2006) as followes:

> According to Steiner's version of anthropocentrism, humans hold

> a special place in the Universe in the sense that one of the central products of the human mind somehow 'tracks down' the deep nomic/structural features of the physical world. (Bangu, 2006, p. 34)

Steiner's claims therefore look an awful lot like metaphysical claims, while he was so careful to point out that he did not want to go into metaphysics. He concludes that the mathematics is used unreasonably in the natural sciences and that it seems to be the case that there is a connection between the human mind and the structural features of the world, which is a metaphysical claim. But he is not willing to make the next step and try to explain how mathematics is related to on the one hand the human mind, and on the other hand the natural world. This is one of the criticism of Simons (2001) as well:

> Despite Steiner's attempt to cast off the metaphysical issues and focus only on epistemology, even if the set-theoretic Platonism he quickly adopts were unproblematic- which it is not - the metaphysics behind the epistemology comes back to bite. (Simons, 2001, p. 184)

Indeed, without wanting it, Steiner has ended up at metaphysics, without willing to acknowledge that himself. As we will see in the next chapter, in an attempt to render the anthropocentric element in science intelligible, I too end up with metaphysical questions.

I conclude for now that Steiner has proved the presence of anthropocentric elements in mathematics that play an important role in the development of physical theories. I do not endorse his conclusion that therefore naturalism can no longer be defended. I will attempt to explain how, though anthropocentric elements influence science, we can still end up with knowledge about the world that can be trusted. This will be the main goal of the next chapter.

Chapter 5

How mapping accounts solve part of the mystery

In this chapter I will follow Steiner's lead and will avoid asking metaphysical questions. I will assume the metaphysical position that mathematics is a human invention. The question that I will answer here is whether the anthropocentric elements in mathematics can be rendered intelligible in the scientific method. To arrive at an answer to this question, I will look at different mapping accounts that explain the relation between mathematics and science. A mapping account is defined here as the process of a mapping between a physical concept and a mathematical concept. The way that this is done, by using abstraction, idealization and representation is called a mapping account.

5.1 Pincock's mapping account

The first mapping account that explains the applicability of mathematics in science is put forward by Pincock (2004). By looking at the methodology of science he wants to find the connection between the physical world and mathematics. He observes that there is always some kind of mapping present from the empirical world to a mathematical structure. This becomes clear when a statement like 'five apples are on the table' is uttered. This statement is a mixed statement that contains mathematical concepts (the number five) as well as physical concepts (apples and a table). The truth of this statement depends on the kind of mapping that is used to equate one apple with a segment on the natural number line. This is of course a very simple example but Pincock claims that the same thing applies for more difficult mappings. His position is accurately summarized by Bueno and Colyvan (2011):

> According to this view of mathematical applications - the "mapping account", as Christopher Pincock has called it - the existence of an appropriate mapping from a mathematical structure to a physical structure is sufficient to fully explain the particular application of the mathematical structure in question. (Bueno & Colyvan, 2011, p. 346)

This is all that Pincock's mapping account entails. If there is an accurate mapping between a mathematical structure and a physical structure, and according to him there always is, it explains why mathematics is so useful in the natural sciences.

However in my opinion, Pincock's mapping account is not equipped to explain the usefulness of mathematical structures in the natural sciences at all. To illustrate this, we can compare his mapping account with a mapping of a city onto a city map (Bueno & Colyvan, 2011). The city map represents the most important structural features of the city such as relative distances and north-south structures. Two questions crop up from this example: first, does the city map represent the city in a faithful manner (does it not forget some streets or are the angles between streets represented well?) and is not some structure from the actual city lost when we simplify the city map further after the initial mapping (by placing neighborhoods closer to the center to fit them on the map, or by adding colors for aesthetic reasons?)?

According to Pincock, the mapping of a physical structure onto a mathematical structure is of the kind exemplified above and he claims that this sufficiently explains the applicability of mathematics because it preserves the structure of the physical world. In his mapping account though, he forgets to answer the two important questions that also arose in the example. Is the mapping itself a good representation and is the structure of the physical situation preserved during the manipulation of the mapped mathematical structure? Moreover, he does not answer Steiner's question. Steiner wondered why it is the case that in particular the formal reasoning with its anthropocentric elements seems to preserve the structure of the physical world. Pincock stays silent about this.

The difficulty can be further exemplified with Dirac's prediction of the positron. The mapping here amounts to the quantization of the classical relativistic equations of motion. This mapping is already problematic since there was no conclusive evidence that this was the correct structure of quantum mechanical particles. Furthermore, Dirac mathematically derived that the solution of the equations should be a four-vector. At that point, there was no reason to suppose this four-vector (spinor) had a physical meaning and that his mathematical conclusions still represented physical structures. He assumed this anyway, for mathematical reasons, and turned out to be right. It seems that the mathematical formulation captured more structure than the physical phenomenon. Pincock's mapping account can describe the process but it cannot explain why this method works.

5.2 The inferential conception of the application of mathematics

Bueno and Colyvan (2011) therefore propose an alternative view of the role of mathematics: an inferential conception. They claim that the goal of using mathematics is to find inferential relations between empirical findings and mathematical structures. They construct a mapping account as can be seen in Fig. 5.1. They start from the empirical set-up or empirical situation. The immersion step then maps the empirical set-up onto a mathematical structure. The derivation step is the mathematical derivation of the mapped mathematical structure. Here, mathematical conclusions from the mathematical formalism are generated. The last step is the interpretation step, where the mathematical conclusions are mapped back and interpreted to fit the empirical world. Until now, it seems that this is just a more thorough version of Pincock's mapping account. However, Bueno and Colyvan (2011) acknowledge unlike Pincock, that in the immersion and interpretation step contextual and pragmatic considerations play a role. These are the considerations that Steiner called anthropocentric. They account for those considerations by adhering to the inferential structure of their mapping account. Via trial and error methods the scientist can infer the right description of Nature from a mathematical structure that started as a mere representation. They exemplify their argument with Dirac's equation (Bueno & Colyvan, 2011, p. 364).

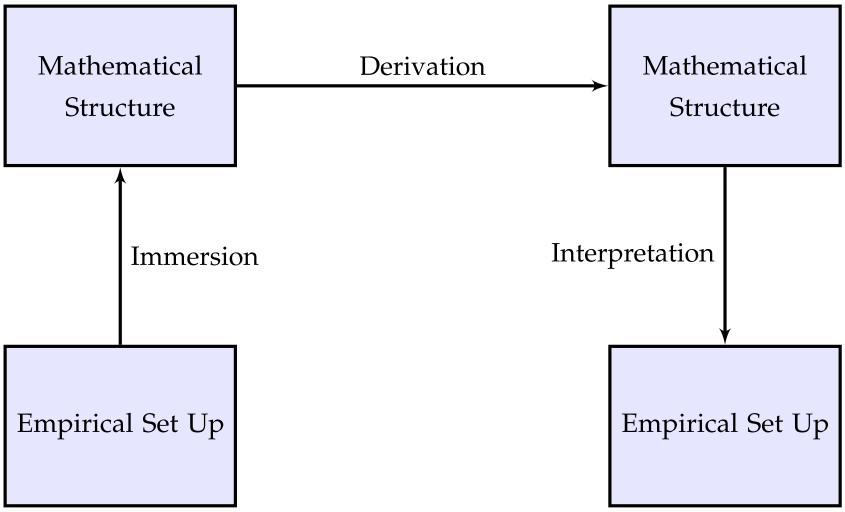

Figure 5.1: Schematic image of the inferential mapping account

Dirac, after finding his mathematically consistent equation, devised three different physical interpretations of the mathematical solutions. First, he denied the negative energy solutions and took them to be non-physical, later he accepted all solutions and proposed that the negative energy solutions were "holes" in a sea of electrons, and finally, he interpreted the negative energy solutions as a new particle, the positron. All these interpretations have a clear anthropocentric element, since they are based on pragmatic and contextual considerations. Dirac's choice for the first interpretation was pragmatic; there was no record of negative energies so he assumed that they didn't exist. His second interpretation was motivated, and later also rejected, by contextual considerations and empirical inadequacy. He inferred therefore, from all former attempts, the third interpretation that was both a correct mapping from the mathematical structure and empirically adequate. This is how the use of mathematical models and methods can result in a correct description of a physical phenomenon. It is the process of finding inferential relations between the empirical set-up and the mathematical structures that accounts for the usefulness of mathematics in the description of physical phenomena.

I think that Bueno and Colyvan here partly solve Steiner's troubles and at the same time point to a weak spot in his considerations. Steiner does not take into account the empirical component of scientific research and does not attempt to establish a relation between the mathematical models used and the experiments performed. The inferential mapping account does this, relieving mathematics from some of its mysteriousness. The anthropocentric elements that Steiner is talking about turn up in the immersion and interpretation step, and the inferential account is capable of making these influences intelligible in the scientific method.

However, there are two problems with the inferential mapping account. The first is that the relation between experiments and the conclusions from mathematical models are not connected in a satisfying way. It remains unclear what the role of experiments is and how mathematical models might be influenced by experiments and vice versa. The second problem is that they do not take into account another step that takes place in the scientific practice: this is what Batterman (2009) and Narens (1990) call the idealization of empirical data. I will show that although they provide a valid criticism of the inferential account, the notion of idealization is too narrow. Considering Dirac's case, the notion of idealization does not cover the method used and I propose that scientists instead impose tractability assumptions, a term borrowed from Hindriks (2006), and that these will turn out to be a crucial and unreasonably effective step in the scientific method.

5.3 Improving the inferential mapping account

According to Batterman (2009), both Pincock's account and the inferential mapping account do not deal with the most difficult aspect of the relation between mathematical structures and physical phenomena: idealizations. As every scientist knows, it is necessary to idealize an empirical situation to be able to do research. For example when we want to find the pressure in a tea kettle the number of molecules is set at infinity. When a scientist uses an idealization, he knows that this is not the real situation and is representing the phenomenon wrongly - however approaching the actual situation. It seems then, that according to Batterman, this needs to be added to the scheme of Bueno & Colyvan since scientists make extensive use of idealizations.

The best known example of an idealization as used in physics is the assumption that an electron is a point particle. When the physicist is concerned with a system of electrons, it is only after this idealization that the second step is taken and the idealized empirical situation is mapped to a mathematical structure (in the case of the electron, we describe the point particle as a Dirac delta function). From then on we can follow the inferential account: the mathematical conclusions are derived from the mathematical model and through inferential methods, we map the mathematical conclusions back to the empirical situation.

Narens (1990) also acknowledges this idealization step but states, unlike

Batterman, that there is a problem here that has remained largely unnoticed:

> [...] actual empirical situations are usually conceptualized as structures on large finite sets. Because of the complexity of such structures and the irregularities and non-homogeneity often necessarily inherent in them, the actual empirical situation is idealized to an infinite "empirical" situation, where the irregularities and non-homogeneities disappear; that is, the actual situation is idealized to a more mathematically tractable structure. A well reasoned account of the conditions under which such idealizations are acceptable is a major unresolved problem in the philosophy of science. (Narens, 1990, p. 134)

What Narens claims is that there are no rules available how to idealize physical 'real world' situations. Still, we seem to achieve the impossible: in pursuing idealizations that have a mathematical structure in them we stumble upon the truth about Nature.

I agree with Narens here but I wish to adjust the notion of idealization somewhat, since it is too narrow for our purposes. Looking at the examples used before of Boyle's law, Newton's law of gravitation and Schrödinger's equation, they don't seem to fall in the same category. Boyle's law is a typical example of an idealization, in which molecules are idealized as perfect spheres with negligible size and without exerting forces on each other. The results of Boyle's law then, are only approximate results. Physicists know that they have to correct for all these influences that they 'idealized away'. But the two other cases are not idealizations in the same sense. Was Schrödinger's decision to treat the quantum system as a quantized classical system an idealization? Was Newton's use of a numerical coincidence to arrive at the law of gravitation an idealization? The predictions that followed from the Schrödinger equation, the energy levels of Helium, turned out to be correct and the law of gravitation has an uncertainty of less than a ten thousandth of a percent. But putting aside the astounding accurate results of these theories, the notion of idealization does not seem to describe the step that Newton and Schrödinger take. What they are really doing is imposing certain assumptions to make the empirical situation tractable for their mathematical methods. I call these assumptions tractability assumptions, a term borrowed from Hindriks (2006).

Hindriks (2006) uses this class of assumptions in relation to economics, where the empirical situation often contains many variables and unknowns so that modeling the situation is very difficult. Economists then use tractability assumptions to obtain a model of the situation which can be described by the mathematics available to them. The difference with an idealization is that these tractability assumptions could be correct or false, whereas idealizations are assumptions for which you know that they are false or incomplete in the real life situation. Note also that an idealization is a subclass of a tractability assumption. Picturing the electron as a point particle, an idealization, is also a tractability assumption. Assuming that the Schrödinger equation has the same differential form as the classical equation of motion is a tractability assumption but not an idealization. A tractability assumption, then, is an assumption that makes it easier to handle the problem mathematically and according to Hindriks,

> A large number of considerations fall under the heading 'tractability'. In the case of theoretical tractability, relevant considerations include the level of sophistication of the mathematics available at the time, the cognitive capacities of scientists or students, more especially the puzzle-solving capacities of scientists, and 'auxiliary' theories in neighboring fields of inquiry. (Hindriks, 2006, p. 414)

In the case of natural science the tractability assumptions are usually imposed to make the problem *mathematically* tractable. Scientists search for ways to employ the mathematical toolbox at their disposal and by imposing tractability assumptions, they are able to use this toolbox.

Therefore, in the light of the criticism given above - that Bueno and Colyvan (2011) fail to accurately describe the unreasonable role of tractability assumptions and to incorporate the role of experiments in their mapping account, I propose a more complete mapping account. This mapping account can be seen in Fig. 5.2 and incorporates both the role of experiments and the tractability assumptions. I will explain the different steps with the help of the already much quoted example from Dirac.

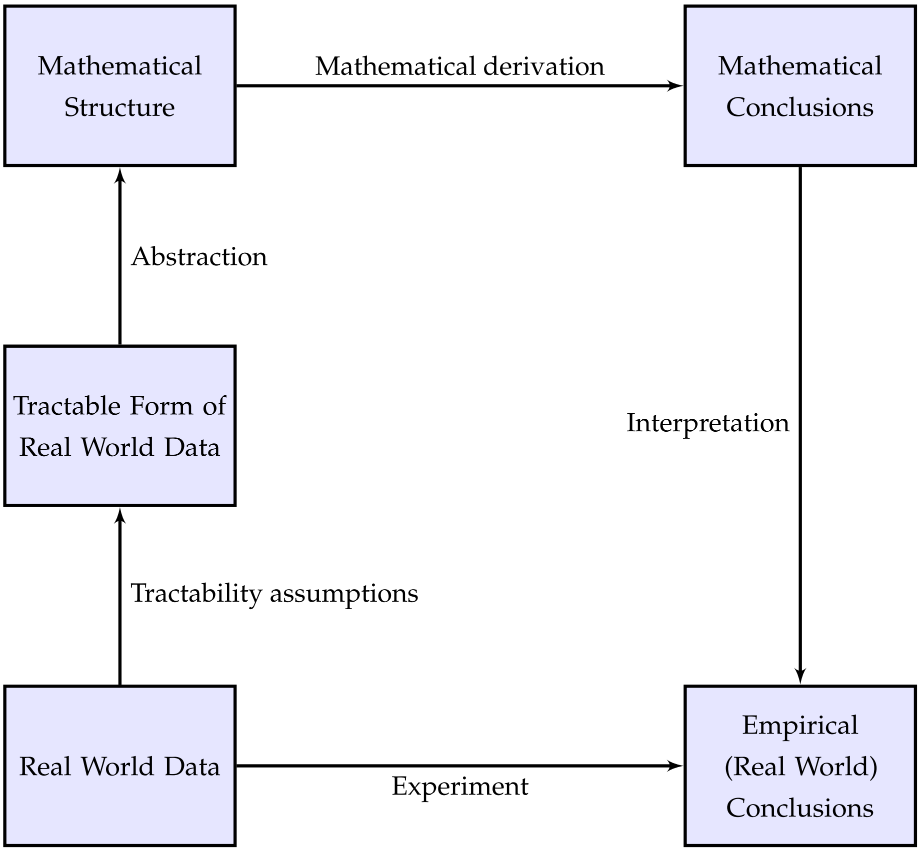

Figure 5.2: Improved mapping account

We start from the empirical 'real world' situation with empirical data. In Dirac's case, this is the situation that the Schrödinger equation is empirically verified for non-relativistic particles and the fact that relativistic particles will behave in a different way - they obey Einstein's mass-energy relation. Dirac's goal is to find the relativistic version of the Schrödinger equation. The first step is to make this situation mathematically tractable by imposing tractability assumptions. In Dirac's case, this corresponds to his assumption that the quantization procedure used in non-relativistic quantum mechanics is also valid for relativistic mechanics. He needs this assumption in order to approach the problem mathematically. Note that Dirac's assumption is not an idealization. He does not simplify the problem by simplifying physical objects into geometrical objects or by eliminating influences from the environment. He guesses the form of the solution (by mathematical analogy) to make the problem mathematically tractable.

The actual mapping (the abstraction step in Fig. 5.2) of the more tractable situation onto a mathematical structure is the next step, in which the quantized version of the energy-mass equation is inserted in the general differential equation for the movement of a particle. From then on, this account follows largely the inferential mapping account; mathematical conclusions are drawn from the mathematical derivation and the conclusions are interpreted by using the inferential relation between the experiments performed and the mathematical conclusions. Crucial in this inference is the experiments that are done on the original empirical 'real world' situation (which is illustrated by the arrow 'experiment' in Fig. 5.2). It is only in this combination that the scientist can arrive at the correct conclusion from his mathematical models. As in the inferential account, I acknowledge that anthropocentric influences play a role in the mathematical abstraction, derivation and interpretation step, but that these can be rendered intelligible by the inferential relation between mathematical models and the experimental results of real world data.

It now becomes clear that the truly mysterious step in the scientific method is the successful application of tractability assumptions. Dirac uses an assumption that is a mere guess and turns out to be correct. But why does he choose this particular assumption? Why does making an empirical situation more mathematically tractable, an anthropocentric process since mathematics is a human invention, results in true knowledge about Nature? Why do the results of this method correspond so well with the results of experiments?

One answer that could be given is that the inferential account is also applicable to this first step. I argue, however, that the assumption imposed on the initial empirical situation is too important a step to be able to be corrected by inferences. Batterman (2009) explains this, though using the notion of idealization, by using an analogy to perturbation series:

> A very helpful, and often quite accurate, way of understanding this mathematically is in terms of regular (analytic/nonsingular) perturbation series: The idealization is the first-order term in the series and we improve upon that by adding correction terms in powers of the relevant parameter. (Batterman, 2009, p. 17)

The first step, imposing tractability assumptions, is an important step that is governed by anthropocentric elements and an inferential conception that uses trial and error methods is not able to eliminate those influences. Wigner's puzzle is still not solved, because what is unreasonable here, is that our assumptions, based on the mathematics we invented and the conceptual capabilities of our brains, seem to be exactly the right ones. Why does it work, when we picture an electron as a point particle or when we assume that the Schrödinger equation has the same form as a classical equation? It becomes clear that the anthropocentric elements that Steiner is talking about also arise in this step; we simplify empirical situations in such a way that the situation becomes tractable *for us*; with our limited brains. Why then do we stumble upon the right theories every time?

Although this more extensive mapping account does not explain the applicability of mathematics, I have made clear where anthropocentric influences enter the scientific method and where these are intelligible and where not. It becomes clear that it is the first step, of imposing tractability assumptions, that is truly unreasonable. It does not make sense that these assumptions are always the correct ones. We seem to be unreasonably accurate in guessing the structures that underly complex physical phenomena. The world looks 'user-friendly', as Steiner phrases it, where there is no reason that this should be the case.

5.4 Why does mathematics only work in the natural sciences?

In the former section, I have borrowed some terminology from economics. Nevertheless there is an astounding difference with the use of tractability assumptions in economics and natural sciences. Economists also employ tractability assumptions, knowing that it will influence their results and will no longer represent the actual situation. They know that their assumptions are usually wrong, and that this will compromise their results and the level up to which they are applicable to real world problems. In the natural sciences, the tractability assumptions of the scientist turn out to be the correct assumptions in 'an uncanny number of cases'. Economists end up with a theory of which they know is only an approximation. Physicists end up with predictions that turn out to be right up to 13 decimals. What is going on here?

This comparison leads to the question why mathematics only works so well in the natural sciences. Compare for example the natural sciences with the social sciences. In social science, many structures are found (general behavior of humans, action and reaction structures, etc) and many experiments are done. So why does the mapping account as I have outlined it not work in the social sciences? This question is both relevant and not easy to answer. There is no reason to suppose that the social sciences cannot map certain structures onto mathematical structures and draw conclusions from those. Moreover, enough experiments are done to be able to check ones mathematical conclusions and use inferential relations. Yet, the field of sociology eschews the use of mathematics: Sociology is characterized by "schools of thought" instead of mathematically formulated laws (Doreian, 1990).

I suggest, with caution, that the difference between the natural sciences and economics and social sciences is twofold. On the one hand natural phenomena have a more structured character than social phenomena. They are less volatile. Social phenomena also exhibit structural features, but in a less clear sense. This makes it harder to apply a mapping. On the other hand the tractability assumptions imposed by physicists are of the same type as are used in the social sciences. It seems that physicists are unreasonably good at guessing the structure of Nature, whereas sociologists seem to be very bad at guessing the structure of social phenomena. So it is in this step that a difference is made between the natural sciences and social sciences. Only the natural world seems 'user-friendly' to our human mathematics.

In conclusion, my improved mapping account only partially solves Steiner's problems. It solves how anthropocentric influences can be made intelligible by adhering to the inferential relation between experiment and mathematical models. However, it fails to explain the first step in which the empirical situation is made mathematically tractable and where considerations such as simplicity, the mathematics available and convenience for the scientist play an important role. This part of Steiner's problem is therefore not solved since these conditions are anthropocentric - we want a mathematically tractable structure of the chaotic natural phenomenon in order to understand it with our limited human brains. Why this works in an uncanny number of cases, why the mathematical conclusions from these assumptions are verified by experiment so often, remains a mystery.

It seems that I am stuck. The question why Nature is user-friendly cannot be explained by closely reviewing the scientific method and how mathematics enters science. So maybe, I am approaching this from the wrong perspective. Until now, I have been able, just as Steiner did, to avoid a metaphysical account of mathematics. However, at the end of Chapter 4 I already commented on Steiner's conclusions that were metaphysical in nature. He also seemed to end up at metaphysics. With the problem that I have left now, I too have to conclude that a metaphysical discussion about the nature of mathematics is needed, in order to understand the relation between mathematics, the human mind and the natural world. Questions as why the natural world looks user-friendly, and why our use of tractability assumptions result in a mathematical model that describes the natural world so accurately, results in the feeling that maybe, my (and for that matter Wigner's and Steiner's) conception of mathematics and its relation to the human mind on the one hand and the natural world on the other is wrong. A complete account of all metaphysical issues is not pursued, but a head-start will be given in the next chapter.

Chapter 6

Metaphysical considerations

Complex mathematics is by and large a human invention and completely detached from the physical realm. This was the conclusion of Chapter 2. However, I have not succeeded in explaining the applicability of mathematics by using this metaphysical default position. Therefore, I have to consider the possibility that the metaphysical position taken by Wigner, Steiner and myself is the wrong position. This means asking more fundamental questions, such as what mathematics really is and what position it has in the world. The most important metaphysical question that needs to be answered is the following: what is the relation between mathematics, the human mind and the natural world? It is in these relations that we have to find an answer to our main question.

The first account discussed in this chapter is the well-known stance of mathematical Platonism, claiming that mathematics and its objects are a part of Nature. Human beings are in this account mere observers who discover these mathematical objects and relations between them. The second solution is a Kantian account, with the claim that mathematics is constituted by the human mind and body; any fit between mathematics and the world is mediated by the human mind and restricted by its capabilities. The third explanation states that mathematics and Nature are not related at all. We simply find what we look for. The final solution is a solution from contemporary physics and is based on the multiple-worlds hypothesis. This hypothesis claims that there are a myriad of universes out there of which we are only one, that happens to have all the right natural constants to provide enough structure to our world. We are not unique in that we can understand the world with our brilliant minds - mathematical comprehensibility is a necessity for a world on which life is possible.

Although these four approaches are not meant as an exhaustive list of all metaphysical positions concerning the relation between mathematics, mind and world, it does show this difficult relation and the possibilities and threats of every position you take.

6.1 Mathematical Platonism

As mentioned in Chapter 2, Wigner's puzzle leads to the suggestion that we should consider the possibility that mathematics is already present in Nature. Subsequently, the conclusion may be drawn that mathematical objects are real, existing objects. This position is called mathematical Platonism and is the realist stance that mathematical objects exist outside our mind.[1] The position can be led back to Frege, who claims that the objects over which we quantify in scientific theories must be existing objects if the scientific theory turns out to be correct (Shapiro, 2000). The argument that naturally followed from Frege's position is the indispensability argument put forward by Putnam and Quine in the 1980's. The argument claims that we should believe in entities that are indispensable to our best scientific theories and therefore, if mathematical entities are indeed indispensable to our best scientific theories, we should believe in their existence. An body of literature is build up in recent years, wherein attempts are made to prove the indispensability of mathematical objects in the explanation of natural phenomena.[2] By proving this, they claim to have proven the existence of mathematical objects, which in turn will solve the applicability of mathematics in the natural sciences. Science is aimed at finding the structure of Nature and if Nature contains mathematical objects and structures, it is only logical that the explanation of natural phenomena is done in mathematical terms.

Yet Benacerraf claims there is an epistemological problem for the Platonist. He says that there is a fundamental difference between the causal structure of our world and the acausal structure of the mathematical realm. Because of this, it is not at all clear how we can equate the truth of theories that make use of mathematical objects with the existence of mathematical objects. In his book *Thinking about mathematics*, Steven Shapiro describes the problem clearly:

> [...] the ontological realist is left with a deep epistemic mystery. If mathematical objects are part of a detached, eternal, acausal mathematical realm, how is it possible for humans to gain knowledge of them? [...] How can we know anything about the supposedly detached mathematical universe? (Shapiro, 2000, p. 28)

[1] for brevity I will call this position simply Platonism from now on.

[2] See for example Daly and Langford (2009), Rizza (2011), Saatsi (2011) and Batterman (2009).

So the challenge for the ontological realist, or Platonist, is to explain how humans, physical beings living in a physical world, can have knowledge about this completely different realm. How can mathematical objects, acausal in principle, have caused something in the physical world which humans picked up and became knowledgeable about? This remains a puzzle for the Platonist.

6.2 Embodied mathematics: A Kantian approach

In Platonism mathematics is present in Nature and the human mind is a mere spectator that discovers mathematical structures. A completely different ontological position is a Kantian approach, in which mathematics is constituted by the human mind. Kant himself was convinced that mathematics was a 'product of reason' but then asked himself the question, in 1783, how 'pure' mathematics was possible:

> Here is a great and established branch of knowledge, encompassing even now a wonderfully large domain and promising an unlimited extension in the future. Yet it carries with it thoroughly apodictical certainty, i.e., absolute necessity, which therefore rests upon no empirical grounds. Consequently it is a pure product of reason, and moreover is thoroughly synthetical. [Here the question arises:] "How then is it possible for human reason to produce a cognition of this nature entirely a priori? (Kant, 1902, pp. 16-17)

Mathematics exemplifies throughout a large part of his work his famous synthetic a priori principle. According to Kant it is impossible to have knowledge about the world 'as it really is' because our brains impose certain structures on the way we perceive the world. For example any description of the natural world has to be done in time and space and the concepts of time and space are constituted by our synthetic a priori propositions of respectively algebra and geometry. In this way, Kant claims that the objective reality is partially constructed by the way our brains are wired, and mathematical concepts play an important role in this. Although Kant's philosophy of mathematics has been discredited, it remains an interesting position in thinking about the applicability of mathematics. A modern Kantian conception of the relation between the human brain and mathematical knowledge is given by Lakoff and Núñez (2000). From their expertise as cognitive scientists they launched the discipline of mathematical idea analysis from a cognitive perspective.

Their goal is to explain human mathematical ideas with the help of cognitive science and as a byproduct, solve the mystery of the applicability of mathematics. They start their theory with the argument that mathematics is constituted by the human mind and body:

> Human mathematics, the only kind of mathematics that human beings know, cannot be a subspecies of an abstract, transcendent mathematics. Instead, it appears that mathematics as we know it arises from the nature of our brains and our embodied experience. (Lakoff & Núñez, 2000, p.xvi)

They claim that mathematics is a product of the way our brains are wired and base this claim on three insights in recent cognitive research. The first insight is what they call the embodiment of mind. This is the non-dualistic insight that our minds and ideas are constituted by the detailed nature of our bodies. The second insight is that of metaphorical thought. A human being makes extensive use of the conceptualization of abstract concepts in concrete terms. An example of a mathematical metaphorical conceptualization is that we conceptualize numbers as points on a line (Lakoff & Núñez, 2000, p. 5). The third insight is that human beings have the ability to subitize (distinguish between different objects or 'count' a small number of objects). However, that is all the mathematical intuition humans are born with.

How can it be, given this last insight, that we are capable of such complex mathematics that is moreover also applicable in the description of Nature? Lakoff and Núñez (2000) claim that any fit between mathematics and the world is mediated by the human mind and restricted by its capabilities. The fit does not occur in the natural world but is only present in the mind of the scientist that cognizes both the world and mathematics. The mathematics he uses is rooted in experiences had by mathematicians before him and by cultural and historical influences. So what they claim is that there is no relation between mathematics and the world; there is only a relation between mathematics and the world as we perceive it. In describing that world mathematically, therefore, we are not representing the world as it really is - we are representing it as we perceive it.

This leads to a few interesting questions and criticisms. First, following a criticism of Dorato (2005), the fact that mathematics has its origin in our brains and in the experiences of all other human beings before us does not explain the fact that mathematics is also applicable when we are dealing with scales that are significantly smaller or larger than our own 'scale': *"Why should evolution have equipped us with the laws of objects that, like atoms, play no role in our ordinary* *life"* (Dorato, 2005, p. 137)? We would need analogies again, which are in turn anthropocentric, to extend the mathematics created through our bodily experiences to for example the quantum world.

Secondly, why is it, of all human capabilities, per se mathematics that does the job? Why is musical intelligence or artistic intelligence for example not able to fulfill the role of explaining natural phenomena? If the inborn capacity of humans for mathematics is only the ability to subitize, why is it the case that only this capacity is so thoroughly developed that we can understand the world around us with it?

Finally, their account does not explain fully why the outcome of experiments in fundamental research, though devised by human beings, leads to such incredible applications that all work. If we only have knowledge about the world as we perceive it, but not about the world as it really is, why do planes fly?

6.3 Do we just see what we look for?

In considering the position discussed in the previous section, another metaphysical position emerges. This is the very simple stance that we just 'see what we look for'. When the glasses that we put on are mathematical, we will find mathematical structure in what we see in the same way that we will see the world purple when we put on purple glasses. The difference with the previous account is that true knowledge about the world as it is, is not per se excluded. We 'see' Nature through the mathematical glasses we have put on, but that does not necessarily mean that we see Nature wrongly - we just only see that part of Nature that our human mathematics can deal with. Examples that support this claim are provided by Hamming (1980), of which the most remarkable is provided below.

> As another example of what has often been thought to be a physical discovery but which turns out to have been put in there by ourselves, I turn to the well-known fact that the distribution of physical constants is not uniform; rather the probability of a random physical constant having a leading digit of 1,2 or 3 is approximately 60%, and of course the leading digits of 5,6,7,8 and 9 occur in total only about 40 % of the time. (Hamming, 1980, p. 88)

This is called Benford's law, and up until today no one has figured out why this is the case. The digits should be uniformly distributed! Why would Nature make a distinction between the number 1 and the number 8 in the leading digits of a physical constant? The one thing of which we were so sure that it was a part of Nature seems to have an essential anthropocentric twist to it. It seems that we have found the physical constants that we were looking for, thereby unconsciously favoring the numbers 1,2 and 3. It suggests the one thing that you will never hear a physicist say out loud: we approach Nature with a certain set of tools, assumptions and presuppositions and in doing so, we see a world according to *our* rules, not Nature's rules. In the words of Hamming:

> [...] we approach the situations with an intellectual apparatus so that we can only find what we do in many cases. It is both that simple, and that awful. (Hamming, 1980, p. 88-89)

Indeed, on the one hand it is that awful, because the ultimate goal of science, to describe all natural phenomena with equations, can never be reached. On the other hand, we seem to have become very good in controlling that part of Nature that has a structure we can understand. Although we may only understand a very small part of Nature, we have put the part that does have a structure that we recognize to very good use. However, this also means that we might miss out on a lot of knowledge, simply because the only glasses we put on are mathematical. It suggests that we have chosen mathematics (or is it a historical fluke?) to understand the natural world around us, and that we would maybe understand different parts of that natural world when we try on other glasses.

The role of experiments in this account becomes less important because it is no surprise that the experimental outcomes match theoretical predictions: we only measure the specific regularity in Nature that we picked out. There is a nice story about this by the astronomer, physicist and mathematician Eddington (1939, p. 17-18) paraphrased by Hamming (p. 89): *"Some men went fishing in the sea with a net, and upon examining what they caught they concluded that there was a minimum size to the fish in the sea."*

6.4 The multiple-worlds hypothesis

This final account originates from the more general question why our world is knowable at all. This question is asked even before we ask the question why the world is knowable through mathematics. In solving this more general mystery, we might come closer to solving the applicability of mathematics. The problem is clearly articulated by Einstein:

> One may say "the eternal mystery of the world is its comprehensibility." It is one of the great realisations of Immanuel Kant that the setting up of a real external world would be senseless without this comprehensibility. (Einstein, 1936, p. 351)

What Einstein is saying here is that there is a necessity to the comprehensibility of the world around us. He hints upon the fact that life would not be possible, had the world not been comprehensible by living beings. In his time, there was no scientific evidence for this conceptual idea but in recent decades, a new hypothesis surfaced that might be a defense of this idea.

Andrei Linde, a theoretical physicist, refers to recent research in cosmology that might point towards the fact that we live in a multiverse in which an almost infinite number of universes exist with all possible physical laws and constants (string theory sets the number at 10^{500} possible choices) (Linde, 2012). He explains that in one of those other worlds, comprehensibility is never possible. He takes as an example a universe in which the Planck density is of the order of $10^{94} g/cm^3$, instead of $10^{93} g/cm^3$. Calculations show that the quantum fluctuations would be so large that everything in that universe would be bending and shrinking in a completely unpredictable way. This makes the universe in principle incomprehensible; moreover, any computation about that universe is impossible and mathematics would be inefficient.

This may make the situation here on earth even more unique and the question crops up how we could have been so lucky to be living in the world in which all physical constants are *exactly* right. But this is asking the wrong question, because life is only possible in the few (or maybe only one) universes where the physical constants and laws make for a stable existence of living things. We are therefore not unique in that we can comprehend the world with our brilliant minds, we are not lucky to be living in a world that is comprehensible - mathematical comprehensibility is necessary for a world on which life is possible. According to Linde, the multiple-worlds hypothesis necessarily implies the existence of stable (mathematical) relations that can be used for long-term predictions:

> To summarize, the inflationary multiverse consists of myriads of 'universes' with all possible laws of physics and mathematics operating in each of them. We can only live in those universes where the laws of physics allow our existence, which requires making reliable predictions. In other words, mathematicians and physicists can only live in those universes which are comprehensible and where the laws of mathematics are efficient. (Linde, 2012, p. 91)

In itself, this is consistent hypothesis. The question is of course if it is really true that we are in living in a 'multiverse'. Theoretical physicists are very much divided. Some say that there is scientific evidence that points in that direction, some say the theory is in principle untestable, making it a pure guess.[3] Another price has to be paid when accepting the multiple-worlds hypothesis. The hopes and dreams for a final 'theory of everything' are shattered. When we live in a multiverse, there is no point in explaining why our laws work as they do. They work because it happens to be the case that these laws of nature are needed to create life. It makes both philosophy of science and theology superfluous. The description of the natural constants and laws are everything we are left with. The explanation of why they work becomes a useless exercise.

To conclude, these four approaches show that different routes can be followed to investigate the relation between mathematics, humans and the natural world. The third explanation, we see what we look for, seems to me the most reasonable. Moreover, this account is compatible with the results of Chapter 5. Tractability assumptions are imposed on natural phenomena and they pick out the regularities of the phenomenon that are tractable by our human mind and human mathematics. We devise an experiment that measures only these regularities, and what do you know: the results match. We see what we look for. That we, without understanding Nature in an objective way, are still able to control Nature, is the next thing bordering on the mysterious, but that should be the topic of future research.

[3]Steven Hawking, Andrei Linde and Leonard Suskind are proponents, David Gross and Paul Davies are for example opponents. See Carr (2007), Susskind (2008) and Davies (2003).

Chapter 7

Conclusion and outlook

In this thesis, the unreasonable effectiveness of mathematics in the natural sciences is discussed. I have shown that this is a deep philosophical problem for which no easy solution is available. The historical narrative has shown that basic mathematics evolved into complex mathematics and that a distinction is made from the scientific revolution on between pure and applied mathematics. The methodological approach showed that Steiner's main argument, that anthropocentric elements in mathematics play a crucial role in the discovery of new physical theories, is a valid observation in need of an explanation. I have provided an improved inferential mapping account that is able to render some parts of the anthropocentric influences intelligible, however the successful use of tractability assumptions cannot be explained by this mapping account. This leads to the, metaphysical, conclusion that the world looks 'user-friendly', because our anthropocentric assumptions result in correct knowledge about the natural world. Therefore, I have concluded that I could not refrain from a metaphysical discussion about the relation between mathematics, the human mind and the natural world. Several accounts are provided, of which the most reasonable is that we simply 'see what we look for'. The adoption of this account means that a price has to be paid: complete knowledge about the world around us will never be possible. Moreover, it remains mysterious that we are able to control natural phenomena in such a detailed way, while according to this account, we only have knowledge of a small part of it.

In reflecting on mathematics and science, one last remark must be made regarding the changing role of mathematics in science. On the one hand, mathematics has become increasingly important in the scientific method since Wigner's article. As Zee (1990) mentions, with the advent of superstring theory around 1983, the role of mathematics has become crucial in a way that not even

Wigner could have foreseen:

> In 1984, a theoretical physicist who had a comfortable familiarity with such concepts as coset spaces, homotopy groups, homology sequences, and exceptional algebras would have been regarded by his colleagues as mathematically sophisticated. Some four years later, that same person would be despised by string theorists as a hopelessly unschooled mathematical ignoramus. Is so much mathematics good for physics? I have no idea. (Zee, 1990, p. 322)

Indeed, we have to ask ourselves Zee's question: are we not missing valuable information by excluding all other methods of inquiry? Can we can still discover new knowledge about the world that cannot be structured mathematically?

On the other hand, mathematics has become less important in the scientific method since the rise of computers in the last 30 years. Computer-based methods have initiated new fields, such as the study of non-linear systems, chaos theory and fractal geometry. Simulation has become an integral part of the scientific method and its role will increase even more in the future. Question is, whether the simulation of Nature by powerful computers is still applied mathematics, or that this has, in turn, become its own discipline.

In conclusion, with the role of mathematics in the natural sciences once again subject to change, reflection upon this role is necessary now more than ever.

Acknowledgments

Many thanks to Victor Gijsbers for supervising and for the helpful discussions and remarks. Thanks also to James McAllister, who helped me a lot during my whole master and supervised me in the very beginning. Last but not least, a special thanks to Maarten, for all his support and constructive criticism.

References

Bangu, S. (2006). Steiner on the applicability of mathematics and naturalism. *Philosophia Mathematica*, *14*(1), 26–43.

Batterman, R. W. (2009). On the explanatory role of mathematics in empirical science. *The British journal for the Philosophy of Science*, 1–25.

Bochner, S. (1966). *Role of mathematics in the rise of science.* Princeton University Press.

Bueno, O., & Colyvan, M. (2011). An inferential conception of the application of mathematics. *Noûs*, *45*(2), 345–374.

Burton, D. M. (2011). *The history of mathematics: An introduction, seventh edition*. McGraw-Hill.

Carr, B. (2007). *Universe or multiverse?* Cambridge University Press.

Colyvan, M. (2001). The miracle of applied mathematics. *Synthese*, *127*(3), 265–278.

Daly, C., & Langford, S. (2009). Mathematical explanation and indispensability arguments. *The Philosophical Quarterly*, *59*(237), 641–658.

Davies, P. C. (2003). *A brief history of the multiverse.* (Published as opinion article April 2003 in NY Times)

Dijksterhuis, E. J. (1961). *The mechanization of the world picture*. Oxford University Press Oxford.

Dorato, M. (2005). The laws of nature and the effectiveness of mathematics. In *The role of mathematics in physical sciences* (pp. 131–144). Springer.

Doreian, P. (1990). Mathematics in sociology: Cindarella's carriage or pumpkin. In *Mathematics and science* (pp. 34–54). World Scientific.

Eddington, S. A. (1939). *The philosophy of physical science*. Cambridge Univ Press.

Einstein, A. (1922). *Sidelights on relativity.* published online.

Einstein, A. (1936). Physics and reality. *Journal of the Franklin Institute*, *221*(3), 349–382.

Feynman, R. P. (1967). *The character of physical law* (Vol. 66). MIT press.

Fresnel, A. (1831). Ueber das gesetz der modificationen, welche die reflexion dem polarisirten lichte einprägt. *Annalen der Physik*, *98*(5), 90–126.

Gross, D. J. (1988). Physics and mathematics at the frontier. *Proceedings of the National Academy of Sciences of the United States of America*, *85*(22), 8371-8375.

Hacking, I. (2011). Why is there philosophy of mathematics at all? *South African Journal of Philosophy= Suid-Afrikaanse Tydskrif vir Wysbegeerte*, *30*(1), 1–15.

Hamming, R. W. (1980). The unreasonable effectiveness of mathematics. *American Mathematical Monthly*, *87*(2), 81–90.

Hardy, G. H. (1940). *A mathematician's apology*. Cambridge University Press.

Hindriks, F. A. (2006). Tractability assumptions and the musgrave–mäki typology. *Journal of Economic Methodology*, *13*(4), 401–423.

Kant, I. (1902). *Prolegomena to any future metaphysics that can qualify as a science* (No. 53). Open Court Publishing.

Katz, V. J. (2009). *A history of mathematics: An introduction, third edition*. Addison-Wesley.

Kline, M. (1985). *Mathematics and the search for knowledge*. Oxford University Press.

Lakoff, G., & Núñez, R. E. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. Basic books.

Linde, A. (2012). Why is our world comprehensible? In *This explains everything* (p. nog opzoeken!). HarperCollins Publishers Incr.

Lützen, J. (2011). The physical origin of physically useful mathematics. *Interdisciplinary Science Reviews*, *36*(3), 229–243.

Mac Lane, S. (1990). The reasonable effectiveness of mathematical reasoning. In *Mathematics and science* (pp. 115–121). World Scientific.

Narens, L. (1990). Three aspects of the effectiveness of mathematics in science. In *Mathematics and science* (pp. 122–135). World Scientific.

Odom, B., Hanneke, D., D'Urso, B., & Gabrielse, G. (2006, Jul). New measurement of the electron magnetic moment using a one-electron quantum cyclotron. *Phys. Rev. Lett.*, *97*, 030801. Retrieved from `http://link.aps.org/doi/10.1103/PhysRevLett.97.030801` doi: 10.1103/PhysRevLett.97.030801

Oldershaw, R. L. (1990). Mathematics and natural philosophy. In *Mathematics and science* (pp. 136–153). World Scientific.

Peat, F. D. (1990). Mathematics and the language of nature. In *Mathematics and science* (pp. 154–172). World Scientific.

Pincock, C. (2004). A new perspective on the problem of applying mathematics.

Philosophia Mathematica, *12*(2), 135–161.

Remmert, R. (1991). Complex numbers. In H. Ebbinghaus et al. (Eds.), *Numbers, volume 123 of graduate texts in mathematics* (p. 55-96). Springer-Verlag.

Rizza, D. (2011). Magicicada, mathematical explanation and mathematical realism. *Erkenntnis*, *74*(1), 101–114.

Saatsi, J. (2011). The enhanced indispensability argument: Representational versus explanatory role of mathematics in science. *The British Journal for the Philosophy of Science*, *62*(1), 143–154.

Shapiro, S. (2000). *Thinking about mathematics: The philosophy of mathematics*. Oxford University Press.

Simons, P. (2001). Mark steiner - the applicability of mathematics as a philosophical problem. *The British Journal for the Philosophy of Science*, *52*(1), 181–184.

Steiner, M. (1998). *The applicability of mathematics as a philosophical problem*. Harvard University Press.

Susskind, L. (2008). *The cosmic landscape: String theory and the illusion of intelligent design*. Back Bay Books.

Todorov, I. (2012). Quantization is a mystery. *arXiv preprint arXiv:1206.3116*.

Wigner, E. P. (1960). The unreasonable effectiveness of mathematics in the natural sciences. *Communications on pure and applied mathematics*, *13*(1), 1–14.

Wilson, M. (2000). The unreasonable uncooperativeness of mathematics in the natural sciences. *The Monist*, 296–314.

Zee, A. (1990). *The effectiveness of mathematics in fundamental physics.* World Scientific.