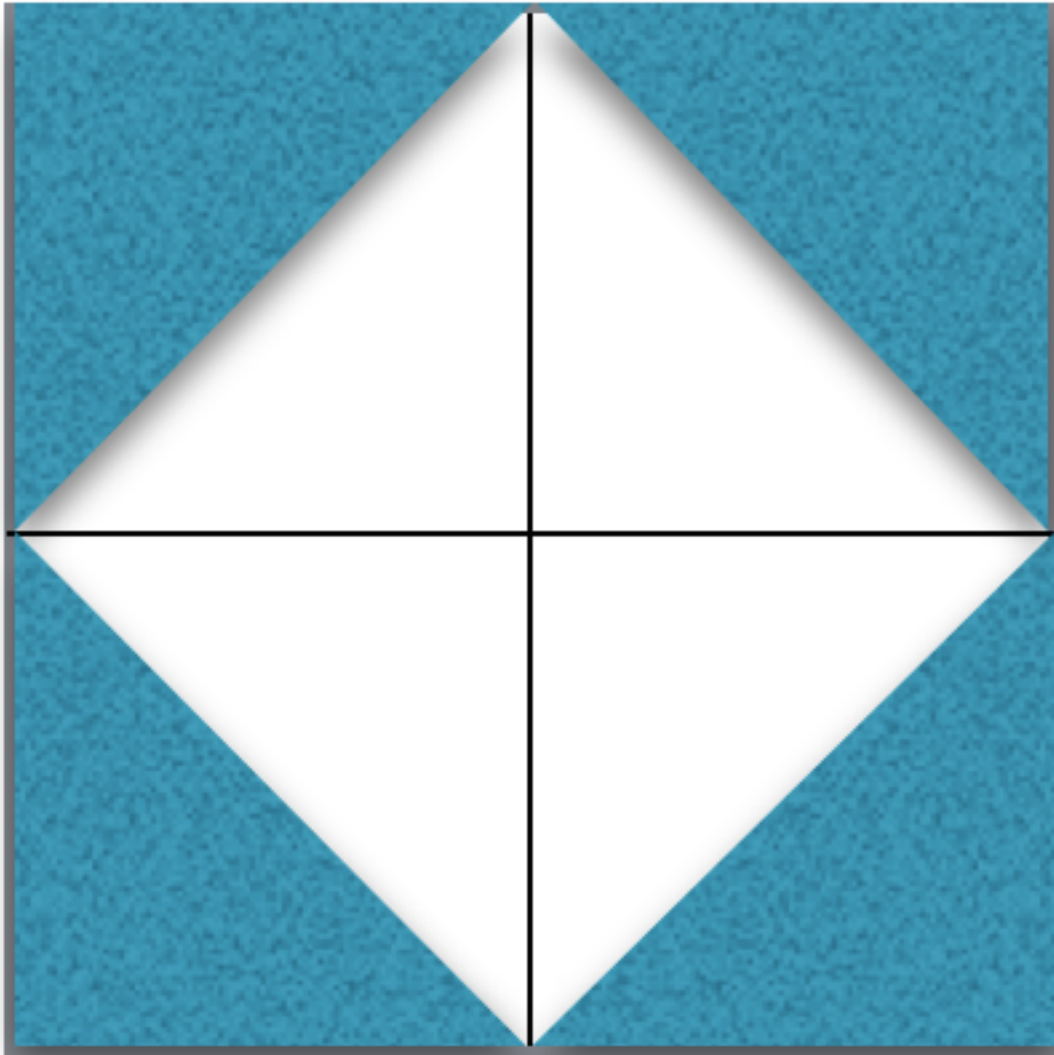


# MYSTERIOUS MATHEMATICS

in Plato's *Meno*



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## Foreword: Where Do Mathematical Theorems Come From?

*“How could you,” began Mackey, “how could you, a mathematician, a man devoted to reason and logical proof ... how could you believe that extraterrestrials are sending you messages? How could you believe that you are being recruited by aliens from outer space to save the world? How could you ... ?”*

*Nash looked up at last and fixed Mackey with an unblinking stare as cool and dispassionate as that of any bird or snake. “Because,” Nash said slowly in his soft, reasonable southern drawl, as if talking to himself, “the ideas I had about supernatural beings came to me the same way that my mathematical ideas did. So I took them seriously.”<sup>1</sup>*

Mathematics, which at its core is all about formulating and proving theorems, has this peculiarity to it: *mathematical theorems come first, and proofs follow*. Examples are Fermat’s “Last Theorem” (asserting that there are no whole-number solutions to any equation of the form  $a^n + b^n = c^n$  when  $n > 2$ ), formulated in 1637 by Pierre de Fermat, and proved only in 1994 by Andrew Wiles; the Prime Number Theorem (describing the asymptotic distribution of prime numbers among the positive whole numbers), first expressed by Gauss in 1792 or 1793, and proved in 1896 by Hadamard and de la Vallée-Poussin; or Goldbach’s Conjecture from 1742 (every integer can be written as the sum of two primes), and the Riemann Hypothesis of 1859 (the  $\zeta$ -function has zeros only at the negative even integers and the complex numbers with real part  $\frac{1}{2}$ ), both of which remain as yet unproved.

Mathematical theorems are, by general consent, non-trivial—they generate new information not contained in earlier theorems, lead to unsuspected new insights, and result in novel applications.<sup>2</sup> But any novelty issuing from a theorem is acceptable to the scientific community only, once a logically rigid, and therefore irrefutable, proof has been established:<sup>3</sup> the theorem then results, as it were, from the proof—there now is, so to speak, a direct route from easy-to-understand definitions, axioms, common notions (if one would take Euclid as a model), and so on, to more complex statements, onwards to the theorem—a route from the trivial to the non-trivial. But this is not how the theorem *originated*: theorems *always* come first, and proofs *always* follow. Since theorems are non-trivial, *and* do not originate from proof, many mathematicians have felt compelled to ask the question: *where do theorems come from?*

“It is a mystery where they come from,” Andrew Wiles said of the intermediate theorems that he came up with in his proof of Fermat.<sup>4</sup> This seems an innocent statement, with a somewhat romantic ring to it, and claiming no more than the odd bit of poetic license – but *is* Wiles’ statement really that innocent? For in the same vein, Villani called the theorem that earned him the Fields Medal a “miracle,”

<sup>1</sup> Nasar 1998, p. 11 (my italics).

<sup>2</sup> Cf. Poincaré 1910, p. 325.

<sup>3</sup> Dedekind 1893, p. vii.

<sup>4</sup> Quoted from a BBC Horizon documentary, directed by John Lynch: *Fermat’s Last Theorem*, broadcasted 15 January, 1996 (quote starts at 22:39).

the result of “divination,” “divine inspiration,” “illumination,” and “magic”;<sup>5</sup> Gauss wrote that he owed a certain theorem, not to his own “painful efforts,” but to “the grace of God”;<sup>6</sup> Ramanujan stated time and again that he received his insights from the Hindu goddess Namagiri;<sup>7</sup> and Nash claimed that theorems came to him in ways similar to how he once believed to have received messages from extraterrestrials.<sup>8</sup> All were serious when they said these things, and in good mental health (including Nash).<sup>9</sup>

These are no isolated examples, nor do they represent a phraseology that is only figurative and therefore innocuous. Literature on the history of mathematics is rife with “mystery talk.” While preparing for this thesis, I read dozens of biographies on mathematicians (Newton, Riemann, Ramanujan, Gödel, Turing, Nash, and Erdős, to name only a portion), autobiographies and miscellanies by mathematicians themselves (Hadamard, Hardy, Littlewood, Poincaré, Ulam, and Villani), and read and watched interviews with mathematicians (such as Conway, Mazur and Wiles). Time and again, the word ‘mystery’ was mentioned, as well as other expressions with a similar drift, and always in reference to the fact that *mathematical theorems come first, and proofs follow*.

What, then, is this so-called “mystery”? As said, a theorem will be welcomed as true only when proof has been established. But once a theorem, often after decades or even centuries of immense labour, finally has its proof, and therefore is demonstrably true... Well then (so mathematicians appear to argue) it must have been true all the way from its inception. This would mean that even though the theorem, before proof was settled, was not admissible *as* a truth according to scientific standards, it *was* actually (and not just potentially) true all along. In that case, the mathematician who came up with the theorem, in some sort of way, will be considered to have “partaken of” something true (i.e. the theorem). But since proof is the *only* way in which members of the scientific community can make sense of a theorem, this partaking can’t be called anything else but “mysterious.”

The “mystery,” then, is inexplicable access to truth before proof. Such a view is of course highly problematic. But my point is *not* that allowing “mystery talk” would open the door to pseudo-scientists who then can claim all kinds of truth without adducing proof. In the 18<sup>th</sup> century, this type of impostor was a serious problem for Kant (for example in *Träume eines Geistersehers* and, decades later still, in *Von einem vornehmen Ton*);<sup>10</sup> but in the 20<sup>th</sup> and 21<sup>st</sup> centuries, such people are simply no longer taken seriously by the scientific community. My point is more subtle: it is that mathematicians today freely utter “mystery talk” *while* strenuously continuing to find proofs for their theorems—that mathematicians today seem to feel that their scrupulous inclination towards rigid proof discharges them from any obligation to give a *serious* answer to the question: *where do theorems come from?*

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<sup>5</sup> Villani 2015, pp. 19, 126, 135, 137, 142-143, 184-185, 200, 203.

<sup>6</sup> Quoted from Hadamard 1945, p. 15.

<sup>7</sup> Kanigel 1991, pp. 4, 20, 30, 36, 281, 283, 335.

<sup>8</sup> *Op. cit.*, *loc. cit.* footnote 1.

<sup>9</sup> Nash was diagnosed with paranoid schizophrenia in 1959: see Nasar 1998, pp. 308-319.

<sup>10</sup> *TG*, e.g. pp. 5, 6, 57; *vT*, e.g. p. 390.

Even Hardy, who may have been the biggest stickler for proof in the history of mathematics,<sup>11</sup> called the origin of mathematical theorems a “mystery”;<sup>12</sup> in the same breath, however, he heartily admitted his “distaste for all sorts of mysticism.”<sup>13</sup> In discussing his collaboration with Ramanujan, Hardy commented how “it seemed ridiculous to worry about how [Ramanujan] had found this or that theorem, when he was showing me a dozen new ones every day”; “anxious to get on with the job [...],” Hardy and Ramanujan “[...] had more interesting things to think about than historical research.”<sup>14</sup> In other words: Hardy believed it was quite fine to call the origin of mathematical theorems a “mystery,” but that it was of no importance to inquire there any further. The moniker “historical research” was intended in a derogatory way (historians should take offence from Hardy, and not from me): this arrogance, I feel, exhibits the ultimate forsaking of the question, *where theorems come from*.

Plato seems to have experienced a similar “mystery” with respect to the fact that theorems come first, and proofs follow. His *Meno* is the earliest surviving written source in which mathematics is expressly linked to the word ‘mystery’: Plato uses it in the sense of the Eleusinian Mysteries (*tôn mustêriôn*, 76E9), and in the context of the main question of the *Meno*, “What is virtue?” (*ti esti aretê*, 71A9). The emphasis lies on Meno’s reluctance to be initiated (*muêtheiês*, 76E10), which, in the context of the dialogue, points to Meno’s unwillingness to face an *aporia*. The *aporia* is an unpleasant, yet inevitable part of a larger quest: finding a *zêtoumenon* (e.g. 79D7-8), i.e. something which one believed to know, but realizes one does not know, and has to search for while not knowing it. The possibility of inquiring into such a *zêtoumenon* is established by the famous mathematical passage, in which Socrates confronts a slave with a problem from geometry (82A-86B). As will be demonstrated below, what happens there—the actual finding of a *zêtoumenon* (or of a theorem, according to one’s taste)—is nothing “mysterious” in the torpid sense of Hardy and Wiles, but the result of Socrates’ unusual treatment of the problem of “Doubling the Square”, in inciting misleading thought tendencies, employing opaque features of mathematical diagrams, and avoiding mathematical vocabulary.

As will be demonstrated, Plato’s references to what could be called “mysterious” about finding the solution to the geometrical problem—his quotation of Pindar (81B-C), which implies a link between the solution on the one hand, and a myth about Persephone (concerning the immortality and remigration of souls) on the other; the explanation of this link within the context of Plato’s theory of remembrance (*anamnêsis*, 81C9 *ff.*); the concept of *alêthês doxa* (85C8 *ff.*), or true opinion, i.e. something that can be called true but not yet knowledge, and which is the result of a divine dispensation (99E8 *ff.*)—all pertain to Socrates’ clever treatment of the mathematical problem, to a degree where it can be said that the

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<sup>11</sup> Kanigel 1991, p. 151,

<sup>12</sup> Hardy 1946, p. 112.

<sup>13</sup> Hardy, *op. cit.*, p. 113.

<sup>14</sup> Kanigel 1991, p. 279.

“mystery” surrounding the mathematics actually issues *from* the mathematics in the very way in which it is employed in the dialogue.

### **Introduction: The Incompleteness of Approaches to the *Meno*, either Strictly Philosophical or Strictly Mathematical**

The main focus of this thesis will be on the mathematical passage in the *Meno*, which deals with the problem of doubling the square.<sup>15</sup> It should, however, be pointed out that it makes little sense to take a strictly mathematical approach to that section, and discuss the problem of doubling the square in separation from the main issue of the dialogue (*ti esti aretê*), as happens in many textbooks on the history of ancient Greek mathematics.<sup>16</sup> Vice versa, it makes just as little sense to take a strictly philosophical approach by discussing the main problem of the *Meno* separately from the mathematical exercise, or by viewing this exercise simply as one example of *anamnêsis* out of many, deriving no significance from the mathematics as such.<sup>17</sup> Both the mathematical approach, or the philosophical approach, when taken independently, will remain incomplete.

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<sup>15</sup> There is of course mention of a second mathematical problem in the *Meno* (87A-B); this appears to revolve around finding a geometrical construction through which a given area (of unspecified geometrical shape) can be transformed into a triangle of identical area, so that the triangle can be inscribed inside a circle with a given diameter. Since Plato's phrasing of the problem is unclear, and historians of mathematics are divided on its exact content, this problem will be ignored here. See Lloyd 1992, pp. 166-175, for a detailed discussion of at least six different interpretations of the problem.

<sup>16</sup> Heath 1921, pp. 297-298, mentions *anamnêsis*, which is described as “the reawakening of the memory of something,” apparently regarding this as a philosophical notion that has no bearing on mathematics, since Heath moves on to discuss the problem of doubling the square in complete separation from Socrates' thoughts on remembrance (as well as from the main question of the dialogue, *ti esti aretê*, which Heath ignores); Knorr 1975, pp. 26, 53 (note 23), 71, 73, 90, 104 (note 72), discusses the historical origins of the problem of doubling the square, and several other historical issues, such as ways in which the solution to the problem of doubling the square can be proved, while such proof is entirely omitted from the *Meno* (we will return to this in Chapter 2); the continuity of magnitude; and the position of mathematics in the *Meno* within the “metrical tradition” (we will return to this in Chapter 1). Fowler 1999 ignores the philosophical content of the *Meno*, to focus on the dialogue as the earliest available written source on ancient Greek mathematics, especially with regard to the development of mathematical terminology; the relationship between arithmetic and geometry; the use of anthypharesis; incommensurability; the relation of the problem of doubling the square to that of the duplication of the cube; and the relation of mathematics in the *Meno* to later developments in Greece, especially Euclid; see Fowler 1999, pp. 3-10, 13-14, 30-31, 33, 65, 70, 101, 114, 148, 366 note 12, 367, 387 (Fowler also describes an entirely fictional dialogue between Socrates and the slave: page numbers on which this invented conversation occurs have been omitted here).

<sup>17</sup> Scott 2009 deals with the mathematics passage entirely as one case of *anamnêsis* out of many, without much regard for the mathematics involved: see Scott 2009, pp. 98-112.

The problem of doubling the square would have to be considered trivial when discussed from a strictly mathematical point of view. As is well known to Plato scholars, historians of mathematics, and philosophers alike, the problem in *Meno* 82A-86B bears on irrational numbers, the so-called “surds” or “immeasurables,” in particular  $\sqrt{2}$ .<sup>18</sup> Allegedly, these numbers caused a “foundation crisis,” which during some period in ancient Greek mathematics seemed insurmountable,<sup>19</sup> but had been sufficiently resolved by the time of Plato.<sup>20</sup> This leads to the question why Plato bothered to discuss the problem of doubling the square at all, since it no longer was fundamental in his day—unless that would have been what he wanted to point out. This, however, does not follow from the *Meno*. Fowler, in his study *The Mathematics of Plato’s Academy*, has forwarded the thesis that irrationals never led to a “foundation crisis”:<sup>21</sup> if he is right, the triviality of the mathematics in *Meno* 82A-86B is even more striking.

The triviality of the problem of doubling the square, or rather of dealing with irrational numbers in general, is underscored by Plato himself in several other dialogues. Most explicitly this happens in the *Laws*, where people with no understanding of irrational numbers, or more precisely, who are helpless in the face of immeasurable line segments, are considered “detestable” (*phaulôs*, 820A), and are even compared to pigs (*huênôn*, 819D).<sup>22</sup> Elsewhere, in the *Theaetetus*, two men handle irrational numbers with ease and with no hint at a “foundation crisis” (147D-148A).<sup>23</sup> But it could be counter-productive to dismiss the notion of a “foundation crisis” too quickly: the *aporia* from which this crisis supposedly resulted may be relevant to the present discussion of the *Meno*. If the problem of doubling the square is considered trivial, and therefore of no mathematical interest, the danger arises that the meaning of *anamnêsis*, as it occurs in the mathematical passage, will be based entirely on the “mysterious” thoughts of Socrates on remembrance, as professed in those parts of the dialogue which immediately precede and follow the mathematical passage, and which have promoted the notion that Plato must have felt there was something truly mysterious about the origin of mathematical thought—which, if correct, would place Plato on a par with the likes of Hardy and Wiles.

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<sup>18</sup> Klein 1965, pp. 99-101, 185; Knorr 1975, p. 26.

<sup>19</sup> Burkert 1972, pp. 455-456; Dunham 1991, pp. 1-2, 8-10; Hardy 1940, pp. 100-101; Heath 1921, p. 155; Struik 1987, p. 43.

<sup>20</sup> Dunham 1991, p. 9; Heath 1921, p. 155; Klein 1965, p. 107; Knorr 1975, pp. 22, 40-41; Woodbridge 1965, p. 39.

<sup>21</sup> Fowler 1999, pp. 356-359; Knorr does not entirely reject the notion of a “foundation crisis,” but is skeptical about its occurrence: see Knorr 1975, pp. 2-4, 39-42, 49-50.

<sup>22</sup> For brief treatments of this passage within the history of mathematics, see Fowler 1999, pp. 290, 360, and Heath 1921, p. 156.

<sup>23</sup> Cf. Fowler 1999, pp. 290, 359-360, 362-365. Another passage in which Plato demonstrates perfect comfort in dealing with irrational numbers, even though little detail is provided and calculations are omitted, can be found in *Republic* XIII, 546C, where the diameter of the square with sides of 5 units in length is mentioned, i.e. the diameter with length  $\sqrt{50}$  or  $5\sqrt{2}$ . For an overview of other examples of comfortable dealings with irrational numbers in Plato, see Ast 1835, vol. I, p. 486 (under the lemma *diametros*).

Perhaps it is hard indeed to see how Socrates' thoughts on *anamnêsis* can be regarded as anything other than "mysterious."<sup>24</sup> They are often called "mysterious" because Socrates announces them by calling on the authority of "priests, priestesses, many other divine poets (*hiereôn te kai hieriôn ... kai alloi polloi tôn poiêtôn, hosoi theioi eisin*, 81A10-B1), and by quoting Pindar (81B9-C4), who sings of a "requital" for an "ancient wrong," in return for which "souls are restored" by Persephone to mankind (this being the parallel to *anamnêsis*). He repeats the same position later on in the dialogue, when he compares the rulers of city states to soothsayers and prophets (*chrêsmôidoi te kai ... theomanteis*, 99C3-4), who are "divine," "enraptured," "inspired," and "possessed by the god" (*theious te einai kai enthousiazein, epipnous ontas kai katechomenous ek tou theou*, 99D3-5). With this in mind, the characterization of virtuous behaviour as a "divine dispensation" (*theiai moirai*, 99E8, 100B3) towards the end of the dialogue quite naturally comes across as "mystery talk" *pur sang*.

Therefore, this thesis faces a twofold task. In order to avoid the shortcomings of either a strictly mathematical or a strictly philosophical approach, it needs to show, firstly, that the geometrical problem is not just one example of *anamnêsis* out of many, and secondly, that Socrates' thoughts on *anamnêsis* amount to more than "mystery." Therefore, one part of the task is to demonstrate that the mathematics passage, as a geometrical exposition, adds to the meaning of *anamnêsis* in such a way that Socrates' thoughts on remembrance are lifted above mere "mystery talk." Vice versa, it needs to show that Socrates' citation of Pindar, and his mentioning of "priests, priestesses, and many other divine poets" together with "soothsayers and prophets," as well as the reference to the Eleusinian Mysteries, carry over into the mathematical passage in such a way that the mathematics can no longer be regarded trivial. Then, by consequence, it should be possible to demonstrate that—the mathematics in the *Meno* being *anamnêsis*, and *anamnêsis* being more than "mystery"—the question where the slave's mathematical insights come from can be answered in a way that steers clear from the torpid "mystery talk" of Hardy and Wiles.

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<sup>24</sup> Klein 1965, pp. 26, 100, 178-183, 186, 188-190, 201, calls Socrates' thoughts on *anamnêsis* "mythical"; Klein remarks that even after the mathematical passage, when talking about *anamnêsis*, "Socrates is merely expanding on the myth of recollection previously reported by him. The mythical way of speaking prevails throughout." For this see Klein 1965, p. 178; the present thesis strongly disagrees with Klein in this respect. Sesonke 1965, p. 91, speaks of the "mystery of recognition" in the context of the slave's *anamnêsis*, without elaborating any further on his use of the word 'mystery'. Brown 1967, p. 76, hints at the possibility of the mathematics passage constituting a "mystery," without elaborating on this any further. Vlastos 1994, pp. 103-104, calls Socrates' references to mythology in the *Meno* "religious" and attributes this religiousness to a personal faith in reincarnation privately held by Plato; we will return to Vlastos' thesis in Chapter 3. Scott 2009, pp. 1, 92-94, also calls Socrates' references "religious," probably because of Socrates' mentioning of priests and priestesses. Shapiro 2000, p. 52, calls the connection of the mathematics in the *Meno* to *anamnêsis* "mythical", which suggests that he too refuses to see any internal connection between *anamnêsis* and the mathematics as such.



In Chapter 1, the role of  $\sqrt{2}$  in the *Meno* will be discussed, as well as several of its mathematical characteristics, such as its so-called “immeasurability,” and the opaque matter of its constructibility. Attention will also be paid to the geometrical representation of numbers as this was common, but also potentially problematic, in ancient Greek mathematics. Chapter 2 will highlight the diagrams drawn by Socrates in the *Meno*, and investigate some particularly surprising features of the final diagram, which are cleverly exploited by Socrates; also, it will be demonstrated how Socrates avoids the use of certain mathematical expressions. In Chapter 3, it will be shown, mainly through a detailed analysis of Plato’s Greek, how Socrates prepares the ground for the slave’s actual *anamnêsis*; most importantly, it will be discussed in what sense this *anamnêsis*, indeed, is a “mystery”—consisting, as it does, in the sudden recognition of the *zêtoumenon* being the diagonal, which, while coming as a stunning surprise to the slave, is the result of Socrates’ peculiar treatment of the mathematical problem.

## Chapter 1: The Slave's *Aporia* and the Square Root of Two

### 1.1 The Slave's *Aporia*, I

In *Meno* 84A, the slave admits that he has no answer to the problem of doubling the square (*egôge ouk oida*, 84A3-4), and enters into an *aporia* (*aporein*, 84A10 and 11):<sup>25</sup> he is at a loss for words, and does not know how to bring the task to an end. In some sense, the slave's *aporia* has to do with  $\sqrt{2}$ , or to be precise, with a multiple of  $\sqrt{2}$ , i.e.  $2\sqrt{2}$  (which, like any multiple of an irrational number, is itself irrational). Because of the apparent triviality of the problem of doubling the square, it is not easy to see why the slave has an *aporia*. As mentioned in the *Introduction*, the problem of doubling the square pertains to an alleged "foundation crisis" in ancient Greek mathematics, which had been resolved in Plato's time. For this reason, the problem was trivial to Plato's contemporaries, and to Plato himself; but it also appears trivial to us, for reasons which are quite different from Plato's. So first, before properly situating the problem of doubling the square within the context of ancient Greek mathematics, we should try to get a sense of why the problem is, or seems, trivial to ourselves.

We nowadays deal with the problem of doubling the square in a straightforward manner, without much ado. Not only does the task appear easy, soluble to anyone possessing basic knowledge of high-school arithmetic, algebra, and geometry: it is, from the point of view of present-day mathematics, not much of a problem at all. The assignment is to double a square with a surface area of 4 units, i.e. to construct a square with an area of 8 units, using the smaller square as a point of departure. The first issue to arise is the computation of the sidelength of the larger square; the second is to determine what line in the original square corresponds in length to the side of the larger square. In order to calculate the side of a square with an area of 8 units, we simply extract the square root of 8. In algebraic notation, this is  $\sqrt{8}$  (the solution  $-\sqrt{8}$  being irrelevant in this case), which can be simplified to  $2\sqrt{2}$ . We determine this root, because we know that the surface area  $A = 8$  results from multiplying two sides of the square,  $a$  and  $b$ ; since  $a$  and  $b$  are identical, we have:  $A = a \cdot b = a \cdot a = a^2 = 8$ , and therefore  $\sqrt{(a^2)} = a = \sqrt{8} = 2\sqrt{2}$ . It follows from the Pythagorean Theorem that this is the length of the diagonal of the smaller square;<sup>26</sup> therefore, we are able to quickly realize that the 8-unit square can be constructed on the diagonal of the 4-unit square.

In the solution above, a problem from geometry was tackled with the help of basic arithmetical and algebraic operations (addition, multiplication, raising numbers to a power, and root extraction). To us, it seems obvious that we can leave the geometrical context of the problem of doubling the square behind; perform algebraic computations using elementary rules of arithmetic, *without* reference to the geometrical figure; and return to the geometrical problem with the solution in hand. *But that is not what we were asked to do.* The task was to *construct* a square

<sup>25</sup> Cf. *aporein*, 84B6; *aporian*, 84C6; *aporias*, 84C11.

<sup>26</sup> Let  $c$  and  $d$  be the sides of the smaller square, and  $e$  its diagonal; then, since  $c = d$ , we get  $c^2 + d^2 = c^2 + c^2 = e^2$ ; and since  $c = 2$ , we have  $4 + 4 = 8 = e^2$ ; hence  $\sqrt{e^2}$  is  $\sqrt{8} = 2\sqrt{2} = e$ .

with an area of 8 units from a square with an area of 4. And construction, in the ancient Greek sense of the word, allows only the use of a straightedge, compass, marker (e.g. a stylus, reed pen, or stick), and a surface on which to draw lines (e.g. a wax tablet, papyrus, or layer of sand).<sup>27</sup> It was by starting from certain given lines (those of the smaller square), and by adding extra lines through the extension or transposition of these given lines (or of lines acquired by extending or transposing the given lines, etc.)—that is: by *constant* reference to, and elaboration of, the geometrical figure—that the problem was supposed to be solved. Such are the rules of the game in ancient Greek geometry.

Meno states explicitly that the slave was never taught geometry,<sup>28</sup> so we can safely assume that the boy was not in the least aware of these rules. But Socrates, throughout the conversation, gently (yet consistently) incites the slave to employ those rules, not by explicitly stating them, or at least by silently ensuring the slave would adhere to them, but rather by immediately connecting words used by him or the slave to elements of the diagrams, in a colloquial, “point-and-see” kind of way. We will return to the colloquial nature of the conversation in chapter 3; for now, the important thing to realize is that Meno’s slave, indeed, never abandons the geometrical context of the problem. Of course, the mathematics in the *Meno* is not devoid of arithmetic: the slave performs arithmetical operations, primarily those of counting and addition, and (perhaps) also multiplication and division. But he does so with *uninterrupted* reference to the diagrams: what he counts, adds up, multiplies or divides are always elements of the geometrical figures drawn by Socrates—and precisely that lies at the heart of the slave’s *aporia* with respect to  $2\sqrt{2}$ . All the slave’s arithmetical calculations are *suggested* by the diagrams, and *verified* or *rejected* with reference to the figures. There is no rigid distinction between numbers on the one hand, and elements of the figures, such as lines and areas, on the other—in short, no distinction is maintained between number and magnitude.<sup>29</sup> Geometry, it seems, is considered a seamless extension of arithmetic, and vice versa. The result is that for specific numbers (the integers), the slave derives arithmetical properties from elements of the geometrical figure; he then carries these properties over into other parts of the diagram, i.e. into particular other numbers (the irrationals)—or into one such number, to be precise:  $2\sqrt{2}$ —to which those properties do *not* apply.

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<sup>27</sup> Netz 2003, pp. 14-17. It is often believed that straightedges used in ancient Greek geometrical proofs were always unmarked. Fowler discusses evidence against this; see Fowler 1999, pp. 283-289.

<sup>28</sup> Socrates asks Meno: “Or has someone taught [the boy] how to do geometry?” (*ê dedidachē tis touton geōmetrein*, 85E1); to which Meno responds: “Well, I know that nobody ever taught him [geometry]” (*All’oida egōge hoti oudeis pôpote edidachen*, 85E7-8).

<sup>29</sup> Knorr 1975, p. 90.

## 1.2: Doubling the Square

In *Meno* 82B, Socrates draws a square, and adds one vertical and one horizontal line, dividing the figure into four smaller squares (fig. 1).<sup>30</sup> He then asks the slave if it is true whether the six lines are equal in size, and if the whole figure can be either smaller or larger: the slave replies, correctly, in the affirmative (82C). The slave is then asked to calculate the area of the entire square, on the premise that the sidelength is 2 feet: he does this accurately by multiplying 2 by 2 feet: 4 feet—or in fact, all he needs to do is count the four smaller squares.<sup>31</sup> Next, Socrates asks to calculate the area of a square double in size: the slave simply multiplies 4 by 2 feet, and arrives at the right answer, 8 feet (82D)—or rather, he mentally adds up four additional small squares to the four original ones, which is to have an important implication soon.

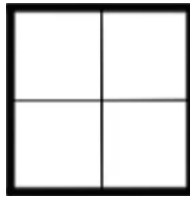


Fig. 1: the initial 4-foot square drawn by Socrates (82B).

Socrates, fully deliberately, then poses a trick question:<sup>32</sup> if the area of the larger square were double to that of the smaller square, what would be the sidelength of this larger square, compared to that of the smaller? The slave gives the wrong answer, by saying that the requested length, clearly (*dêlon*), must also be double (82E), that is 4 feet (fig. 2):

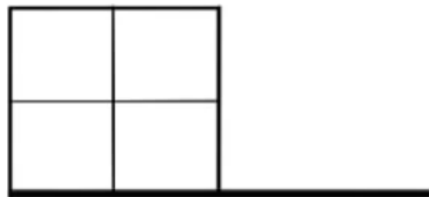


Fig. 2: the slave proposes to generate the 8-foot square from a side twice the length of the side of the original 4-foot square (82E).

<sup>30</sup> According to Boter 1988, pp. 209 *ff*, Socrates does not draw two perpendicular transversals inside the initial square, but two diagonals. Boter was apparently unaware of Ebert's earlier (but less elaborate) rendering of the same opinion; see Ebert 1973, pp. 178-179. I fully disagree with Ebert and Boter, for reasons that will become apparent in the course of this thesis.

<sup>31</sup> In line with the text in the *Meno* itself, I will take the liberty to quantify areas as surfaces of 1 foot, 4 feet etc., without each time adding the word 'square' (as in "8 square feet"), which would make the present text rather unpleasant to read.

<sup>32</sup> Cf. Vlastos 1994, p. 98.

Socrates demonstrates that the square resulting from sides with a length of 4 feet has an area of 16 feet, for it consists of sixteen 1-foot squares (83C, fig. 3). So if the sidelength of the square with a surface area of 8 is larger than 2 feet, Socrates says, and smaller than 4, what must it be? This, again, is a trick question, and once more the slave answers incorrectly: 3 feet (83E, fig. 4). And this, together with the preceding answer, reveals the problem with the slave's calculations: he is looking for a *whole-number solution* to the problem of doubling the square, and that is what twice has led him into falsehood.

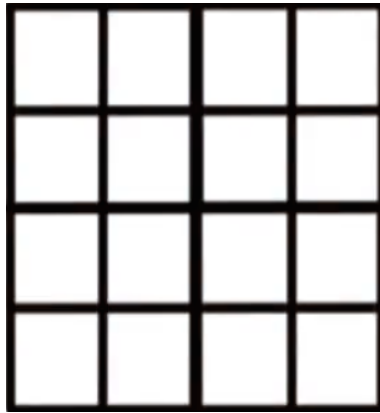


Fig. 3: the square with sides of 4 feet, as proposed by the slave (82E).

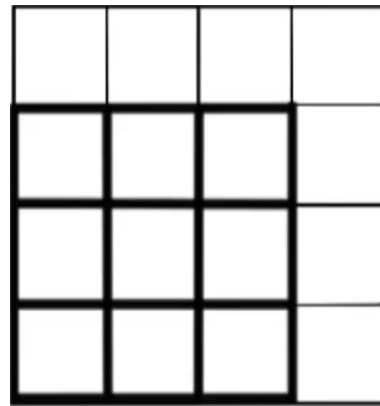


Fig. 4: the square with sides of 3 feet, as proposed by the slave (83E).

The slave's tendency to go after whole-number solutions can be represented geometrically. As mentioned earlier, the slave does not maintain a strict distinction between number and magnitude. Where this distinction is not upheld, whole numbers can be expressed as geometrical shapes consisting of units representing the number 1, for example as squares or oblong rectangles built from unit squares (figs. 1-3); using these shapes, problems of arithmetic and algebra can be formulated and solved, as in the following example, in which the equation  $3^2 + 4^2 = c^2$  is solved for  $c$  (fig. 5):

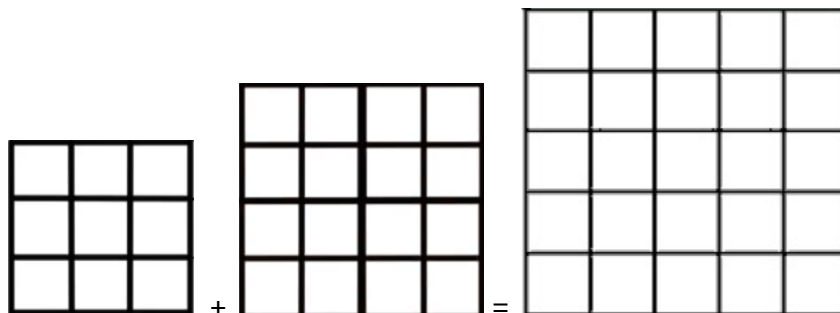



Fig. 5: an example of a whole-number solution in geometry ( $3^2 + 4^2 = 5^2$ ).

In the problem of doubling the square, algebraically speaking, the slave is looking for a solution to the following equation:  $l \cdot u + (m \cdot u) \cdot (n \cdot u) = p \cdot u$ , where  $u$  is the unit, or

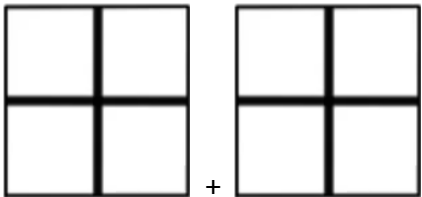
number 1,  $l \cdot u$  equals four, and  $m$ ,  $n$  and  $p$  are whole numbers; all products ( $l \cdot u$ ,  $m \cdot u$ ,  $n \cdot u$ , and  $p \cdot u$ ) share the common unit  $u$ , that is: they are measurable by the same unit, and can be represented as squares or oblong rectangles. Remember that  $p \cdot u$  is a square, i.e. should be represented as a square in a diagram.<sup>33</sup> Stepwise, this means that:



$$+ (m \cdot u) \cdot (n \cdot u) = p \cdot u = 8.$$

Fig. 6.

If we now solve  $(m \cdot u) \cdot (n \cdot u)$ , it is clear that this product must be equal to 4, which can again be expressed as a square, so that we have:



$$= p \cdot u = 8.$$

Fig. 7.

Because  $u$  is the unit,  $p$  equals 8;  $p$  is also the sum of two squares that each consist of 4 units; and  $p \cdot u$  is a square: from this it should follow that we can construct a square from 8 units. But of course, from 8 units, no square can be formed (fig. 8):

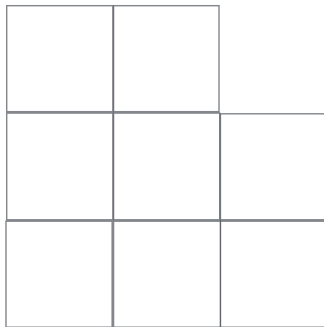


Fig. 8: a square with an area of 8 square units cannot be assembled from eight unit squares.

<sup>33</sup> The equation contains a product  $(m \cdot u) \cdot (n \cdot u)$  with two unknowns,  $m$  and  $n$ , because the slave's tendency to look for whole-number solutions, as suggested by the initial diagram (fig. 1), does not necessarily imply that he is looking (or must look) for the sum of two squares; the second figure, described by  $(m \cdot u) \cdot (n \cdot u)$ , could (in principle) be an oblong, i.e. a figure with sides  $m$  and  $n$  differing in length.

This exposition may seem contrived and roundabout, but the point is to bring out the basic assumptions that lead the slave in his calculations; only once these have been exposed, will we be able to understand why the boy enters into an *aporia*. In summary: the initial diagram drawn by Socrates, and consisting of four unit squares, suggests to the slave that he must look for a whole-number solution to the problem of doubling the square (fig. 1); the boy's first two attempts at calculating the sides of the square with an area of 8 feet confirm that, indeed, he does so (figs. 2, 3, 4). Soon, after having performed only a few calculations, the slave runs into an *aporia*: the side of the 8-foot square has no unit  $u$  in common with the smaller, initial square that should be doubled (fig. 8). This means that the slave's *aporia* centers on the irrational number  $2\sqrt{2}$ . Since  $2\sqrt{2}$  is only a multiple of  $\sqrt{2}$ —i.e. twice  $\sqrt{2}$ —and for its properties depends entirely on  $\sqrt{2}$ , we will, for now, continue by focusing on some characteristics of  $\sqrt{2}$ .

### **1.3: Locating $\sqrt{2}$ on the Number Line**

Several historians of mathematics have pointed out that Plato's thoughts about numbers, at least until the *Theaetetus*, are characteristic of what can be called "the metrical tradition."<sup>34</sup> The lack of distinction between number and magnitude, as at least is witnessed by the calculations of the slave in the *Meno*, certainly places the boy in that tradition. His ready acceptance of the foot as a unit, instead of an abstract unit, further strengthens the point: the unit equally represents 1, a number, and the foot, a magnitude.<sup>35</sup> The metrical tradition almost certainly evolved from applied mathematics, likely from land measurement;<sup>36</sup> from this connection with measurement, and the use of the foot as a concrete unit, it could follow that the problem regarding  $\sqrt{2}$  is one of "immeasurability." Numbers that can be measured against the foot, then, would be rational and measurable numbers; numbers which cannot be measured against the foot, would be irrational and immeasurable. The notion of immeasurability will be discussed below, using a concept from current-day

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<sup>34</sup> Knorr 1975, pp. 25-26, 90-91. On Plato's development away from the metrical tradition as from the *Theaetetus*, see Knorr 1975, pp. 91-92. Fowler goes further than Knorr in calling ancient Greek mathematics as a whole "non-arithmetised," naming the *Meno* in particular as an example; see Fowler 1999, pp. 10, 366.

<sup>35</sup> Knorr, *op. cit.*, p. 90.

<sup>36</sup> Cf. Dunham 1991, pp. 2-3. The supposed connection of ancient Greek geometry (and of the "metrical tradition") to land measurement likely stems from Herodotus, *Histories* II, 109, in which the Egyptian practice of re-measuring land after a flood is discussed ("It seems to me that from this, the Greeks found out about geometry, *dokeei de moi entheuten geômetriê heuretheisa es tēn Hellada epanelthein*"). Fowler attempts to discredit Herodotus' opinion; see Fowler 1999, pp. 279-281. Whether or not the origins of Greek geometry lie in Egyptian land measurement, it is certain that the Greeks in general considered the Egyptians as the ultimate source of Greek mathematics; it is also certain that Egyptian mathematics evolved from entirely practical concerns, as is for example witnessed by the Papyrus Rhind (nowadays also known as the Ahmes Papyrus): see Heath 1921, pp. 120-128.

mathematics: the number line. But let us first look at how Greek mathematicians conceived of the irrationality of  $\sqrt{2}$ .

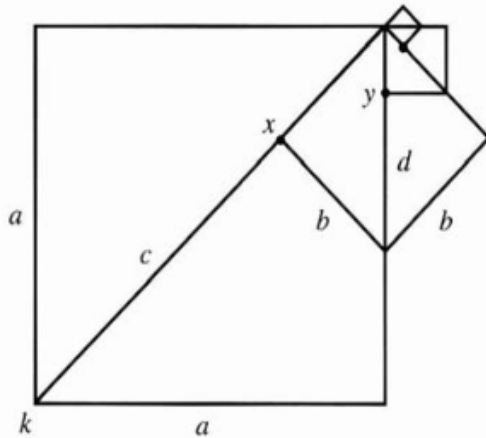


Fig. 9: an infinite-descent proof demonstrating the irrationality of  $\sqrt{2}$ .

The proof represented in fig. 9 works by infinite descent. The diagram shows several squares and their diagonals; we know that the ratio of each diagonal to the side of its square is  $\sqrt{2} : 1$ . Let us assume that  $\sqrt{2}$  is rational. Based on that assumption, and starting with the largest square, there must be a smallest number  $u$ , representable as a line segment of length  $u$ , such that the magnitude of the side as well as that of the diagonal are both multiples of  $u$ . Let the side of this square have magnitude  $a$ . Next, a point  $x$  is chosen on the diagonal such that the segment of the diagonal below  $x$  has magnitude  $c$ , and  $c = a$ . Let the segment of the diagonal above  $x$  have magnitude  $b$ : since the magnitude of the diagonal as a whole is considered a multiple of  $u$ , it can be deduced that  $b$  must also be a multiple of  $u$ . From this segment with magnitude  $b$ , a second square is constructed. Since the magnitude of the diagonal of this second square is  $\sqrt{2} \cdot b$ , and  $\sqrt{2}$  is assumed to be rational, the magnitude of the diagonal must again be a multiple of  $u$ . Next, a point  $y$  is chosen on this diagonal, such that  $d$  is a segment of the diagonal with magnitude  $b$ . From the segment above  $y$ , a third square is constructed, and by the same reasoning as before, the sides and diagonal of this square must be multiples of  $u$  too. Continuing the procedure, the fourth square is constructed; a fifth square can be formed, and a sixth square, and so on: at some point, a square will be constructed with sides of a magnitude smaller than  $u$ . This contradicts the initial assumption that  $a$  and  $c$  are multiples of a smallest line segment  $u$ , and that  $\sqrt{2}$  is rational. Therefore,  $\sqrt{2}$  must be irrational.<sup>37</sup>

The irrationality of  $\sqrt{2}$ , as announced earlier on, can also be demonstrated using the concept of the number line. Suppose that we were asked to indicate  $\sqrt{2}$  on the number line, using only a sheet of paper, a pencil, and a marked ruler. Let us posit familiarity with  $99/70$  as an approximation of  $\sqrt{2}$ . We would locate  $99/70$  on

<sup>37</sup> Cf. Knorr 1975, pp. 35-36, for a (too) brief treatment of this proof; Fowler 1999, pp. 33 and 300, for an alternative version of this proof.



the number line by drawing a horizontal from 0 to  $99/70$  with the aid of our pencil and ruler; this can be done easily if the ruler at hand were precise enough (let's assume it pre-scaled to a sufficient degree). We could then say: " $\sqrt{2}$  is just to the left of  $99/70$  on the number line." To that, of course, it may be objected that "to the left of  $99/70$ " is not an indication of  $\sqrt{2}$  *at all*:  $\sqrt{2}$  must be a fixed point on the number line, and it is this point which must be located *exactly*. Therefore, it would be useless to find a more precise approximation of  $\sqrt{2}$ , such as  $577/408$ ; nor would it help to say that  $\sqrt{2}$  lies between  $576/408$  and  $577/408$ , since there are infinitely many other numbers between those two fractions.<sup>38</sup> In short: as long as we continue to extend a line towards the left, e.g. from  $99/70$  to  $577/408$ , from  $577/480$  to  $665857/470832$ ,<sup>39</sup> and so on, we will never reach the fixed point  $\sqrt{2}$ .

The question is: why? Is it because  $\sqrt{2}$  is "immeasurable"? Perhaps  $\sqrt{2}$  could be called "immeasurable" in the sense that there exists no ruler, however finely scaled to whatever unit—say unit  $C$ —so that  $C$  goes a  $p$  number of times into a length of line  $AB$ , this length  $AB$  being a whole number or fraction, and into a line of length  $\sqrt{2}$  a  $q$  number of times,  $p$  and  $q$  being whole numbers. In other words: there is no unit  $C$  so that, when  $p \cdot C$  (being  $AB$ ) divided by  $q \cdot C$  (being  $\sqrt{2}$ ), and  $C$  is factored out,  $AB$  divided by  $\sqrt{2}$  equals  $p/q$ .<sup>40</sup> It is for this reason that, since no such ratio  $p/q$  can be found,  $\sqrt{2}$  is called an *irrational number*. This immediately leads to another mathematical characteristic of  $\sqrt{2}$ . Since there is no product  $q \cdot C$  which equals  $\sqrt{2}$  ( $q$  and  $C$  defined as above),  $\sqrt{2}$  cannot be expressed as a whole-number fraction,<sup>41</sup> however large its numerator and denominator; and any number which cannot be expressed as a whole-number fraction has an infinite number of decimals, which also happen to continue unpredictably (i.e. they contain *no repetend*).<sup>42</sup> Therefore,

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<sup>38</sup> Derbyshire 2004, p. 179.

<sup>39</sup> The large fraction  $665857/470832$  may seem a bit of a jump from  $577/480$ , but it is the approximation of  $\sqrt{2}$  which immediately follows  $577/408$  if one uses the computation algorithm  $((a_n + 1) = ((a_n/2) + (1/a_n)))$ , which in consecutive calculations, starting with a guess and then using the outcome as an input to generate a new outcome (e.g. uses  $3/2$  to generate  $17/12$ , which in turn generates  $577/408$ , and so on), yields ever more accurate approximations of  $\sqrt{2}$ .

<sup>40</sup> Compare Dunham 1991, pp. 8-9. This algebraic proof of the irrationality of  $\sqrt{2}$  basically repeats the argument of the geometrical proof represented in fig. 9.

<sup>41</sup> For it is always possible to rewrite  $q \cdot C$  as a fraction (with  $q$  and  $C$  as defined above). For another way to demonstrate that  $\sqrt{2}$  cannot be written as a fraction, see Hardy 1940, pp. 94-6, who discusses a proof mentioned by Aristotle, *An. Pr.*, I.23, 41a21-30.

<sup>42</sup> The fraction  $1/3$  can be written as  $0,333\dots$ , with the decimal 3 occurring an infinite number of times; therefore,  $1/3$  has 3 as a repetend. The fraction  $3226/555$  can be written as  $5.8144144144144\dots$  and has the repetend 144 after the first decimal 8 that occurs only once.  $\sqrt{2}$  cannot be written as a fraction, i.e. a common fraction such as in the two preceding examples (with one numerator, being a whole number, and one denominator, being a whole number); but  $\sqrt{2}$  *can* be represented as a nested (or continued) fraction, which goes on without end:

there is a sense in which we will *never know* the number  $\sqrt{2}$ , since we will never be able to list all its decimals (or, as far as the ancient Greeks were concerned, never be able to draw both the side of a square and its diagonal as multiples of a smallest common unit  $u$ , see fig. 9). All we will ever know, in a strictly numerical sense, are approximations of  $\sqrt{2}$ .<sup>43</sup>

The observations made so far may be helpful in taking the slave's *aporia* seriously—with regard to  $\sqrt{2}$ , especially in the compelling context of mathematical diagrams, it is pretty easy to get confused; and as we will see in the next paragraph, the way to dispel this confusion is not very evident.

#### **1.4: Constructing $\sqrt{2}$ on the Number Line**

We cannot indicate  $\sqrt{2}$  on the number line using only a sheet of paper, a pencil, and a straightedge. In other words: while we are able to draw lines of 1 unit in length, or of 2 units, or of any integer or whole-number fraction, we appear to be *unable* to draw a line of length  $\sqrt{2}$  (or any other irrational number), even with the help of a marked ruler. But it can be demonstrated that  $\sqrt{2}$ , even though irrational, is *not* “immeasurable.” The reason is that  $\sqrt{2}$  *can* be indicated on the number line, but the way in which this is to be done is *not very obvious*: this cannot be emphasized too strongly, for this lack of self-evidence is crucial to interpreting the mathematics passage in the *Meno*. Besides the concept of the number line, we need to add some plane geometry to our endeavour; also, we need the aid of an additional instrument, being a compass. The number line becomes an x-axis; an orthogonal is constructed on the point 0 on the x-axis, and this orthogonal will function as a y-axis. From this, a plane results, allowing us to construct a diagonal from the origin 0 (0, 0) to the point (1, 1) in the plane. One end of the compass is to be placed on this point, and the other on the origin. From (1, 1), with the help of the compass, a circle segment can be constructed: this cuts the x-axis at *exactly*  $\sqrt{2}$  (fig. 10). So it is possible to locate  $\sqrt{2}$

$$\sqrt{2} = 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}}$$

<sup>43</sup> Kanigel 1991, p. 59. As of June 28, 2016, a total of 10,000,000,000,000 decimals of  $\sqrt{2}$  had been calculated; for this, and examples of other irrationals, see <http://www.numberworld.org/y-cruncher/records.html>. Of course,  $\sqrt{2}$  can be defined as the limit of the sequence 1/1, 3/2, 7/5, 17/12, 41/29 etc.; see Derbyshire 2004, p. 16. This, however, presupposes the infinitesimal, of which the Greeks had no concept. For numerical approximations of  $\sqrt{2}$  in ancient Greek mathematics, see Heath 1921, pp. 91-93, 308.

on the number line—but the technique used, as forecasted, is not immediately obvious.<sup>44</sup>

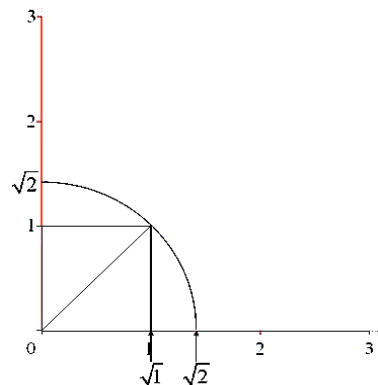


Fig. 10: the construction of  $\sqrt{2}$  on the number line.

With the help of the number line, it has now been established that  $\sqrt{2}$  is *constructible*. The ancient Greeks did not have the concept of the number line, and quite possibly did not even consider  $\sqrt{2}$  a number.<sup>45</sup> Rather than as a number, they viewed  $\sqrt{2}$  as a continuous magnitude, always mentioning it in relation to geometrical figures (usually as the *dunamis*, i.e. the side, of the 2-foot square, or as the diagonal, the *diametros*, of the 1-foot square). In any case, the fact that  $\sqrt{2}$  is constructible as a magnitude was established before Plato’s time. And because of this, the alleged “foundation crisis” was not the reason why Plato chose to center the conversation between Socrates and the slave on the irrationality of the side of the 8-foot square in the *Meno*. It is the *constructibility* of  $\sqrt{2}$  that Plato was interested in, and which Socrates and the slave will be seen to utilize in their solution to the problem of doubling the square—but, more importantly, Socrates will also be found to deliberately exploit the opaque, non-evident character of the constructibility of  $\sqrt{2}$ .

<sup>44</sup> The length of the diagonal in the *Meno* is not  $\sqrt{2}$  but  $2\sqrt{2}$ , as indicated; however, if  $\sqrt{2}$  is constructible, then so is  $2\sqrt{2}$ , or any multiple of  $\sqrt{2}$ . For the length of the diagonal of a square with sides of length  $n$  is  $n\sqrt{2}$ , which follows from the Pythagorean Theorem (let the diagonal be  $p$ ; then  $p^2 = n^2 + n^2$ , or  $p^2 = 2n^2$ ; therefore,  $p = n\sqrt{2}$ ).

<sup>45</sup> Aristotle says in the *Physics* that “there can be nothing between 2 and 1 (*ouden gar metaxu duados kai monados*, 227a31),” that is: there are no other numbers between 1 and 2, ruling out fractions and irrationals (such as  $\sqrt{2}$ , which according to current-day number theory is a number which lies between 1 and 2) as numbers. In the *Metaphysics*, Aristotle states that “number is commensurate, and one does not speak of the incommensurate as number (*ho gar arithmos summetros, kata mē summetrou de arithmos ou legetai*, 1021a5)”; with the “incommensurate” Aristotle means irrational numbers.

## Chapter 2: Socrates' Peculiar Employment of the Diagrams

### 2.1: The Slave's *Aporia*, II

As discussed in the previous chapter, the first square drawn by Socrates was divided into four unit squares (fig. 1); the task was to double it to an area of 8 square feet. When the slave suggested to extend the side of the original square by doubling it in length, Socrates transformed the smaller square to a 16-foot square (fig. 2), demonstrating that the slave was wrong. Socrates now again draws a 16-foot square (84D), this time not subdividing it into 16 unit squares, but into four quadrants only<sup>46</sup>—which will lead to an important result soon (fig 11):

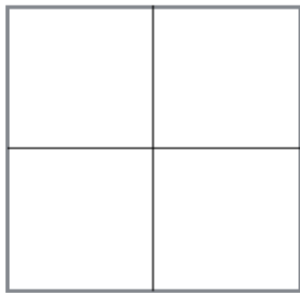


Fig. 11: the second 16-foot square, subdivided by Socrates into four quadrants.

Socrates asks how much larger this square is compared to the 4-foot square: the slave answers, correctly, that it is four times larger (84E)—all he needs to do is count the four quadrants. Socrates repeats that the figure they are searching for—the 8-foot square—is two instead of four times larger; the slave consents. *Without* first telling or asking the slave how much smaller the 8-foot square would be compared to the 16-foot square (i.e. half), Socrates draws an oblique line in the lower left quadrant, from top left to bottom right (fig. 12):

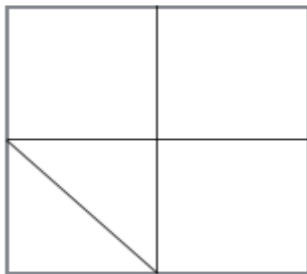


Fig. 12: Socrates draws an oblique line in the lower left quadrant.

He prompts the slave if “this line” (*hautê grammê*, 84E8)—one should duly note that Socrates does *not* mention the word ‘diagonal’ at this stage—cuts the lower left

<sup>46</sup> Cf. Brown 1967, p. 61.

quadrant in half: the slave agrees. Socrates then draws three more oblique lines in the other quadrants, each time turning the line by 90 degrees (fig. 13):

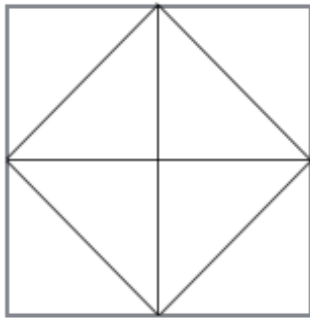


Fig. 13: in each quadrant, Socrates draws an oblique line from corner to corner, turning each line at an angle of 90 degrees.

Again without using the word ‘diagonal’, Socrates asks if there are now four such lines (*tettares hautai grammai isai*, 85A4-5); while pointing at the figure, he inquires whether these “contain this area” (*periechousai touti to chôrion*, 85A5): here, one should observe that Socrates does not call the resulting figure—a tilted square—a *square*. The boy confirms that the oblique lines contain the area at which Socrates just pointed. Socrates then asks how large (*pêlikon*, 85A7) the area is. The slave replies that he does not understand (*ou manthanô*, 85A9). This lack of understanding should—however difficult this may be for us—be taken seriously; the reason for this will be explained in the next paragraphs. For now, it can at least be surmised that the answer of the slave indicates that, although Socrates has now completed the final diagram, the boy’s *aporia* has not evaporated.

## **2.2: The Diagonal as the Solution to the Problem of “Doubling the Square”**

Socrates informs if in each of the quadrants “that line” (*hekastê hê grammê*, 85A11)—again, he refrains from using the word ‘diagonal’—cuts the quadrant in half. The slave confirms. Socrates again points at the diagram, and asks how many half-squares are contained within the tilted area, again not calling the area a square, but simply “this [figure]” (*toutôi*, 85A13), nor naming the half-squares, merely calling them “[figures] of such size” (*posa [...] têlikauta*, 85A13), whereas he could have easily called them “isosceles triangles” (*isoskelês*), for example.<sup>47</sup> The slave counts the half-squares and replies, that there are four (*tettara*, 85A14). Socrates then informs how many there are in the first quadrant; the slave, of course, says two (*duo*, 85A16). The slave is asked, how much four is with respect to two (*ta de tettara toin duoin ti estin*, 85A17); the slave answers that four is double (*diplasia*, 85A18). At *that* very moment, the sudden realization dawns on the slave that the tilted area

<sup>47</sup> Plato uses the geometrical expression ‘isosceles’ in *Timaeus* 54B6, for example. For other examples in Plato, see Ast 1835, vol. II, p. 106.

must be twice the first quadrant, and has to be the square he had been looking for all along. When asked to calculate the surface of the tilted area, the slave provides the right answer: 8 feet (*oktôpoun*, 85B2).

The boy's identification of the tilted area is not where the problem of doubling the square ends—Socrates wants to point out one more thing. He asks through what line (*apo poias grammês*, 85B3) the area of 8 feet was generated (*gignetai*, 85B1). The slave points at the oblique line in the lower left quadrant and, not knowing the name of the line, answers: "From that one" (*apo tautês*, 85B4). Socrates wants to hear this again: "From the line drawn from corner to corner in the figure measuring 4 feet?" (*apo tês ek gônias eis gônian teinousês tou tetrapodos*, 85B5-6)? The slave confirms, and only then, towards the very end of the mathematical exercise, Socrates says that the line is named "diagonal" (*diametros onoma*, 85B9).

The slave's calculation of the area of the 8-foot square requires proof, as Socrates indicates by mentioning the necessity of additional *akribeia* (*akribôs*, 85C13).<sup>48</sup> For one thing, it must be demonstrated that the four half-squares indeed form a square (no matter how evident this may seem to us): the four half-squares could, in principle, form an oblong rectangle or a variety of parallelograms rather than a square. In other words, what stands in need of demonstration is that the four sides of the tilted area are equal; also, and most importantly, the result (in being a theorem—which, in short, can be formulated as: all squares can be doubled along their diagonals) needs to be demonstrated for *every* square, and not just for the 4-foot square; therefore, a rigid proof would involve, among other things, Pythagoras' Theorem.<sup>49</sup> However, verification in the *Meno* stops short once the word 'diagonal' has been mentioned, and nowhere in the dialogue do we find rigid proof for the conclusion that the tilted area, indeed, is a square twice the size of the initial square. Such *akribeia*, as far as Socrates is concerned, is for another moment—as it appears, rigid proof is not relevant to what he has been trying to bring across.

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<sup>48</sup> Cf. Moravcsik 1994, p. 126. What in addition requires proof is the implied assertion that the diagonal bisects the square into two exactly equal parts.

<sup>49</sup> With reference to the diagram on p. 298 in Heath 1921, let the base of the initial square be AD, the orthogonal side on the left be AB, and the diagonal of the initial square be BD. Then  $AD^2 + AB^2 = BD^2$  is true. Since the lengths of AD and AB are identical (i.e. 2),  $AD^2 + AB^2 = AD^2 + AD^2 = (2 \cdot AD^2) = BD^2$  also holds. Therefore,  $\sqrt{BD^2} = BD = \sqrt{(2 \cdot AD^2)} = (\sqrt{2} \cdot AD)$ . Also, since  $AD = AB$ , the surface area of the initial square is  $(AD \cdot AB) = (AD \cdot AD) = AD^2$ . Let BL be the orthogonal side to BD in the tilted square DBLM. Then the surface area of the tilted square can be calculated by multiplying BL and BD. Since BL and BD are equal in length,  $(BL \cdot BD) = (BD \cdot BD) = BD^2$ . We now substitute  $(\sqrt{2} \cdot AD)$  for BD, which gives us  $(\sqrt{2} \cdot AD)^2 = 2AD^2$ . Since the surface area of the initial square was established as  $AD^2$ , this proves that the tilted square is twice the initial square.

### **2.3: The Element of Surprise**

Similar to the construction of  $\sqrt{2}$  on the number line (see §1.4), the construction of the 8-foot square through the diagonal is not obvious, but remains an opaque feature throughout almost the entire conversation between Socrates and the slave, and finally comes as a surprise—Fowler remarks on this as “Socrates [...] conjuring a clever figure out of thin air.”<sup>50</sup> Most readers of the *Meno* will tend to underestimate this surprise element, probably because they never set themselves the task of solving the problem of doubling the square *before* reading the solution, and because the solution, once provided, is so handsome, and so intuitively easy to comprehend. But the solution is simple only with the benefit of hindsight. While writing this thesis, I asked more than 40 people—friends, relatives, and colleagues, all of them well-educated adults—to double a square I had jotted down on a napkin, tablecloth, or piece of paper. I told them to work under certain assumptions (without telling them, these were the same as in the *Meno*: in short, I told them to work from the 4-foot square). Nearly *none* were able to find the answer, and almost everybody made the same mistakes as the slave did (in particular the first, that of doubling the side of the smaller square, 83A and fig. 2), except two people who had read the *Meno*, and remembered the solution—but the fact that they remembered it probably means that the diagram stuck with them because it is so surprising (and strikingly shaped).

Why is the solution surprising? For several reasons. For instance, all diagrams drawn before the final construction were “stacked” diagrams (figs. 1-4, 11): they resulted from subdividing and dissolving figures into unit squares, and reassembling those into new figures—a process similar to playing with Lego bricks (but then, only *square* Lego bricks).<sup>51</sup> More to the point: those first diagrams all contained straight lines, i.e. horizontals and verticals, and because of that, a bias was created towards identifying, handling and creating shapes that are contained within straight lines, i.e. squares and rectangles, themselves positioned at straight or parallel angles to the other elements of the diagrams. The final diagram, however, contains *oblique* parts: the diagonals, and a tilted figure, the 8-foot square. As said, Socrates enabled the slave to use the half-squares as a unit for counting, but the boy did not realize what he was doing; to see the half-squares as such would have gone against the prevailing bias, because they were partly based on the oblique diagonals, and shaped, not as squares, but as triangles; but the only units, as far as the boy is concerned, are the stackable, straight-angled 1-foot squares.

Several more things can be said about the surprising nature of the final diagram. As indicated before, Socrates returned to the 16-foot square, and *covertly suggested* that what the slave could attempt, is not to double the 4-foot square, but cut the 16-foot square in half (84E-85A). But, as discussed, a bias had been induced towards stacking unit squares, and towards identifying straight lines, i.e. horizontals and verticals, and objects contained within such lines, by Socrates’ initial diagram of the 4-foot square (fig. 1). Cutting the 16-foot square in half is naturally conceived of as cutting the 16-foot square into two equal parts through the middle, either

<sup>50</sup> Fowler 1999, p. 367.

<sup>51</sup> In the same vein, Klein speaks of the “assemblage” of units; see Klein 1965, p. 100.

vertically or horizontally. But this leads to either of the following two figures (figs. 14 and 15):



*Fig. 14: the 16-foot square cut in half across the middle.*



*Fig. 15: the 16-foot square cut in half along its length.*

These figures are oblongs and not squares, and as we know from fig. 8, they cannot be dissolved into unit squares and reassembled into a square composed of 8 units. Most people would briefly consider (but quickly reject) cutting the 16-foot square along the *full* diagonal (from the lower right to the upper left, or from the lower left to the upper right), without giving this idea any further and more refined consideration. Cutting the 16-foot square in half seems, at first sight, to present no opportunity, and the slave certainly doesn't catch on to the occasion: but as Socrates shows, it can be done by cutting the four quadrants in half along the diagonals, turning each diagonal by 90 degrees. But, again because of the prevailing bias, this doesn't occur naturally to people as "cutting a square in half."

#### **2.4: Diagrammatic Construction as a Case of *Anamnésis***

The final diagram in the *Meno* was found to have several surprising aspects. A similar phenomenon can be gathered from the *Theaetetus*: there, the surprising nature of particular diagrams (*schêmati*, 148A5) is underscored by the verb *eiserchomai*. Theaetetus relates how, one day, the mathematician Theodorus was drawing diagrams in order to demonstrate that the sides or "roots" (*dunameôn*, 147D3) of several squares, such as those measuring 3 and 5 square feet (*tripodos*



[...] *kai pentepodos*, 147D4), are incommensurable with the unit of the foot (*podiaiai*, 147D5); in other words, the point was to show that the sidelengths of said figures are quantifiable only as irrational square roots. Theaetetus, witnessing Theodorus at work, decided to make an attempt at “embracing” all irrational square roots of whole numbers under one name (*peirathênai sullabein eis hen*, 147D9-E1). He soon realized that integers such as 3 and 5, the square roots of which are irrational, can only be resolved into factors by multiplying “either a larger [whole number] by a less[er], or a less[er whole number] by a larger” (*ê pleiôn elattonakis ê elattôn pleonakis*, 148A2-3): when constructed from the unit square, such numbers can only be geometrically represented by oblong rectangles (rectangles with unequal sides, compare figs. 14 and 15)—hence, a particular feature of these diagrams, namely that they are oblong (*promêkê*, 148A5), suddenly *jumped at* him (*eisêlthe*, 147C8 and D8), as Theaetetus phrases it. There was no need to determine every single *dunamis* by going through all the integers one by one and extracting their square roots (which is impossible, since there are infinitely many integers, and infinitely many irrational square roots of whole numbers):<sup>52</sup> all that was necessary was to allow the oblong shape to emerge, and have it “jump at one.”

In the *Meno*, direct parallels to *eiserchomai* are the verbs *tunchanein* (to hit upon, encounter; *tunchaneis*, 86B2),<sup>53</sup> and *paragignomai* (to advance, come towards; *paragignomenê*, 99E8).<sup>54</sup> An even more significant verb, used in connection with the surprising occurrence of a particular, previously overlooked trait of the final diagram in the *Meno*—of this trait suddenly emerging, and “jumping at” the slave—is the verb *analambanein*, ‘to relevate,’ which Socrates associates directly with *anamnêsis*: “And is this relevating of knowledge by himself in himself not [the same as] remembrance?” (*to de analambanein auton en hautôi epistêmên ouk anamimnêskesthai estin*, 85D7-8)? In the sentence quoted just now, we find a prepositional prefix, *ana-* (in *analambanein* and—crucially—in *anamimnêskesthai*), as well as the preposition *en*; these can be considered as equivalents of (or at least as relatable to) the prefix *eis-* (from *eiserchomai*) in the *Theaetetus*, and should be understood in relation to the diagrams in the *Meno*, as well as to the slave’s *anamnêsis*. They signify a movement from *inside* something, *towards* something else: that is, they relate to the sudden and unexpected “leaping to the eye” of certain, previously overlooked features of diagrams, as much as they do to the slave’s sudden awareness that the tilted area drawn by Socrates is the 8-foot square. More such prepositions and prefixes, interchangeably related by Socrates to the diagrams as well as to the slave’s *anamnêsis*, abound in the *Meno*: *prosana-* (‘additional to,’ *prosanaplêrôsaimeth’*, 84D12); *en-* (*entos*, 85A11; *enestin*, 85A13); *en* (‘in,’ 85A13 and 15); *apo* (‘from,’ 85B3-5); *ek* (‘from within,’ 85B5); *eis* (‘through,’ 85B5), and again *apo* (85B9)—all these referring to the diagram—and *ape-* (*apekrinato*, 85B14), *en* and *en-* (*enêsan*, 85C5; *eneisin*, 85C8; *en*, 85D7), *ana-* and *ane-* (*anakekinêntai*, 85C11; *anerêsetai*, 85C12; *analabôn*, 85D4; *analambanein*, 85D7;

<sup>52</sup> Their number, as Theaetetus himself notes, is “unbounded” (*apeiroi*, 147D8).

<sup>53</sup> Also *entuchois*, 80D9.

<sup>54</sup> Also *paragignêtai* (100A1); *paragignomenê* (100B3); *paragignetai* (100B6).

*anamimnêskesthai*, 85D8); and *ex* ('from,' 85D4)—all those referring to the slave's remembrance.

This intricate patchwork of prepositions and prefixes indicates that what happens in and develops from the diagrams, simultaneously happens with and within the slave; what happens in the geometrical constructions, is equally applicable to the slave's remembrance; the constructions run parallel to the *anamnêsis*, or rather, they seem to *be* the *anamnêsis*. But from the things said previously about Socrates' treatment of the diagrams, it has become clear that this construction does not take place in a straightforward manner. In synopsis: Socrates, after having asked the slave to quantify the side of the 8-foot square, subdivides the initial diagram into four squares of 1 square foot each, and thus induces a tendency in the boy to look for a whole-number solution to the mathematical problem. This leads the slave to presuppose a common unit between the side of the 4-foot square and that of the 8-foot square to be constructed, which causes an *aporia* in the boy, revolving around the so-called "immeasurability" of  $2\sqrt{2}$ . Silently forcing the slave to remain within the confinements of the strict rules of construction in ancient Greek geometry, Socrates creates a bias towards identifying, handling, and creating shapes contained within straight lines, thus exploiting the opaque character of the constructibility of  $2\sqrt{2}$ . Finally, Socrates avoids mentioning the names of certain elements of the final diagram.

Because what happens in the diagrams is charged with particular opacities and thought tendencies, and since what occurs in the diagrams transpires in the slave's *anamnêsis* too, this *anamnêsis* must, somehow, be simultaneously suffused with similar tendencies and opacities. This may be revealing as to what is "mysterious" about the mathematical passage in the *Meno*; therefore, the next chapter will further consider how Socrates' treatment of the problem of doubling the square is abnormal, and an attempt will be made to connect Socrates' treatment of the mathematical problem with his allusions to mystery. Crucially, these allusions will be shown to be tied up with a particular paradox mentioned in the dialogue—the "Learner's Paradox," or "Meno's Paradox"—which, ultimately, will be dissolved by the slave's utterance of *alêtheis doxai* or true opinions, in finding a "whole" (the 8-foot square) before one of its "parts" (its side, the diagonal). In that way, this thesis will finally deal with the task of showing that the geometrical problem is not treated by Plato as one mere example of *anamnêsis* out of many, and that, vice versa, Socrates' thoughts on *anamnêsis* amount to more than sheer "mystery talk." To be precise: it will be demonstrated that the "mystery" in the *Meno* effuses from *within the mathematics itself*, that is: the mathematics as this *includes* Socrates peculiar treatment of the diagram.

## Chapter 3: Meno's Paradox and the Slave's Manifestation of True Opinions

### 3.1: The Slave's *Aporia*, III

After having discussed the final diagram with the slave, Socrates turns to Meno, in order to assess the slave's achievements as a case of *anamnêsis* (85B *ff.*). Socrates' wording—especially in contrasting the slave's *nun bios*, his current life as a member of Meno's household (*tôi nun biôi*, 85D14),<sup>55</sup> with “some other time” (*allôi tini chronôi*, 86A1), “when [the slave] was not [yet] a human being” (*ho chronos, hot'ouk ên anthrôpos*, 86A4-5)—indicates that this happens in light of his earlier citation from Pindar (81B *ff.*). In this quote, the poet relates how men become glorious kings as a consequence of Persephone returning souls to the living; this “becoming glorious” is interpreted by Socrates as *anamnêsis*. The slave's *anamnêsis* therefore is not “simply” remembrance, but indicates an involvement of the soul, which so far has not been discussed in the present thesis. Yet it is crucial to investigate the significance of the soul in the mathematics passage, since Socrates' exposition of the Pindar quote in 85B-86C, more than the citation itself, have given rise to countless “mysterious” interpretations of Plato's view on *anamnêsis* that completely ignore the mathematics in the very way in which Socrates employs geometry in the *Meno*.

In order to understand what share the soul has in the slave's *anamnêsis*, it is not only necessary to remember that the mathematics exercise serves as an elucidation of the Pindar quote, but also that, in turn, this quote is Socrates' reply to what has become known as “the Learner's Paradox” or “Meno's Paradox.” This paradox came about as follows. One of Meno's answers to the question “What is virtue?” was: “Virtue is the ability to procure things honourable” (*egô touto legô aretên, epithumounta tôn kalôn dunaton einai porizesthai*, 77B5-6). In reply, Socrates asked if the procuring must be done righteously and devoutly (*dikaiôs kai hosiôs*, 78D4), to which Meno responded that such has to be the case (78E3); hence, justice and wisdom (*tên dikaiousunên kai sôphrosunên*, 79A 4-5) both became an example of virtue, or a “part of virtue” (*morion aretês*, 79A4). Socrates concluded that Meno claimed to know what certain parts of virtue are (namely justice and wisdom), without yet knowing what virtue *itself* is: “But now, dear sir, you should not, while still searching for what virtue as a whole is, explain [virtue] by replying through its parts” (*Mê toinun, ô ariste, mêde su eti zêtoumenês aretês holês ho ti estin oiou dia tôn tautês moriôn apokrinomenos dêlôsein autên hotôion*, 79D7-9). Meno's Paradox revolves around the relationship between whole and parts; and as we will see in the next paragraphs, this relationship is crucial in interpreting the mathematics passage.

Socrates' remark causes Meno to enter into an *aporia* with respect to the question *ti esti aretê* (“now I have nothing at all to say about what it is,” *nun de oud' ho ti esti to parapan echô eipein*, 80B4-5):<sup>56</sup> he feels as if his soul and tongue are stunned (*egôge kai tên psuchên kai to stoma narkô*, 80A10-B1). Meno accuses Socrates of bewitching and intoxicating him (*goêteueis me kai pharmatteis*, 80A3),

<sup>55</sup> Also *tôi nun biôi*, 85E11.

<sup>56</sup> Also *aporeis*, 80A2; *aporein*, 80A2; *aporias*, 80A4.

and compares Socrates to an electric ray (*homoiotatos einai to te eidos kai talla tautêi têi plateiai narkêi têi thalattiai*, 80A 6-7). In turn, Socrates says that he causes *aporiai* in others because he himself, more than anyone else, is in doubt about what virtue is (*pantos mallon autos aporôn houtôs kai tous allous poiô aporein*, 80C9-D1).<sup>57</sup> Instead of feeling even more despondent, Meno replies sarcastically: “Then in what way will you search, dear Socrates, for that of which you do not at all know what it is?” (*Kai tina tropon zêtêseis, ô Sôkrates, touto, ho mê oistha to parapan ho ti esti*, 80D6-7). Meno tries to rub this in deeper: “What sort of thing, then, among those that you do not know, will you search for and place before us? Or even if, when you are lucky, you will hit upon something, how will you know that it is the thing that you did not know?” (*poion gar hôn ouk oistha prothemenos zêtêseis? ê ei kai hoti malista entuchois autôi, pôs eisêi hoti touto estin, ho su ouk êidêstha?*, 80D7-10).

Socrates calls this a “highly contentious argument” (*eristikon logon*, 80E1), and rephrases Meno’s “contention” into the Learner’s Paradox: “That a human is not to search for what he knows or does not know—for he either knows it, in which case it is not necessary to search for it; or does not know it, in which case he does not know what it is that he would be searching for” (*ouk ara esti zêtein anthrôpoi oute ho oiden oute ho mê oiden [...] oute gar an ho ge oide zêtoi: oide gar, kai ouden dei tôi ge toioutô zêtêseôs: oute ho mê oiden: oute gar oiden ho ti zêtêsei*, 80E1-5). Since Socrates answers this paradox by citing Pindar, and illustrates his interpretation of the citation by confronting the slave with the problem of doubling the square, it follows that the slave’s *aporia* should be understood in light of Meno’s Paradox. Meno seems hopelessly stuck with respect to the question *ti esti aretê*, but he attempts to dispel the gravity of his *aporia* by pointing out the paradox. As we have seen, the slave appears to be hopelessly stuck too, with respect to the irrationality of  $2\sqrt{2}$ . But this does not describe the full extent of the comparison between the *aporiai* of Meno and the slave. Socrates, as we have seen, states that Meno claims knowledge of certain “parts” of virtue, without knowing what virtue itself is as a whole (*kata holou*, 77A7). The situation of the slave will be similar, albeit in an *inverted* sense. The slave utters what Socrates will call *alêtheis doxai*: without claiming to know what a certain “part” is (the side of the 8-foot square, i.e. the diagonal), and in spite of not knowing that “part,” he will encounter the *zêtoumenon* by first being confronted with a “whole” (of which the side is a “part”)—the 8-foot square. Precisely that event, as Socrates wants to point out, answers Meno’s Paradox.

Before discussing how the utterance of *alêtheis doxai* sheds light on the role of the slave’s soul, we will first look into the meaning of *alêthês doxa*; this will happen with respect to how the mathematics develops in a peculiar “orderly” fashion.

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<sup>57</sup> Also *aporein*, 80C9-10.

### **3.2: (Ouk) Eidenai Versus Doxa and Epistêmê**

Many commentaries on the *Meno* argue that the conversation between Socrates and the slave exhibits the pattern of mathematical proof. Socrates, according to that interpretation, takes the slave through a proof in steps, starting from “logically primitive propositions,”<sup>58</sup> and “axiomatic truths,”<sup>59</sup> via the discovery of some “logical relations of concepts” through “the rules of inference,”<sup>60</sup> onwards to the theorem,<sup>61</sup> all this happening, of course, “in the right order.”<sup>62</sup> But this interpretation is problematic, even though certain pronouncements by Socrates can be considered “primitive propositions” (e.g. that the sides of a square are equal in length, 83C1-2), and the slave indeed makes logical inferences (e.g. that the tilted area measures 8 square feet, 85B2). Without denying the logical nature of certain events in the mathematical passage, it is my opinion that no interpretation of *Meno* 84B-85B can afford to ignore “the mystery in the maths.” If this passage is interpreted as following the pattern of mathematical proof—without accounting for the possibility that the mathematics, in the very way in which it takes place in the *Meno*, offers crucial clues as to what is “mysterious” about *anamnêsis*—the exegesis will end up off track.

This kind of interpretation will either entirely ignore the diagrams, or treat them as mere illustrations of what is essentially an arithmetical procedure, or even qualify the diagrams as dispensable.<sup>63</sup> But the diagrams are of crucial importance to the entire dialogue, since by Socrates’ employment of their opaque features, they form the indispensable core of his reply to Meno’s Paradox. Also, the interpretation of the mathematical passage in terms of proof may interpret the slave’s *anamnêsis* either strictly as a process of logical inference, reducing the part played by the dream-like state of the slave and his soul to mere embellishments; or it will explain away the “mysterious” character of the slave’s *anamnêsis* by reference to a religious belief held by Plato, about which we have no direct statements from Plato himself,

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<sup>58</sup> Moravcsik 1994, p. 119; Vlastos 1994, p. 96.

<sup>59</sup> Vlastos 1994, p. 99.

<sup>60</sup> Vlastos 1994, pp. 92, 97, 99; Scott 2009, pp. 102, 105.

<sup>61</sup> Knorr 1975, p. 90.

<sup>62</sup> Scott 2009, p. 102.

<sup>63</sup> Vlastos claimed that if the diagrams were deleted from the *Meno* entirely, and the geometrical problem were replaced by a strictly arithmetical one, this would not make “any material change” to the meaning of the dialogue (Vlastos 1994, p. 90). Apart from the fact that Vlastos apparently was not aware of the crucial role of diagrams throughout ancient Greek mathematics, he clearly missed the significance of the final diagram in the *Meno* in answering Meno’s Paradox. An interesting side-note here is the following. Vlastos asserts that the content of the *Meno* would have remained the same if, instead of the slave going through a geometrical problem, Plato would have staged a blind person solving an arithmetical problem (Vlastos 1994, p. 95). This would hold if the implicit suggestion that blind people cannot solve geometrical problems were true. There is, however, a famous proof of the Pythagorean Theorem by a blind girl, Emma Coolidge, which makes use of diagrams: see Kaplan & Kaplan 2011, pp. 103-107.

and which, because his was apparently an unavowed “personal faith,”<sup>64</sup> is supposed to have no bearing on the mathematical passage in such a way that it would undermine the “orderliness” of the geometrical process transpiring independently from that faith in 84B-85B.<sup>65</sup>

That the mathematical passage follows an orderly path is usually gathered from the adverb *ephexês* and the impersonal verb *dei* in 82E,<sup>66</sup> where Socrates remarks to Meno: “And now watch his remembering next, as one should be remembering” (*Theô dê auton anamimnêskomenon ephexês, hês dei anamimnêskesthai*, 82E13-14). Since *ephexês* is often used to describe an orderly succession of things or events, such as the succession of natural numbers in counting (1, 2, 3, 4, 5 etc.),<sup>67</sup> the adverb is taken to signify the orderly succession of first principles,<sup>68</sup> inferences made from those, and certain analyses (e.g. the solving of unknown variables); *dei* is considered to denote the logical necessity involved in this step-by-step process leading towards the solution of the geometrical problem,<sup>69</sup> as if one gradually moves from darkness to light. But first of all, there is no such gradual progress, as the slave remains in darkness, and perplexed, until the meaning of the final diagram (fig. 13) strikes him unsuspectedly and at once. Furthermore, in what is *ephexês*, i.e. in what follows next in the passage of the dialogue where the adverb is used, we do not witness a step towards the light, but yet another mistake by the slave (his statement that the side of the 8-foot square is 3 feet, 83E and fig. 4), followed by the ineluctible *aporia*. Rather than expressing the order and necessity of rigid logical reasoning, *ephexês* and *dei* signify that the slave is *bound* to make mistakes, and will *have to* end up in an *aporia*.

The fact that *ephexês* does not imply a gradual ascent from darkness to light, but rather a sudden turn of events, i.e. an abrupt shift from one stage, sharply marked off from a next, can be followed in Plato’s Greek. In the quotations discussed in §3.1, and taken from 77B-80D, forms of (*ouk*) *eidenai* were used for ‘knowing’ and ‘not knowing’, while neither Meno nor Socrates used *doxa* and *epistêmê* (or words derived from those nouns) anywhere in these passages. This use of (*ouk*) *eidenai* to the exclusion of *doxa* and *epistêmê* is continued in the first part of the mathematical passage, up until, and including, 84C. This is significant, for the use of *doxa* will not occur until immediately before the final diagram is drawn (fig. 13), in 84D, and there refers to the *alêtheis doxai* manifested by the slave *after* the final diagram has been

<sup>64</sup> Vlastos 1994, pp. 103-105.

<sup>65</sup> Vlastos 1994, pp. 103-105.

<sup>66</sup> Scott 2009, p. 101.

<sup>67</sup> Cf. Aristotle, *Physics* V.iii, 226b35-227a9.

<sup>68</sup> Klein 1965, p. 173.

<sup>69</sup> According to Klein, one does not witness a mathematical proof in *Meno* 84B-85B, since the slave, as Klein notes, does “not ascend to those more comprehensive suppositions from which he would have been able to demonstrate, step by step, in a strictly regulated and transparent manner, the construction presented to him by Socrates.” Even so, the mathematical passage neither represents “an initial ‘analytical’ exercise in mathematics,” as Klein then moves on to claim, since the point of the mathematical passage is not the mathematics as such, but answering Meno’s Paradox through the employment of certain misleading or otherwise opaque features of the diagrams. See Klein 1965, pp. 176-177.

drawn, while *epistêmê* will surface only as from 85C, and there refers to the slave's future knowledge, which, as said before, does not materialize in the *Meno*. The use of (*ouk*) *eidenai* to the exclusion of *doxa* and *epistêmê* is best illustrated by the two interjections in which Socrates turns his attention away from the boy, and remarks to Meno on the slave's development (82E3-14 and 84A5-D3).

The first interjection occurs after the slave has given Socrates his initial wrong answer (that the side of the 8-foot square must be double that of the original square, i.e. 4 feet; 82E, fig. 2), and right before Socrates will draw the diagram of the 16-foot square (fig. 3). The adverb *nun* (82E5) describes the timespan between these two moments: "He now supposes he knows of what kind the line is from which one begets the 8-foot square" (*kai nun houtos oietai eidenai, hopoia estin aph' hês to oktôpoun chôrion genêsetai*, 82E5-7). As in 77B-80D, *doxa* is avoided (one would consider the use of *pseudês doxa* appropriate), and instead, *eidenai* is used. The next interjection takes place right after Socrates has demonstrated to the slave that the 3-foot line generates a 9-foot square (fig. 4), and the boy, when asked what line generates the 8-foot square, exclaims: "But by Zeus, Socrates, I don't know" (*Alla ma ton Dia, ô Sôkrates, egôge ouk oida*, 84A3-4); this is characterized by Socrates as an *aporia*, as already noted above in §1.1 (*aporein*, 84A10 and 11). Immediately after this exclamation, Socrates asks if Meno has seen how the boy is already moving along in remembering (*êdê badizôn hode tou anamimnêskesthai*, 84A5-6)—the adverb *êdê* here performing the same function as *nun* just now, marking off a stage in the slave's development, a "moving along" (*badizôn*), which can hardly be considered progress, but rather indicates that the boy's predicament is intensifying: besides not knowing, he no longer believes he knows (*kai hôsper ouk oiden, oud' oietai eidenai* (84A11-B1).

The use of (*ouk*) *eidenai* to the exclusion of *doxa* and *epistêmê* is continued throughout 84B and C: at first the slave did not know (*êidei ou*, 84A7), and he does not know now (*oude oiden*, 84A8), but believed he knew (*eidenai*, 84A9); he answered as if he knew (*hês eidôs*, 84A9-10), whereas now he does not know (*ouk oiden*, 84A11) nor assumes he knows (*eidenai*, 84B1); he is better off now, not knowing (*ouk êidei*, 84B4), for he will continue searching while not knowing (*ouk eidôs*, 84B11), not assuming that he knows (*eidenai*, 84C5) while not knowing (*ouk eidôs*, 84C5-6), because after having entered into an *aporia* and being faced with not knowing (*mê eidenai*, 84C6-7), he will be spurred on by a desire to know (*eidenai*, 84C7). At this stage in the dialogue, the slave does not have opinions, not even false ones—he simply does not know (*ouk eidôs*). Only once the final diagram has been drawn, will he begin to utter true opinions, *alêtheis doxai*. That event, even though logical inferences play their part in it, is not the result of a process that exhibits the pattern of mathematical proof: in its dependency on particular opaque features and surprising aspects of the diagram, it marks an abrupt change instead.

### **3.3: The Slave and His Soul – A Game of Musical Chairs**

It could be argued that there is "progress" in *Meno* 82A-86B, not as a gradual ascent from darkness to light, or from first principles to a theorem, but as "progress"

involving the following necessary (but not *logically* necessary) steps: (1) believing one knows and giving wrong answers, i.e. not knowing (*ouk eidenai*); (2) entering into an *aporia*; (3) besides not knowing, no longer believing one knows. These “progressive” steps do not represent “intermediary results” in the mathematical sense: they do not mark progress along the road towards proof, in which one can “perceive each successive logical relationship,”<sup>70</sup> i.e. the logically necessary relationships between primitive statements, intermediate results, and the theorem. They are necessary in a different sense, to be understood within the context of Meno’s Paradox: only after giving wrong answers, entering into an *aporia*, and besides not knowing, no longer believing he knows, the slave is forced into a situation in which, while not knowing, he is to search for what he does not know, the *zêtoumenon*. The step in the slave’s progress where the *zêtoumenon* is about to be found, marks a sudden sweep towards a different stage, which is characterized by the slave manifesting *alêtheis doxai*. Rather than discussing this stage as an intermediary result in the mathematical sense, Plato characterizes it as a “dreamlike state,” bringing the slave’s manifesting of *alêtheis doxai* on a par with the way in which *alêtheis doxai* are discussed later on in the dialogue, in the context of virtue and with regard to statesmen: as being “enraptured” (*enthousiazein*, 99D4), “inspired” (*epipnous*, 99D4), “possessed” (*katechomenous*, 99D4), and as an instance of “divine dispensation” (*theiai moirai*, 99E8, 100B3).

In what sense the slave is in a “dreamlike state” can be gathered from Plato’s Greek. As from 85B, in the passage where the second mention of *doxa* occurs, we can witness Socrates playing a “game of musical chairs,” speaking about the slave and the slave’s soul, while much of the time not explicitly distinguishing between the two. The manifesting of *alêtheis doxai* marks the stage that is current in 85C, as indicated by the adverb *nun* (85C10)—the stage where the final diagram has been drawn, and the tilted area is recognized as the 8-foot square. Precisely there, the use of (*ouk*) *eidenai* is abolished, except when—significantly—Socrates discusses Meno’s thoughts instead of the slave’s (*oisth’*, 85C13), or when Meno expresses his own thoughts (*oïda*, 85E7). Now that (*ouk*) *eidenai* is dropped with respect to the slave, *doxa* is introduced; and *epistêmê* will be spoken of in the future tense, as a stage in the slave’s development that, as indicated in §2.2, will not transpire in the *Meno*. First, Socrates asks Meno if the slave has given any *opinion* that is not *his* (*estin hêntina doxan ouch hautou houtos apekrinato*, 85B13-14)—the indicative present tense *estin* and the noun *doxa* clearly referring to the phase after the final diagram has been drawn, indicated, as noted before, by *nun* (85C10). Meno replies: “No, on the contrary: [any opinion he gave is] his” (*ouk, all’heautou*, 85C1). We note how Meno agrees that the opinions, which were “just now” uttered by the slave (such as his enunciation that the tilted square measures 8 square feet), are the slave’s opinions—it is, however, not said that he *has* them. Then, Socrates interjects that the slave, “as agreed a little earlier, *did not know* (*kai mên ouk êidei ge, hês ephamen oligon proteron*, 85C2-3): the “little earlier” referring to the phase *before* the tilted area was recognized as the 8-foot square, in which forms of *eidenai* were used instead of *doxa* and *epistêmê* (the continued use of *êidei* in 85C2 being

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<sup>70</sup> Scott 2009, p. 102.



consistent with this, and the past perfect tense in *êidei* referring back in time). With this interjection, Socrates' game of musical chairs kicks off properly.

Next, Socrates asks if "these opinions were in the slave" (*enêsan de ge autôi hautai hai doxai*, 85C5): the indicative past tense referring back to the phase before the tilted area was recognized as the 8-foot square—this, however, being the phase from which the use of the noun *doxa* was advertently barred. So the slave did not *have* any opinions at that stage: the opinions *were in* him—we note the role of *ev*, here as a prepositional prefix, and sense that *enêsan* expresses that the *doxai* were in the *soul* of the slave, so that when Socrates says that the opinions were "in him," i.e. in the slave, he is not being precise, speaking of the slave while actually referring to the slave's soul. With this lack of *akribeia*, Socrates' game of musical chairs is gaining momentum. From *doxai* "being in" the slave, Socrates draws a first conclusion: "So in [the slave], not yet [having reached the stage of] knowing about the things which he does not know, are true opinions about the things which he does not know?" (*Tôi ouk eidoti ara peri hôn an mê eidêi eneisin alêtheis doxai peri toutôn hôn ouk oiden*, 85C7-8). The several conjugations of *eidenai* (the single dative participle (*tôi*) *eidoti*, the third person single subjunctive *eidêi*, and the third person single indicative *oiden*), in spite of the present tense of my translation, refer back to the stage before the tilted area was recognized as the 8-foot square (as indicated, again, by the use of *eidenai*), and stress (also through the adverb *ara*) that true opinions, though he *absolutely* did not know (*eidenai*), were *already* in the slave. But again, they "were in" him, i.e. in his soul: he did not "have" them—for then he would have had *epistêmê*.<sup>71</sup> But as said, the use of *epistêmê* is reserved for the slave's future development, which we do not witness in the *Meno*.

From the phase before the tilted area was recognized as the 8-foot square, when the slave did not know (though true opinions were in him), Socrates shifts attention to the actuality of what happened just now (indicated by the adverb *arti*, "just now": the moment after the slave realized that the tilted area is the 8-foot square), and of what is going on right now (indicated by the adverb *nun*, "at present"): "And now, to him it is as if [he is] in a dream, after those opinions have been brought into movement just now" (*Kai nun men ge autôi hôsper onar arti anakekinêntai hai doxai hautai*, 85C10-11). The *doxai* that were in the slave have been stirred up: one should note the prefix *ana-* in *anakineô*, suggesting, as noted in §2.4, a movement from something towards something else, a relevating similar to

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<sup>71</sup> None of the translations that I am aware of maintain the dative in *tôi ouk eidoti* in respect of its dependence on the prefix *en-* in *eneisin*: they therefore do not distinguish between the slave and his soul, i.e. do not appreciate the fact that the slave does not *have* true opinions, whereas rather the true opinions are "in him," i.e. in the boy's soul: thus, these translations erase Socrates' game of musical chairs. These translations neither respect the difference between *eidenai* on the one hand, and *doxa* and *epistêmê* on the other, preferring instead to translate the sentence in 85C7-8 as the slave "having", i.e. being in full possession of, true opinions. But this, of course, begs the question of what the difference is between having true opinions and having knowledge: there may be none. But there *is* a difference between *manifesting* true opinions and having knowledge, and in order to appreciate this, the dative in *tôi ouk eidoti* should be respected, as it communicates a peculiar "disconnect" within the slave; this "disconnect" will be discussed in the next chapter.

the significance of the prefix *eis-* in *eiserchomai*, used in the *Theaetetus* to indicate the “jumping at one” of a previously unrealized feature of a diagram. In the *Meno*, as we have seen, what “jumps at” the slave is the tilted area, which is ultimately recognized as the 8-foot square. The dreamlike state of the slave, the being brought into movement of *doxai*, and the recognition of the 8-foot square are connected, as underscored by Socrates’ game of musical chairs: the boy’s *doxai* are not his, he does not “have” them, but they are—somehow—in his soul. They “jump at him” from the diagram as much as from his soul, so that the boy is now able to *manifest* truths—but in such a way that, when he points to “that line” (*apo tautês [grammês]*, 85B4), he almost speaks as if he were an object of ventriloquism.

From the above observations, an opportunity arises to connect Socrates’ game of musical chairs—a playful hodgepodge that stirs together the utterances of the slave with events taking place in the slave’s soul as much as in the diagrams—to Socrates’ numerous manipulations of the slave. These manipulations, as will be discussed in the final paragraph, bring about a peculiar disparity within the slave: they cause a “disconnect” between the slave on the one hand, and the boy’s soul on the other.

### **3.4 The Slave’s Manifestation of *Alêtheis Doxai***

The point of Socrates’ game of musical chairs is that the slave does not *have* true opinions (which would imply knowledge on the boy’s part), but *manifests* them: inducing the slave to manifest *alêtheis doxai* constitutes Socrates’ reply to Meno’s Paradox. How does this manifesting take shape, and first of all, what precisely *are* the slave’s *alêtheis doxai*? One could assume these to be “The tilted area measures 8 feet,” and “The tilted area is the 8-foot square.” Those statements, however, are inferred from what is said by *both* Socrates *and* the slave—the latter’s assumed *alêtheis doxai* are not what the slave himself literally expresses. What the slave verbally says goes no further than mentioning outcomes of a number of calculations (such as *diplasia*, “double,” and *oktôpoun*, eight feet), and, more importantly, brief expressions of consent (such as *panu men oun*, “well indeed”; and *nai*, “yes”). These are words of acquiescence with respect to what Socrates says—but Socrates’ treatment of the diagram, as pointed out previously, is peculiar, and therefore one could expect the slave’s consent to be affected by this. In this final paragraph, we will discuss one further peculiarity; at the core of this lies that the words used by Socrates in discussing the geometrical problem, and to which the slave consents, were *already familiar* to the slave in a colloquial sense. In the course of the mathematics passage, the significance of those words will be stretched by Socrates beyond their everyday use, as the slave finds out twice, to his surprise—first in the *aporia*, and next in the sudden realization that the tilted area must be the 8-foot square.

The colloquial sense of the vocabulary used is prominent at the beginning of the conversation, when Socrates asks the slave: “Tell me, boy, do you know that a square is like this?” (*Eipe dê moi, ô pai, gignôskeis tetragônnon chôrion hoti toiouton estin*, 82B8-9), to which the boy answers: “I do indeed” (*egôge*, 82B10). Because we

are reading about a geometrical problem, we are inclined to assume that we should read some strict mathematical significance into words such as *tetragônon chôrion*, and all other subsequent words with a mathematical ring to them: the mathematical passage would not make sense—we tend to believe, at least—if these words were not used (or not intended by Socrates to be used) with some strictly defined geometrical meaning (as is the case, for example, in Euclid’s *Elements*). But the sense in which the slave consents to (or utters) “mathematical” expressions need not be understood within the context of rigid, proof-driven mathematics; rather, their sense should be gathered from remarks exchanged between Socrates and Meno right before Socrates begins to address the slave. Socrates corroborates whether the boy is Greek, and speaks Greek (*Hellên men esti kai hellênizei*, 82B3), to which Meno responds: “Very much so, as he was born in the house” (*Panu ge sphodra, oikogenês ge*, 82B4). This, and not the first principles of geometry, is the initial background to the sense in which the words are used in the mathematics passage: the colloquial Greek language, by which the slave was surrounded from the moment he was born in Meno’s house.

The expressions used throughout the mathematical exercise are words that we learn for the first time, not in geometry class, but on our mother’s (or father’s) lap. In order of first occurrence, the following are the “mathematical” expressions used by the slave, or gathered by the slave from Socrates: ‘square’ (*tetragônon chôrion*, 82B8-9); ‘equal’ (*isas*, 82C1); ‘line’ (*tas grammas*, 82C2); ‘four’ (*tettaras*, 82C2); ‘middle’ (*mesou*, 82C4); ‘larger’ and ‘smaller’ (*meizon* and *elallon*, 82C7-8); ‘two’ (*duoin*, 82C10); ‘foot’ (*podoin*, 82C10); ‘one’ (*henos*, 82C12); ‘double’ (*diplosion*, 82D10); ‘eight’ (*oktô*, 82D14); ‘how much’ (*pêlikê*, 82D15); ‘long’ and ‘short’ (*makron* and *brachu*, 83A2-3); ‘half’ (*hêmiseas*, 83C10); ‘three’ (*tri[poda]*, 83E3); ‘to count’ (*arithmein*, 84A1); ‘corner’ (*gôniai*, 84D13). *That is all* – what the slave expresses or consents to, are words he learned as an infant, as elements of the colloquial Greek language. But colloquial Greek, like any colloquial language, is ill-defined, vague, and adaptable to the pressures of what happen to be current circumstances. But because colloquial words are vague, they are also taken to be transparent, i.e. easily comprehensible in a point-and-see kind of way, which, as already discussed in §1.1, is the way in which they are used in the *Meno*, and in which they had always been used by the slave in his *nun bios*. While using the words in Meno’s household, the slave believed he *knew* their meaning; due to their transparency, he assumed that their meaning stretched no further than their everyday use; and this is what the slave continues to assume throughout his conversation with Socrates. Through the diagrams, however, Socrates will twice be seen to infuse the words with diverging meanings that the slave was not yet aware of in his *nun bios*; this “infusion” is possible precisely *because* the words are used colloquially, i.e. as ill-defined and adaptable vocabulary. Yet the slave, initially at least, does not notice this: when asked, if he understood what a square was, the boy looked at the initial diagram (fig. 1)—and what he saw, suited (or at least did not conflict with) the boy’s colloquial sense of the words *tetragônon chôrion*, *isas*, *grammê*, and *tettaros* (82B8-C2)—a transparency that Socrates, of course, had anticipated.

In what manner does Socrates infuse the colloquial Greek with diverging meanings? The first thing to notice is that the words continue to refer to one and the same subject matter, applying unambiguously to the diagrams and their elements, such as is the case with “four equal spaces” (*tettara isa chôria*) in 84D15, referring to the four quadrants of the 16-foot square, and “those lines” (*hautai grammi*) in 85A4, referring to the four oblique lines drawn by Socrates in each quadrant; the same goes for all other “mathematical” words used throughout the conversation, and which are immediately verifiable in the “point-and-see” manner of transparent, colloquial Greek. The second thing to notice is that, due to the vagueness of everyday Greek, the words are open to becoming infused with two very different meanings: these can be brought out by again looking at Plato’s choice of words, especially at two different verbs, *gignomai* on the one hand, and *eneinai* on the other. Roughly speaking, the verb *gignomai* is applicable to the slave’s ill-starred attempt at constructing the 8-foot square by doubling the 4-foot square with the unit squares as a point of departure, while the verb *eneinai* encompasses Socrates’ concealed strategy of arriving at the 8-foot square by cutting the 16-foot square in half. From the start of their exchange, Socrates infuses the slave’s colloquial Greek with the first meaning, that of *gignomai*; but from a certain moment in the conversation, Socrates stealthily adds the second meaning, that of *eneinai*. The use of *gignomai* continues, but the vocabulary has now become infused by *both* meanings: this goes especially, as we will see, for the word ‘line’ (*grammê*). This simultaneous infusion by two meanings will bring us to the core of why the slave *manifests* true opinions instead of simply *having* them—and to what, in a sense, can be called “mysterious” about the mathematical passage.

As discussed in chapters 1 and 2, Socrates induced particular thought tendencies in the slave through the first series of diagrams (figs. 1-4). One tendency, established by the unit squares from which the first diagrams were assembled, led the boy to assume that the 8-foot square would share a common unit with the initial 4-foot square drawn by Socrates. This tendency can be termed *compositive*; in the words of Meno’s Paradox, it can be understood as attempting to arrive at a “whole” (the 8-foot square) through assumed “parts” (the unit squares) *before* having determined the whole in and by itself, i.e. as assembling a whole by simply adding *preselected* parts to one another (as if they were Lego bricks) until the whole comes about. This tendency almost completely marks the first stage of the conversation between Socrates and the slave (82B8-84A4), in which a total number of 13 inflections of the verb *gignomai* occurs, always having the first, compositive sense.<sup>72</sup> For example, in the first occurrence of *gignomai*, Socrates discusses how a figure assembled from two unit squares measures 2 square feet, and if two unit squares were added, the whole (*to holon*, 82C11) would become (*gignetai*, 82D2) 4 square feet. This immediately leads Socrates to suggest that a larger figure, twice the size of this 4-foot square (*toutou tou chôriou heteron diplasion*, 82D9-10), can come about (*genoit’*, 82D9) as well, i.e. can be assembled compositively as with the 4-foot square

<sup>72</sup> *Gignetai*, 82D2; *gignetai*, 82D3; *gignetai*, 82D4; *genoit’*, 82D9; *genêsetai*, 82E7; *gignesthai*, 83A2; *gignetai*, 83A7; *genôntai*, 83A11; *gignetai*, 83B8; *gignetai*, 83C2; *gignetai*, 83E7; *gignetai*, 83E10; *gignetai*, 83E17.

just before—a suggestion that ultimately leads to the slave’s *aporia*. The compositive character of *gignomai* as marking the first meaning, i.e. that of the colloquial Greek exchanged between the slave and Socrates, is further underlined in 83A7-8, where *gignomai* is coupled with *prostithêmi* (‘to add’; *prosthomen*, 83A8) in discussing the doubling of the side of the 4-foot square in the first of the boy’s failed attempts to arrive at the 8-foot square—a compositive attempt that, as Socrates demonstrates, brings forth (*genêsetai*, 82E7; *gignesthai*, 83A2; *genôntai*, 83A11; *gignetai*, 83B8 and 83C2) the 16-foot square instead (fig. 3).

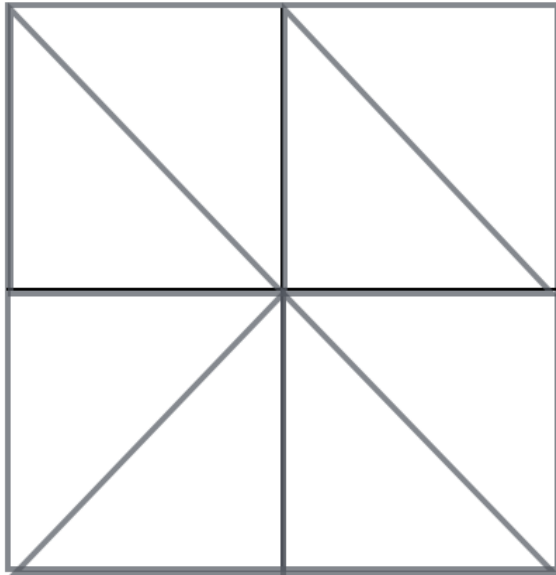
The diagram of the 16-foot square gives rise to the only use of the verb *eneinai* in the first stage, there already indicating the second meaning, that of *first* arriving at a whole to *then* determine an included part, which, in opposition to the compositive tendency, can be called *resolutive*. The discussion on how the 16-foot square contains four equal smaller squares of 4 square feet (*en autôti esti tauti tettara* etc., 83B5-6) prefigures the resumed discussion of the 16-foot square in the second phase of the conversation between Socrates and the slave (84D4-85B11). In this resumed discussion of the 16-foot square, the colloquial Greek is charged with the second meaning as designated by the preposition *en*, occurring twice, and by the adverb *entos*, the verb *eneinai*, and the verb *periechomai* (‘to contain, to embrace’), each occurring once.<sup>73</sup> Simultaneously, however, Socrates continues to infuse the words with the first meaning, as is indicated by the prolonged use of *gignomai*, inflections of which occur 7 times in the second phase of the conversation.<sup>74</sup> When Socrates, after having drawn the first quadrant and using the same everyday language as before, asks the boy: “Is this before us indeed a 4-foot area? Do you understand?” (*ou to men tetrapoun touto hêmin esti chôrion, manthaneis?* 84D4-5), the boy’s “understanding,” as far as he himself is concerned, is still in line with the colloquial meaning of ‘four,’ ‘foot,’ and ‘area,’ and in no way dissimilar to his understanding, earlier on, of the first square drawn by Socrates (fig. 1)—for again, nothing that he sees conflicts with his colloquial use of the words, in spite of the fact that Socrates this time, contrary to the first diagram, does not divide this 4-foot square into unit squares: the slave simply accepts that he is, again, looking at a 4-foot square. Socrates uses that as an opportunity to show the slave how the three other quadrants are “added” (again, *prostithêmi*; *prostheimen*, 84D7), from which, in sum total, four quadrants “come about” (*genoit’*, 84D15), which together “assemble” (*gignetai*, 84E3; *genesthai*, 84E5) the 16-foot square. Socrates teasingly adds: “But we were supposed to bring about only twice [the 4-foot square], or don’t you remember?” (*Edei de diplasion hêmin genesthai, ê ou memnêsai*, 84E5-6).

This use of *mimnêskein* plays on the fact that the mathematical exercise is supposed to be a case of *anamnêsis*; and it is with this taunting remark of Socrates, and his drawing of the four oblique lines, that the *anamnêsis* properly starts. In one and the same sentence, still in colloquial Greek, Socrates expresses the two meanings, doing so sequentially: he first intimates how the oblique lines add up to a total of four (*gignontai*, 85A4), using the first meaning; and then asks if the oblique

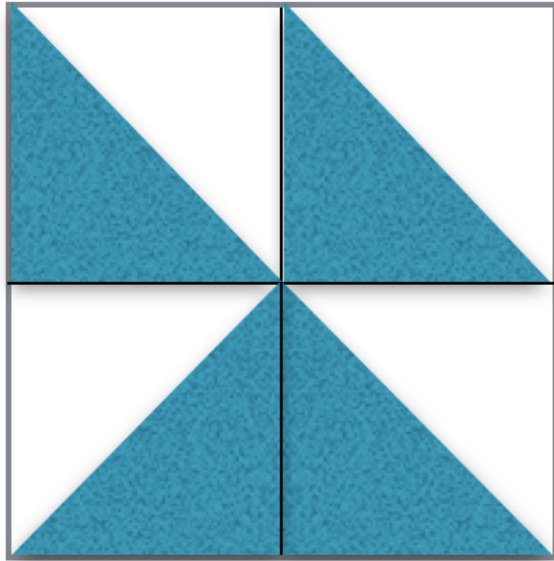
<sup>73</sup> *Periechousai*, 85A5; *entos*, 85A11; *enestin*, 85A13; *en*, 85A13 and 15.

<sup>74</sup> *Genoit’*, 84D15; *gignetai*, 84E3; *genesthai*, 84E5; *gignontai*, 85A4; *gignontai*, 85A6; *gignetai*, 85B1; *gignoit’*, 85B10.

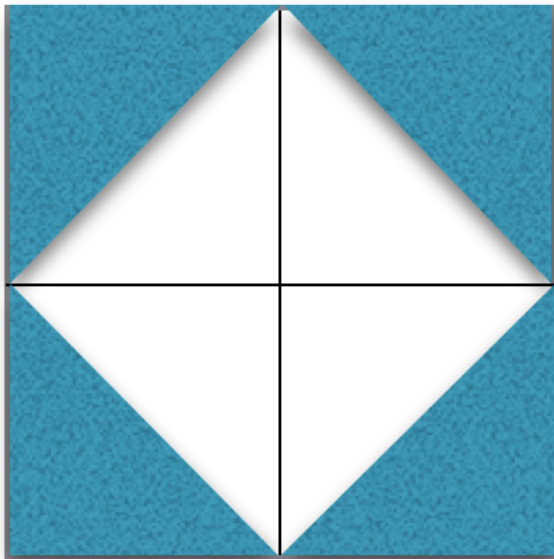
lines contain a tilted area (*periechousai*, 85A5), using the second meaning. Revealingly, the slave answers by repeating “*gignontai*” (85A6), and not by consenting to *periechousai*, thus staying with the first meaning. Nonetheless, the slave has just been shown that the four lines are not randomly distributed throughout the diagram (compare fig. 16). They are arranged in a particular way, as discussed in §2.1: Socrates has turned them at an angle of 90 degrees for every successive quadrant. Thus, the slave has been made aware of the fact that the lines are not only *divisors*, dividing each quadrant in two (as is indicated by *temnousa*, 85A1), but also *boundaries* (indicated by *periechousai*, 85A5), which outline a figure that has now emerged and “jumps at him” (compare fig 18 as opposed to fig. 17). As we have seen, the slave does not yet recognize the tilted area as the 8-foot square, for when asked what the area of the tilted figure is, he answers “I do not understand” (*ou manthanô*, 85A9).



*Fig. 16: one example of a random distribution of the oblique lines in the four quadrants of the 16-foot square.*



*Fig. 17: from a random distribution of diagonals, no figure emerges.*



*Fig. 18: from Socrates' distribution of the diagonals, a figure emerges: the tilted area.*

Socrates, as we have seen, leads the slave towards the realization that the tilted area has four units (half-squares), twice the number of the initial 4-foot square, and so the slave finally identifies the tilted area as the 8-foot square. This should be interpreted in terms of Meno's Paradox. The crucial tipping-point in the conversation was the slave's unwitting discovery of the half-square as a counting unit. But the possibility to use the half-square as such depended entirely on *first* having seen the tilted area *as* an area, and this depended on the word 'line' having the second meaning in addition to the first, i.e. that of boundary, and not only of

divisor. Because of the first meaning, it was possible to see how “that line” divides each quadrant into two; because of the second meaning, it was possible to see that in each quadrant, one half falls within the respective quadrant *as well as* within the tilted area. The identification of the tilted area as the 8-foot square in turn led to the determination of the oblique line as the side of the 8-foot square, which only from there on was called by a “technical,” i.e. non-colloquial word, the name ‘diagonal’ (*diametros*, 85B8).<sup>75</sup> In short: first a whole was found (the tilted area), and then a part (the side)—in entirely the opposite working order from when compared to how the slave dealt with the first series of diagrams. But because the slave had found the whole, the tilted area, without first recognizing that the tilted area was in fact the 8-foot square, it can be said that he found the whole without (yet) knowing it. Precisely this proves Socrates’ point with respect to Meno’s Paradox: that it is possible to find a whole before the part.

## Conclusion

In what sense is the mathematical exercise in the *Meno* “mysterious”? This boils down to asking how the slave’s soul participates in the actual mathematics—which, in turn, acuminates as the question about how the boy manifests *alêtheis doxai* while dealing with the problem of doubling the square. As demonstrated, what happens in the diagrams simultaneously happens in the slave—what becomes relevated in the geometrical figures and “jumps at” the slave, coincidentally emerges from the boy’s soul. The part played by the soul signifies a peculiar “disconnect” within the slave: when the boy speaks, his words express more than he himself realizes, since his vocabulary is stretched by Socrates beyond its colloquial use in the boy’s *nun bios*. This goes especially for when the final diagram is drawn: from that moment, it is as if not only the slave has the word, but something else speaks through him simultaneously. The boy starts speaking polyphonically, as it were, with two voices that act largely independently: the first voice, ringing with the meaning of *gignomai*, is induced by the first series of diagrams, and continues to sound throughout the entire conversation. But with the final diagram, it is joined by another voice, tuned to the second meaning, that of *eneinai*: this provides a counterpoint, so to speak, to the first voice. This contrapuntal occurrence of two meanings at once marks how the slave manifests true opinions.

The slave, however, remains oblivious to this. Once Socrates has drawn the final diagram, the boy’s attempt at solving the geometrical problem continues as a compositive endeavour: the slave still counts or adds up preselected elements (the half-squares, this time), hoping that these will somehow, haphazardly, gel together into the 8-foot square—but to the boy’s great surprise, they *had already* been assembled into the 8-foot square. The slave sees this belatedly, only once he realizes that the tilted area must be the 8-foot square: this occurrence of a delay, of a *lag*, creates a sense of there having been something independent from the slave, yet active within him—“something” that led the slave towards the solution, “something”

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<sup>75</sup> Cf. Brown 1967, p. 71.



affecting his calculations by providing them with a sense of unity and direction, and “something” which finally, after the first abortive attempts (figs. 2-4), breathed life into his calculations. This “something independent” is what I described as the second, resolute meaning; and that, as far as I am concerned, is what Socrates comes to call the slave’s soul. Thus what happens to, and *within*, the slave—the manifesting of *alêtheis doxai* as the coincidental occurrence of two meanings, one signifying the slave’s compositive endeavour, the other an “independent” resolute activity—is the result of several peculiarities in Socrates’ treatment of geometry: Socrates seizes on thought patterns that lead the slave into an *aporia*; employs opaque features of the diagrams; avoids the use of certain words; and utilizes the ostensible transparency of colloquial Greek, thus infusing the slave’s vocabulary with two different meanings coincidentally. So the “mystery” in the mathematics passage of the *Meno* issues *from* the mathematics, i.e. results from certain peculiar features of the geometry as these are seized upon by Socrates.

But still: doesn’t Socrates *literally say* that the slave’s true opinions “were in” the boy already “at the time when he was not a human (*ho chronos hot’ouk ên anthrôpos*, 86A4-5), i.e. when he was soul, and before he became the boy that was born into Meno’s household? Well, not quite—or not necessarily. At the end of the mathematical passage, Socrates remarks to Meno: “And don’t you agree that, if the truth of the things is always with us in the soul, the soul must then be immortal? So that you should be confident, that what there is to know, but do not hit upon now—that is, what is not being remembered [i.e. the *zêtoumenon*—you should endeavour to search and remember?” (*Oukoun ei aei hê alêtheia hêmin tôn ontôn estin en têi psuchêi, athanatos an hê psuchê eîê, hôste tharrounta chrê, ho mê tunchaneis epistamenos nun, touto d’estin ho mê memnêmenos, epicheirein zêtein kai anamnêskesthai*, 86A14-B4). The crucial thing to notice here is that, while in 81C-D, Socrates tried to answer Meno’s Paradox by *deriving* the possibility of remembrance *from* the immortality of the soul (i.e. from the Pindar quote), he now argues *for* the immortality of the soul by reasoning *from* the actual remembrance that just took place in the slave. Several commentators have noticed this turnabout, without however providing a plausible explanation for it.<sup>76</sup> The reason for this lack is probably the failure to notice in what way the mathematics passage is a reply to Meno’s Paradox—yet this is the crucial point. The fact that the slave was able to find a “part”, the *zêtoumenon* (the side of the 8-foot square)—“that what there was to know, but [he] did not hit upon” first—through first finding a “whole” (the tilted area, surprisingly turning out to be the 8-foot square), takes *precedence* over the immortality of the soul, i.e. over the mystery, in the *Meno*.<sup>77</sup>

<sup>76</sup> Klein notices the turnabout without explaining it, arguing that the result in either case (i.e. whether or not the slave’s *anamnêsis* takes precedence over Pindar’s Persephone myth) is the same: see Klein 1965, p. 180. Ebert notices the turnabout too: see Ebert 1973, pp. 179-180, and note 78 below. Scott mentions it, and offers no further comment: see Scott 2009, p. 93.

<sup>77</sup> This point is strengthened by Ebert’s analysis of Socrates’ Greek in the latter’s elaboration on the Pindar citation right before and right after the geometrical exercise. Ebert notices how in the context of yet another, earlier citation of Pindar (76D6-7), Socrates had already

Even though the theorem on doubling the square had long been proved by Plato's time, the *Meno* does not discuss it in terms of proof; nor does it describe its origin as a mere mystery. And as far as there is talk of mystery, the mystery issues, first and foremost, from Socrates' peculiar employment of mathematics, which seizes on misleading thought tendencies, the apparent transparency of colloquial Greek, opaque features of diagrams, and the avoidance of certain words—and so makes it possible to find a “whole” before its “part”, to hit upon an *alêthês doxa*, i.e. upon something true that cannot yet be called knowledge, and to find a theorem before proof. As such, and in a more serious fashion than mathematicians seem to do nowadays, Plato gave an answer to the question *where mathematical theorems come from*.

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remarked how “the style of tragedy” (*tragikê [...] hê apokrisis*, 76E3) seemed agreeable to Meno (*areskei soi mallon*, 76E4), while not being preferable to himself (*all'ekeinê beltîon*, 76E7). Through a careful examination of Socrates' use of homoioteleuta and alliterations, Ebert is able to forward the hypothesis that Socrates, while discussing the slave's *anamnêsis* with Meno, assumes “the style of tragedy” in order to cater to Meno's taste, providing a “parody of a Gorgian epideictic logos”: like my analysis of Socrates' smart employment of mathematics, Ebert's (very different) approach explains why the actual *anamnêsis* in the slave takes precedence over the myth as derived from the Pindar quote in the *Meno*. See Ebert 1973, pp. 176-178.

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