# From Micro to Macro: Multilevel modelling with group-level outcomes

by

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#### Abstract

Most of the research regarding multilevel modelling focusses on so-called macro-micro models, where the outcome is measured at the lowest level of the model, and these models are generally well understood. Much less is known about methods to analyse data where the outcome is measured at the highest level of the model (micro-macro models), with one or more predictors at the lowest level. In this study, we demonstrate not only that the analysis of such data is possible using standard software, we perform a simulation study to compare the performance of a full information maximum likelihood multilevel structural equations modelling (ML-SEM) approach to a limited information 2-step latent covariate regression method (2-step LCM). We will show that both methods are viable alternatives for the analysis of micro-macro models. The 2-step LCM method generally provides less biased estimates of the model parameters (except the intercept) when the independent variables are correlated. The ML-SEM method provides more accurate estimates of the standard errors for some parameters, except when the number of groups included in the analysis is small.

#### Samenvatting

Een groot deel van het onderzoek naar multilevel modellen draait om de zogenaamde macromicromodellen, waarbij de uitkomst wordt gemeten op het laagste niveau van het model. Over deze modellen is vrij veel bekend. Veel minder weten we over methodes voor het analyseren van data waarbij de uitkomst wordt gemeten op het hoogste niveau van het model met één of meer predictoren op het laagste niveau (micro-macromodellen). Met het huidige onderzoek laten we zien dat de analyse van dit soort data mogelijk is met standaard software en we voeren een simulatiestudie uit met als doel de prestaties van twee methodes te vergelijken. De eerste methode is multilevel structural equations modelling (ML-SEM), wat een full information maximum likelihood benadering is. De tweede methode is de 2-step latent covariate regression method (2-step LCM), wat een limited information benadering is. We zullen aantonen dat beide methodes geschikt zijn voor de analyse van micro-macro modellen. De 2-step LCM methode zorgt over het algemeen voor minder bias van de geschatte parameters (met uitzondering van het intercept) wanneer de onafhankelijke variabelen met elkaar correleren. De ML-SEM methode geeft betere schattingen van de standaardfouten voor sommige parameters, behalve wanneer het aantal groepen in de analyse klein is. From micro to macro: multilevel modelling with group-level outcomes

Multilevel modelling (MLM) is a name for a range of techniques that allow for the analysis of data with a hierarchical, or nested, structure. Such data is guite common in the social and behavioural sciences. The hierarchical structure arises when one level of observations is nested in another, for example students who are nested in school, or employees who are nested in firms. It is also possible for observations to be nested in individuals, as is the case with repeated measurements. Sometimes, the nesting arises as a side effect of the design of a study, for example when participants in a survey are nested in interviewers. Regardless of the source of the nesting, what these data structures have in common is that the observations at the lowest level (the students, the employees, the repeated measures) are not independent. Observations within the same level-2 cluster (group) have more in common with one another than with observations from a different level-2 cluster, because they are all influenced by the same group characteristics, and because the level-1 units within the same cluster can (potentially) influence one another. Because of this, the analysis of multilevel data calls for specialized models that can take the dependency of the observations into account, and that can separate the within group variation from the between group variation. When the dependency of the observations is not taken into account, the analysis may lead to incorrect inferences about the relationships in the data. There are two common mistakes that may occur when the relationship between units at different levels is not taken into account. The first mistake is generalization of relationships at the group level, expecting them to hold at the individual level. This is known as the ecological fallacy (Robinson, 1950). The second mistake is generalizing findings from the individual level to the group level, which is known as the atomistic fallacy (Diez-Roux, 1998).

Before more sophisticated methods were available, the analysis of hierarchically structured data was handled by aggregating or disaggregating variables to a single level of analysis and essentially ignoring the hierarchical structure of the data. Analysing data this way, without taking the level of the observations into account, can lead to problems when the relationship between variables is not the same at the individual level and the group level. When aggregating the lower level variable(s) to the higher level (e.g. by computing the group average), researchers run the risk of succumbing to the ecological fallacy. Additionally, this approach reduces the variability in the data, resulting in inappropriate estimates of the standard errors of the regression parameters (Croon & Van Veldhoven, 2007), reduced statistical power, unreliable group-level information, and the incorrect weighting of groups during parameter estimation (Lüdtke et al., 2008; Preacher, Zyphur & Zhang, 2010). When disaggregating the higher level variable(s) to the lower level (e.g. by assigning all members of the group the same score on the group-level variable as if it was measured at the individual level), researchers may succumb to the atomistic fallacy. This may also lead to inaccurate estimates of the standard errors of the regression parameters due to confounding of between group

and within group variation (Croon & Van Veldhoven, 2007). In short, methods that are appropriate for analysing multilevel data should take the hierarchical structure of the data and the level of the observations into account to lead to correct inferences.

Traditional approaches to MLM focus mainly on so-called macro-micro models (Snijders & Bosker, as cited in Croon & Van Veldhoven (2007)). In a macro-micro model, the dependent variable is included at the lowest level of the model, and independent variables can be included at any level. Say, for example, that the outcome we are interested in is children's reading ability (an individual-level variable), and that we have two independent variables that may predict the outcome: a measure of intelligence (an individual-level variable), and the number of hours each week that the teacher spends exercising reading with the class (a group-level variable, measured at the level of the class). This is a simple macro-micro model that can be analysed using traditional MLM. Models for this type of data are generally well understood (see Hox (2010) for a good introduction to MLM).

Models for micro-macro data, on the other hand, have until recently been largely neglected in the literature on MLM (Croon & Van Veldhoven, 2007). There are many areas of research, however, where such models would be of substantial interest. Sometimes, this is because we are interested in the group-level outcome specifically. We may want to predict the performance of schools, organizations, or teams using both group-level and individual-level characteristics. At other times, we may be dealing with data that is only available at the aggregated level. For example when the outcome of interest is students' absenteeism rate, team productivity, or (country-level) pollution indices. Recent studies show that methods using latent variables, or structural equation modelling (SEM) can be used for the analysis of micro-macro data. We will start this paper with a short general overview of SEM, and then proceed to discussing the specific models we will be investigating in the current study.

#### **Structural Equations Modelling**

Structural equations modelling is a statistical modelling technique that can be described as a combination between factor analysis and regression, or path, analysis. When using SEM, it is possible to represent theoretical constructs by latent factors. Latent factors are variables that have not been directly observed, but that are inferred from the observed (measured) variables. These latent variables are free of random error, because error has been estimated and removed, leaving only a common variance (Hox & Bechger, 1998). Both the independent (or endogenous) variables and dependent (or exogenous) variables can be either measured or latent. The SEM consist of two parts: a measurement model, which relates the measured variables to the latent variables, and a structural model, which relates the latent variables to one another. SEM does not test the hypothesis that two (or more) variables are related directly by comparing the measured values of the variables, instead it

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tests the hypothesis indirectly by comparing the variances of the variables. Throughout this paper, we will use an adapted version of the SEM notation of Muthén and Asparouhov (2008). The measurement model of the SEM is:

$$\boldsymbol{y}_i = \boldsymbol{\beta}_0 + \boldsymbol{\beta}_1 \boldsymbol{\eta}_i + \boldsymbol{\beta}_2 \boldsymbol{x}_i + \boldsymbol{\epsilon}_i$$

where *i* indexes individuals.  $y_i$  is a *p*-dimensional vector of measured endogenous variables for individual *i*;  $\beta_0$  is a *p*-dimensional vector of variable intercepts;  $\beta_1$  is a  $p \times m$  loading matrix, where *m* is the number of random effects (latent variables), this matrix also includes random slopes if any are specified;  $\eta_i$  is an  $m \times 1$  vector of random effects for individual *i*; and  $\beta_2$  is a  $p \times q$  matrix of regression coefficients (slopes) for the *q* exogenous covariates in  $x_i$ ; and  $\epsilon_i$  is a *p*-dimensional vector of error terms for individual *i*.

The structural model is:

$$\boldsymbol{\eta}_i = \boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 \boldsymbol{\eta}_i + \boldsymbol{\alpha}_3 \boldsymbol{x}_i + \boldsymbol{\zeta}_i$$

where  $\alpha_0$  is an  $m \times 1$  vector of intercept terms;  $\alpha_1$  is an  $m \times m$  matrix of structural regression parameters;  $\alpha_3$  is an  $m \times q$  matrix of slope parameters for exogenous covariates; and  $\zeta_i$  is an mdimensional vector of latent variable regression residuals for individual *i*. Residuals in  $\varepsilon_i$  and  $\zeta_i$  are assumed to be multivariate normally distributed with zero means and covariance matrices  $\Theta$  and  $\Psi$ , respectively. Additionally, the residuals in  $\varepsilon_i$  and  $\zeta_i$  are assumed to be independent.

The assumption of independent residuals seems to make SEM inherently unsuitable for modelling multilevel data. However, in the early 1990's, researchers have shown that that for data involving repeated measures, SEM and MLM are analytically equivalent (Meredith & Tisak, 1990; Willet & Sayer, 1994; MacCallum, Kim, Malarkey & Kiecolt-Glaser, 1997). The main difference is that for MLM the time variable is used as a predictor variable, while for SEM time is incorporated into the factor loading matrix, thereby relating the repeated measures to the underlying latent factors (Curran, 2003). Others later capitalized on this equivalence and described that any level-1 predictor could be incorporated into the SEM factor loading matrix (Curran, 2003; Mehta & Neale, 2005), making SEM suitable for a broader range of multilevel models and not just those that model individual change over time. This application of SEM regards the cluster as the unit of analysis, and the individuals as variables, and is also known as the people-as-variables approach (Mehta & Neale, 2005). In its early stages, this method was very impractical and data management intensive, because it involved manually rearranging the data matrix to properly relate the level-1 units to the level-2 units, while grouping the level-1 units that had the same values on the level-2 predictors. Currently, multilevel models can be estimates in some standard SEM software, such as Mplus, using full information maximum likelihood estimation without the additional data management steps being necessary.

For our current research, we are interested in models suitable for the analyses of micro-macro multilevel data. By means of a simulation study, we will investigate the performance of two different methods for the analyses of such data and determine whether or not either of the methods performs better than the other. In the remainder of this chapter, we will describe the methods, starting with a latent covariate regression method developed specifically for the analysis of micro-macro data, and then we will take a look at a multilevel SEM (ML-SEM) method.

### Latent Covariate Model

As we discussed in the previous section, using the observed group mean of an individual-level variable as a covariate in an ordinary least squares (OLS) regression leads to biased parameter estimates. We can eliminate this bias by treating the unobserved group mean ( $\xi_g$ ) as a latent variable, with the individuals as indicators. The first method we describe in more detail is based on an aggregation approach, and uses a correction procedure to adjust the group mean for bias (Croon & Van Veldhoven, 2007). We will refer to this method as the 2-step latent covariate method (2-step LCM).

The SEM we described in the previous section, as well as the multilevel SEM we will describe in the next section are general models. The ML-SEM allows any variable to be decomposed into latent group and individual-level variation, while the 2-step LCM we discuss here only allows for the independent variables to have both within group and between group variation, because it was developed specifically for the analysis of micro-macro data (although a similar procedure could be used for other types of data). Because of this, we use a slightly different notation here to distinguish more clearly between the independent variables measured at the individual-level and those measured at the group-level.

Consider the simple micro-macro model shown in figure 1, which includes one group-level independent variable (here denoted as Z) and one individual-level independent variable (X). This model can be expressed as a set of linear equations. The first equation relates the group scores on the explanatory variables Z and  $\xi$  to the outcome variable Y:

$$y_g = \beta_0 + \beta_i \xi_g + \beta_g z_g + \epsilon_g$$

where g indexes groups.  $y_g$  is the outcome for group g;  $\xi_g$  is the latent group mean of the individuallevel variable for group g;  $z_g$  is the value of the group-level variable for group g; and  $\epsilon_g$  is the error (or

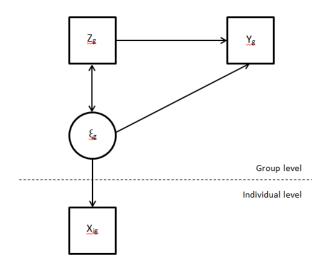


Figure 1. A micro-macro model with correlated independent variables. A simple micro-macro model with one independent variable at the group level, and one independent variable at the individual level. The latent group mean of the individual-level variable is represented by  $\xi_g$ .

disturbance) for group g. The error variable  $\epsilon_g$  is assumed to be homoscedastic (to have constant variance  $\sigma_{\epsilon}^2$  for all groups).

The regression parameters in this model satisfy the following relationships:

$$\begin{pmatrix} \sigma_{\xi}^2 & \sigma_{\xi z} \\ \sigma_{\xi z} & \sigma_z^2 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} \sigma_{\xi y} \\ \sigma_{z y} \end{pmatrix}$$

and

$$\beta_0 = \mu_y - \beta_1 \mu_\xi - \beta_2 \mu_z$$

where  $\mu_y$ ,  $\mu_{\xi}$ , and  $\mu_z$ , are the grand means of the variables *Y*,  $\xi$ , and *Z*, respectively.

The second equation relates the group score  $\xi_g$  to the individual score  $x_{ig}$  of each individual in group g:

$$x_{ig} = \xi_g + v_{ig}$$

The variance of the disturbance term  $v_{ig}$  ( $\sigma_v^2$ , which is the within-group variance of variable X) is assumed to be constant for all subjects and groups. Additionally,  $\epsilon_g$  and  $v_{ig}$  are assumed to be mutually independent and to be independent from the group variable  $\xi$ .

Croon and Van Veldhoven (2007) have shown analytically that regression of  $y_g$  on the observed group mean  $\bar{x}_g$  (instead of  $\xi_g$ ) and  $z_g$  leads to biased estimates of the regression parameters, unless the within-group variability of the individual-level variable (*X*) is zero. The relationship between the regression parameters for the unadjusted model and the parameters for the latent covariate model are an expression of this bias. We indicate the intercept for the unadjusted model by  $\beta_{u0}$ ; the parameter for the  $\xi$  variable by  $\beta_{ui}$ ; and the parameter for the *Z* variable by  $\beta_{ug}$ . The relationships between the parameters for the unadjusted model and the parameters for the 2-step LCM are (see Croon & Van Veldhoven (2007) for a derivation of these results):

$$\begin{split} \beta_{u0} &= \beta_0 + ((1 - w_{g1})\mu_{\xi} - w_{g2}\mu_z)\beta_1, \\ \beta_{ui} &= w_{g1}\beta_i, \\ \beta_{ug} &= w_{g2}\beta_i + \beta_g, \end{split}$$

with

$$w_{g1} = \frac{\sigma_{\xi}^2 \sigma_{z}^2 - \sigma_{\xi z}^2}{\left(\sigma_{\xi}^2 + \sigma_{v}^2 / n_g\right) \sigma_{z}^2 - \sigma_{\xi z}^2}$$

and

$$w_{g2} = \frac{\sigma_{\xi z} \, \sigma_{v}^{2} / n_{g}}{\left(\sigma_{\xi}^{2} + \sigma_{v}^{2} / n_{g}\right) \sigma_{z}^{2} - \sigma_{\xi z}^{2}}$$

where  $n_g$  is the size of group g.

Now that we know the extent of the bias introduced by regressing  $y_g$  on the observed group mean  $\bar{x}_g$  and  $z_g$ , we can correct for it using the following formula to compute the adjusted group mean  $\tilde{x}_g$  (Croon & Van Veldhoven, 2007):

$$\tilde{x}_g = (1 - w_{g1})\mu_{\xi} + w_{g1}\bar{x}_g + w_{g2}(z_g - \mu_z)$$

In this formulation, the adjusted group mean  $\tilde{x}_g$  is the expected value of  $\xi_g$ , taking into account all the observed scores on both the individual and group-level explanatory variables.

Regression of the group-level outcome  $y_g$  on the adjusted group mean  $\tilde{x}_g$  and the group-level variable  $z_g$ , leads to unbiased estimates of the regression parameters. However, the model satisfies homoscedasticity only when group sizes  $(n_g)$  are equal between groups. When group sizes are not equal, the weight matrices  $w_{g1}$  and  $w_{g2}$  have to be determined for each group separately, and the standard errors can be corrected using the heteroscedasticity-consistent covariance matrix estimator defined by White (1980), and further developed by Davidson & MacKinnon (1993).

In a simulation study Croon and Van Veldhoven (2007) examined the difference between the latent covariate method and an unadjusted regression analysis (using the observed group means  $\bar{x}_g$ ) in their effect on the relative bias of the parameter estimates. For the parameter estimate ( $\beta_g$ ) of the group-level explanatory variable  $z_g$ , both methods performed about equally well, except when the  $\xi_g$  and  $z_g$  variables were correlated and the ICC was low, in which case their latent covariate method produced less biased estimates. For the parameter estimate of the (adjusted) group mean of the individual-level variable  $x_{ig}$  ( $\beta_i$ ), the latent covariate method performed better in all examined conditions. No differences were found regarding bias for the intercept.

#### **Multilevel Structural Equations Modelling**

The 2-step latent covariate method described in the previous section is a limited information approach, in contrast to the ML-SEM method we will be discussing in the current section, which allows for full information maximum likelihood estimation and requires only a single analysis step.

Multilevel structural equation modelling, as the name implies, combines multilevel modelling with structural equations modelling. ML-SEM has, in a short period of time, come a long way from the people-as-variables approach discussed previously. Work by (among others) Muthén & Asparouhov (2008) and Lüdtke et al. (2008) has led to the proposal of a general ML-SEM framework (Preacher, Zhang & Zyphur, 2010) which allows for the estimation of a multitude of ML-SEM models using standard SEM software. Though developed in the context of mediation models, the authors touch on the possibility of using ML-SEM to estimate micro-macro models.

The basis for the ML-SEM approach is that each variable is decomposed into unobserved components, which are considered latent variables (Asparouhov & Muthén, 2006), using the observed data to infer the latent group mean and the latent within component. Using ML-SEM, the unreliability of the group mean is taken into account when estimating the corresponding model parameter (Lüdtke et al., 2008). For the model illustrated in figure 1, the decomposition of the individual-level variable  $x_{ig}$  into separate between and within components looks like this:

$$x_{ig} = \xi_g + v_{ig}$$

where  $\xi_g$  is the latent group mean, and  $v_{ig}$  is the latent individual deviation from the group mean. Note that this is the same formula we have seen before when we discussed the 2-step LCM.

The single level SEM described in the beginning of this chapter can be extended to the multilevel case and can accommodate several different designs, including mediation models and micro-macro models (Preacher, Zhang & Zyphur, 2010). Following the adapted version of the SEM notation by (Muthén & Asparouhov, 2008), the measurement model for the ML-SEM is:

$$\mathbf{y}_{ig} = \boldsymbol{\beta}_{0g} + \boldsymbol{\beta}_{1g}\boldsymbol{\eta}_{ig} + \boldsymbol{\beta}_{2g}\boldsymbol{x}_{ig} + \boldsymbol{\epsilon}_{ig}$$

where  $y_{ig}$  is a *p*-dimensional vector of measured variables;  $\beta_{0g}$  is a *p*-dimensional vector of variable intercepts;  $\epsilon_{ig}$  is a *p*-dimensional vector of error terms;  $\beta_{1g}$  is a  $p \times m$  loading matrix, where *m* is the number of random effects (latent variables), this matrix includes random slopes if any are specified;  $\eta_{ig}$  is an  $m \times 1$  vector of random effects; and  $\beta_{2g}$  is a  $p \times q$  matrix of slopes for the *q* exogenous covariates in  $x_{ig}$ . Residuals in  $\epsilon_{ig}$  are assumed to be multivariate normally distributed with a mean of zero and covariance matrix  $\Theta$ . Elements in  $\Theta$  are not permitted to vary across clusters. The within structural model for the ML-SEM is:

$$\boldsymbol{\eta}_{ig} = \boldsymbol{\alpha}_{0g} + \boldsymbol{\alpha}_{1g}\boldsymbol{\eta}_{ig} + \boldsymbol{\alpha}_{3g}\boldsymbol{x}_{ig} + \boldsymbol{\zeta}_{ig}$$

where  $\alpha_{0g}$  is an  $m \times 1$  vector of intercept terms;  $\alpha_{1g}$  is an  $m \times m$  matrix of structural regression parameters (with zeros on the diagonal, indicating the latent variables cannot cause/influence themselves);  $\alpha_{3g}$  is an  $m \times q$  matrix of slope parameters for exogenous covariates; and  $\zeta_{ig}$  is an *m*dimensional vector of latent variable regression residuals. Residuals in  $\zeta_{ig}$  are assumed to be multivariate normally distributed with a mean of zero and covariance matrix  $\Psi$ . Elements in  $\Psi$  are not permitted to vary across clusters. In both equations, the group (g) and individual (*i*) subscripts indicate that some elements of these matrices are allowed to vary within and between clusters, but it is not necessary that they do, as this is a general ML-SEM model (for the micro-macro models we are currently considering, the outcome variable *Y* is measured at the group level, which means that not within variation of *Y* is possible).

The multilevel part of the model is expressed in the level-2 (between) structural model:

$$\boldsymbol{\eta}_g = \boldsymbol{\gamma}_0 + \boldsymbol{\gamma}_1 \boldsymbol{\eta}_g + \boldsymbol{\gamma}_2 \boldsymbol{x}_g + \boldsymbol{\zeta}_g$$

Note that  $\eta_g$  is different from the  $\eta_{ig}$  in the first two equations. The vector  $\eta_g$  contains all the random effects. It stacks the random elements of all the parameter matrices with g subscripts from the equations for the measurement and structural model. Also,  $x_g$  is different from  $x_{ig}$ .  $x_g$  is an s-dimensional vector of all cluster-level covariates. The vector  $\gamma_0$  ( $r \times 1$ ) and matrices  $\gamma_1$  ( $r \times r$ ) and  $\gamma_2$  ( $r \times s$ ) contain estimated fixed effects. Where  $\gamma_0$  contains means of the random effect distributions and intercepts of between structural equations,  $\gamma_1$  contains regression slopes of random effects (i.e., latent variables and random intercepts and slopes) regressed on each other, and  $\gamma_2$  contains regression slopes of random effects regressed on exogenous cluster-level regressors. Cluster-level residuals in  $\zeta_g$  have a multivariate normal distribution with mean zero and covariance matrix  $\psi$ .

In a simulation study by Lüdtke et al. (2008), the ML-SEM method was compared to the 2-step LCM. However, they considered a macro-micro model and were only interested in the estimate of the contextual effect. They found that both methods performed about equally well in estimating the contextual effect, except when the sample size at both levels of the model was small and the ICC was low, in which case the ML-SEM estimates showed less bias and were less variable. To date, there has been no systematic comparison of the 2-step LCM and ML-SEM for micro-macro models. The present study is primarily aimed at investigating how both methods perform for the analysis of micro-macro data, while we also wish to show that there are viable options available for the analysis of micro-macro models in general.

#### Method

We will perform a simulation study to compare the 2-step LCM to the general ML-SEM method for the analysis of micro-macro models. Several considerations come into play when analysing data for both MLM and SEM. One of these is sample size, which in MLM concerns the sample size at all levels of analysis. For the current study, we restrict ourselves to two-level models, which means we need to take into account the total sample size (the number of individuals / level-1 units), and the number of groups (level-2 units). It is generally recommended that the total sample size for SEM should be at least 200 units (Hoogland & Boomsma, 1998). The number of level-2 units for MLM and ML-SEM is recommended to be at least 50, as a lower number of groups can lead to biased estimates of the level-2 standard errors (Hox, 2010; Maas & Hox, 2004; Meuleman & Billiet, 2009). Some software supports Bayesian estimation for ML-SEM. When this estimation method is used, as few as 20 groups may suffice for accurate estimation (Hox, Van de Schoot & Matthijsse, 2012), while some other estimation methods may need even more than 50 groups for accurate estimation (Hox, Maas & Brinkhuis, 2010). A last consideration regarding sample size concerns the effect one wishes to test. According to Snijders (2005), the level-1 sample size is most important when we are interested in testing the effect of the level-1 variable, and the level-2 sample size is most important when we are interested in testing the effect of the level-2 variable. Generally though, when using MLM or ML-SEM, group size is not as important as it is to include a sufficient number of groups (Snijders, 2005; Hox, 2010).

Next, we should consider the intraclass correlation (ICC). For the model in figure 1, the ICC of the individual-level variable  $x_{ig}$  is defined as:

$$\rho = \frac{\sigma_{\xi}^2}{\sigma_{\xi}^2 + \sigma_{v}^2}$$

which is the ratio of the between group variance of  $x_{ig}$  ( $\sigma_{\xi}^2$ ) and the total variance of  $x_{ig}$ . The ICC is a measure for the proportion of variance that is explained by the grouping structure in the population. ICCs of 0.1, 0.2, and 0.3 are commonly found in practice (Maas & Hox, 2004).

Lastly, we will follow the example of Croon & Van Veldhoven (2007) and consider the correlation between the individual-level X variable and the group-level Z variable as a condition to be manipulated in the simulation.

Based on the considerations outlined above, we choose the following conditions for our simulation study: (1) the number of groups is varied at five levels, including a condition with a number of groups that is generally considered too small for multilevel modelling: N<sub>g</sub> = 20, 50, 100, 200, or 500; (2) the number of individuals per group is varied at five levels: N<sub>i</sub> = 5, 10, 25, 50, and we include a condition where the group sizes are mixed, with each group having an equal probability to have either 5, 10, 25, or 50 individuals (resulting in an average group size of about 22.5); (3) the ICC is varied at four levels: ICC = 0.05, 0.1, 0.2, or 0.3; (4) lastly, we consider a condition with no correlation between  $z_g$  and  $\xi_g$ , and a condition where these variables are correlated:  $\rho_{z\xi} = 0$ , or 0.3. This leads to a design with 5\*5\*4\*2 = 200 conditions.

Our simulation study assess the performance of both methods using a simple model with a group-level dependent variable  $(y_g)$ , a single independent variable at the group level and a single independent variable at the individual level (like the model illustrated in figure 1). The population regression equation for  $y_g$  is:

$$y_a = 0.6 + 0.3z_a + 0.3\xi_a + \epsilon_a$$

with  $\epsilon_g$  normally distributed with a mean of zero and variance 0.35, and  $z_g$  and  $\xi_g$  normally distributed with a mean of zero and a variance of 1. The individual score  $x_{ig}$  was obtained from  $x_{ig} = \xi_g + v_{ig}$ , with  $v_{ig}$  normally distributed with a mean of zero. The variance of  $v_{ig}$  was varied to create conditions with different ICCs.

For each manipulated condition, 1000 datasets were drawn from the population described above. We used R software version 3.2.1 to generate the data, as well as for performing the analyses with the 2-step LCM method. The generated datasets were saved to be used in an external Monte Carlo in Mplus version 6.11 for the analyses using the ML-SEM method. An external Monte Carlo does not provide output for each dataset separately, but instead it gives the averages over all replications. Mplus offers maximum likelihood estimation using an accelerated EM-algorithm for ML-SEM models. By default, standard errors are computed with a Huber-White sandwich estimator to correct for heteroscedasticity introduced by unbalanced group sizes. The syntax for both methods is included in appendix II and III.

We want to compare the 2-step LCM method and the ML-SEM method with respect to the bias of the parameter estimates and the accuracy of the standard errors. The bias of the parameter estimates will be determined by computing the difference between the parameter in the population and the estimate (raw bias). For the accuracy of the standard errors, we take the observed coverage of the 95% confidence interval (the proportion of times the 95% confidence interval correctly contains the population parameter).

To determine which of the simulated conditions contribute most to differences between the methods regarding bias and coverage values, we conduct mixed between-within subjects ANOVAs (with the method being the within factor) using the relative bias and the coverage as dependent variables and the manipulated conditions as (between) factors. These ANOVAs are conducted at the cell level (so that the five-way interaction cannot be separated from the error), treating each cell average as a single observation. For the sake of interpretability (and to not end up with small numbers of observations in each cell), we do not investigate interaction effects higher than three-way interactions. To describe the results of the ANOVAs, we calculate the partial- $\eta^2$  effect size for all main effects and for the two- and three-way interactions. We will also investigate whether the data meets the required assumptions of having normally distributed residuals by inspecting QQ-plots and we will test for homogeneous variances using Levene's test.

### Results

No problems were encountered in estimating the model coefficients using the 2-step LCM method. A few convergence issues were encountered when using ML-SEM, and these were mostly restricted to the conditions with a low ICC or a small group size. With the exception of one condition

where two of the generated datasets did not converge, we were in all cases dealing with a single dataset for which the estimation did not converge (a total of nine datasets over eight conditions did not converge).

Table 1 gives the gives the average raw bias for the parameter estimates for each condition, and the coverage value (the proportion of replications where the 95% confidence interval correctly included the population parameter). Due to the size of this table, the results of the simulation are included in appendix I. Note that in a few cases, the bias for the parameter estimates for  $\beta_g$  and  $\beta_i$  is very high when the 2-step latent covariate method is used (a bias as large, or larger than the parameter value itself), and in some cases the bias for the estimate of  $\beta_i$  is not only high, but it is negative, which means the estimate has the wrong sign and completely misrepresents the relationship between  $\xi_g$  and  $y_g$  in the population. Looking at the coverage values, we can see a few extreme values concerning the coverage for the parameter estimate for  $\beta_g$ , which is less than 50% in a few conditions when using ML-SEM method.

We conducted mixed between-within subjects ANOVAs with the bias and coverage values as dependent variables and each manipulated condition as a factor to further investigate which conditions contributed most to differences between both methods regarding bias and coverage. The ANOVAs were conducted at the cell level, with each cell average being treated as one observation. Inspections of the residuals using QQ-plots showed moderate deviations from normality for all residuals of the independent variables related to bias, and large deviations from normality (skewed distributions) for all residuals of the independent variables related to coverage. We used Levene's test to check for homogeneity of variances (for each three-way interaction separately). For most of the independent variables, we found that in at least eight of the ten three-way interactions, the homogeneity assumption was violated, but this was not related to any specific factor. The exception was the independent variable measuring bias for the intercept, here we found that only the six threeway interactions that included method had groups with non-homogeneous variances. In spite of the violations of the assumptions underlying the ANOVA we will discuss the results in the next section, and briefly came back to this issue in the discussion section. We will limit our discussion of the results to those effects that show a difference in performance of both methods (the strictly between subjects effects are not discussed here), and to the highest order significant interactions (in case of both a significant two-way interaction and a significant three-way interaction that contain the same factors).

### **Effects That Influence Bias for the Parameter Estimates**

**Bias for the estimate of the intercept.** None of the examined conditions, or their interactions, had a significant effect on the bias for the estimate for the intercept (see table 2).

|  | Tal | ole | 2 |
|--|-----|-----|---|
|--|-----|-----|---|

*Results of a mixed between-within subjects ANOVA on the source of bias for the estimate of the intercept* 

| Source  | SS    | df  | MS    | F     | р    | partial $\eta^2$ |
|---|-------|-----|-------|-------|------|------------------|
| Between subjects                                    |       |     |       |       |      |                  |
| Correlation   | 0.002 | 1   | 0.002 | 2.106 | 0.15 | 0.0              |
| ICC   | 0.004 | 3   | 0.001 | 1.084 | 0.36 | 0.0              |
| N <sub>g</sub>                                      | 0.003 | 4   | 0.001 | 0.564 | 0.69 | 0.0              |
| Ni  | 0.003 | 4   | 0.001 | 0.753 | 0.56 | 0.0              |
| Correlation $\times$ ICC                            | 0.008 | 3   | 0.003 | 2.253 | 0.09 | 0.1              |
| Correlation $\times N_g$                            | 0.002 | 4   | 0.001 | 0.479 | 0.75 | 0.0              |
| Correlation $\times$ N <sub>i</sub>                 | 0.004 | 4   | 0.001 | 0.967 | 0.43 | 0.0              |
| $ICC \times N_g$                                    | 0.006 | 12  | 0.000 | 0.442 | 0.94 | 0.1              |
| $ICC \times N_i$                                    | 0.012 | 12  | 0.001 | 0.87  | 0.58 | 0.1              |
| $N_g 	imes N_i$                                     | 0.022 | 16  | 0.001 | 1.248 | 0.27 | 0.2              |
| Correlation $\times$ ICC $\times$ N <sub>g</sub>    | 0.007 | 12  | 0.001 | 0.548 | 0.87 | 0.1              |
| Correlation $\times$ ICC $\times$ N <sub>i</sub>    | 0.014 | 12  | 0.001 | 1.018 | 0.45 | 0.2              |
| $Correlation \times N_g \times N_i$                 | 0.014 | 16  | 0.001 | 0.793 | 0.69 | 0.2              |
| $ICC \times N_g \times Ni$                          | 0.057 | 48  | 0.001 | 1.072 | 0.41 | 0.5              |
| Error (BS)  | 0.054 | 48  | 0.001 |       |      |                  |
| Within subjects                                     |       |     |       |       |      |                  |
| Method  | 0.002 | 1   | 0.002 | 1.861 | 0.17 | 0.0              |
| Method $\times$ correlation                         | 0.003 | 1   | 0.003 | 2.71  | 0.10 | 0.0              |
| Method $\times$ ICC                                 | 0.003 | 3   | 0.001 | 1.121 | 0.34 | 0.0              |
| Method $\times$ N <sub>g</sub>                      | 0.002 | 4   | 0.001 | 0.561 | 0.69 | 0.0              |
| Method $\times$ N <sub>i</sub>                      | 0.004 | 4   | 0.001 | 0.866 | 0.49 | 0.0              |
| Method $\times$ correlation $\times$ ICC            | 0.007 | 3   | 0.002 | 2.222 | 0.09 | 0.0              |
| Method × correlation × $N_g$                        | 0.003 | 4   | 0.001 | 0.637 | 0.64 | 0.0              |
| Method $\times$ correlation $\times$ N <sub>i</sub> | 0.004 | 4   | 0.001 | 1.022 | 0.40 | 0.0              |
| Method $\times$ ICC $\times$ N <sub>g</sub>         | 0.005 | 12  | 0.000 | 0.439 | 0.95 | 0.0              |
| $Method \times ICC \times N_i$                      | 0.011 | 12  | 0.001 | 0.937 | 0.51 | 0.0              |
| $Method \times N_g \times N_i$                      | 0.021 | 16  | 0.001 | 1.276 | 0.22 | 0.1              |
| Error (WS)  | 0.138 | 136 | 0.001 |       |      |                  |

**Bias for the estimate of**  $\beta_i$ . Table 3 summarizes the results of the ANOVA on the bias for the estimate of the  $\beta_i$  parameter. There was no significant difference between the amount of bias generated by the two methods overall, F(1,136) = 0.44, p > .05. There was, however, a medium effect (partial  $\eta^2 = .09$ ) of the three-way interaction between method, correlation and number of groups, F(4, 136) = 3.44, p < .05. When the independent variables are uncorrelated, both methods estimate  $\beta_i$  with little to no bias, regardless of the number of groups included in the simulation (figure 2, top). When the independent variables are correlated, using the ML-SEM method results in a consistently biased estimate of  $\beta_i$ . With only 20 groups included in the simulation (and when the independent variables are correlated), the 2-step LCM method performs about the same as ML-SEM, but when more groups are included in the simulation, the bias for the estimate of  $\beta_i$  approaches zero (figure 2, bottom).

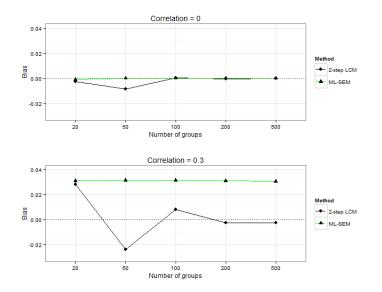


Figure 2. Interaction plot. An illustration of the three-way interaction between method, correlation and number of groups on the bias for the estimate of the  $\beta_i$  parameter.

| Source  | SS    | df  | MS    | F     | р    | partial $\eta^2$ |
|---|-------|-----|-------|-------|------|------------------|
| Between subjects  |       |     |       |       |      |                  |
| Correlation   | 0.000 | 1   | 0.000 | 0.009 | 0.93 | 0.00             |
| ICC   | 0.001 | 3   | 0.000 | 0.044 | 0.99 | 0.00             |
| N <sub>g</sub>  | 0.029 | 4   | 0.007 | 0.686 | 0.61 | 0.05             |
| Ni  | 0.030 | 4   | 0.007 | 0.71  | 0.59 | 0.06             |
| Correlation $\times$ ICC                                    | 0.017 | 3   | 0.006 | 0.542 | 0.66 | 0.03             |
| Correlation $\times$ N <sub>g</sub>                         | 0.178 | 4   | 0.045 | 4.221 | 0.01 | 0.26             |
| Correlation $\times$ N <sub>i</sub>                         | 0.010 | 4   | 0.003 | 0.241 | 0.91 | 0.02             |
| $ICC \times N_g$  | 0.149 | 12  | 0.012 | 1.18  | 0.32 | 0.23             |
| $ICC \times N_i$  | 0.073 | 12  | 0.006 | 0.573 | 0.85 | 0.13             |
| $N_g \times N_i$  | 0.097 | 16  | 0.006 | 0.577 | 0.89 | 0.16             |
| Correlation $\times$ ICC $\times$ N <sub>g</sub>            | 0.214 | 12  | 0.018 | 1.691 | 0.10 | 0.30             |
| Correlation $\times$ ICC $\times$ N <sub>i</sub>            | 0.099 | 12  | 0.008 | 0.782 | 0.67 | 0.16             |
| Correlation $\times$ N <sub>g</sub> $\times$ N <sub>i</sub> | 0.288 | 16  | 0.018 | 1.703 | 0.08 | 0.36             |
| $ICC \times N_g \times Ni$                                  | 0.517 | 48  | 0.011 | 1.02  | 0.47 | 0.50             |
| Error (BS)  | 0.506 | 48  | 0.011 |       |      |                  |
| Within subjects   |       |     |       |       |      |                  |
| Method  | 0.005 | 1   | 0.005 | 0.437 | 0.51 | 0.00             |
| Method $\times$ correlation                                 | 0.007 | 1   | 0.007 | 0.577 | 0.45 | 0.00             |
| Method $\times$ ICC   | 0.050 | 3   | 0.017 | 1.358 | 0.26 | 0.03             |
| Method $\times$ N <sub>g</sub>                              | 0.018 | 4   | 0.005 | 0.365 | 0.83 | 0.01             |
| Method $\times$ N <sub>i</sub>                              | 0.009 | 4   | 0.002 | 0.176 | 0.95 | 0.01             |
| Method $\times$ correlation $\times$ ICC                    | 0.025 | 3   | 0.008 | 0.675 | 0.57 | 0.01             |
| Method × correlation × N <sub>g</sub>                       | 0.170 | 4   | 0.043 | 3.443 | 0.01 | 0.09             |
| Method $\times$ correlation $\times$ N <sub>i</sub>         | 0.019 | 4   | 0.005 | 0.377 | 0.82 | 0.01             |
| Method $\times$ ICC $\times$ N <sub>g</sub>                 | 0.169 | 12  | 0.014 | 1.136 | 0.34 | 0.09             |
| $Method \times ICC \times N_i$                              | 0.144 | 12  | 0.012 | 0.97  | 0.48 | 0.08             |
| $Method \times N_g \times N_i$                              | 0.147 | 16  | 0.009 | 0.742 | 0.75 | 0.08             |
| Error (WS)  | 1.682 | 136 | 0.012 |       |      |                  |

Results of a mixed between-within subjects ANOVA on the source of bias for the estimate of the  $\beta_{i}$  parameter

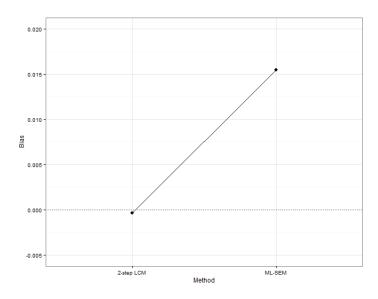
Table 3

**Bias for the estimate of**  $\beta_g$ . Table 4 summarizes the results of ANOVA on the bias for the estimate of the  $\beta_g$  parameter. There was a was a medium effect (partial  $\eta^2 = .08$ ) of method, F(1,136) = 12.24, p > .001, with the 2-step LCM method providing a less biased estimate of  $\beta_g$  overall (figure 3). Additionally, there was a medium effect (partial  $\eta^2 = .06$ ) of the two-way interaction between method and correlation, F(1,136) = 9.34, p > .001. When the independent variables are uncorrelated, both methods estimate  $\beta_g$  with little to no bias. When the independent variables are correlated, the 2-step LCM method provides an accurate estimate of  $\beta_g$ , but the estimate obtained with ML-SEM is biased upward (figure 4).

| Tabl | le 4 |
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Results of a mixed between-within subjects ANOVA on the source of bias for the estimate of the  $\beta_q$  parameter

| Source  | SS    | df  | MS    | F      | р    | partial $\eta^2$ |
|---|-------|-----|-------|--------|------|------------------|
| Between subjects  |       |     |       |        |      |                  |
| Correlation   | 0.031 | 1   | 0.031 | 13.524 | 0.00 | 0.2              |
| ICC   | 0.010 | 3   | 0.003 | 1.528  | 0.22 | 0.0              |
| N <sub>g</sub>  | 0.008 | 4   | 0.002 | 0.932  | 0.45 | 0.0              |
| N <sub>i</sub>  | 0.004 | 4   | 0.001 | 0.48   | 0.75 | 0.0              |
| Correlation $\times$ ICC                                    | 0.009 | 3   | 0.003 | 1.335  | 0.27 | 0.0              |
| Correlation $\times$ N <sub>g</sub>                         | 0.006 | 4   | 0.001 | 0.662  | 0.62 | 0.0              |
| Correlation $\times$ N <sub>i</sub>                         | 0.009 | 4   | 0.002 | 1.016  | 0.41 | 0.0              |
| $\rm ICC 	imes N_g$   | 0.037 | 12  | 0.003 | 1.361  | 0.22 | 0.2              |
| $ICC \times N_i$  | 0.016 | 12  | 0.001 | 0.581  | 0.85 | 0.1              |
| $N_g \times N_i$  | 0.029 | 16  | 0.002 | 0.794  | 0.68 | 0.2              |
| Correlation $\times$ ICC $\times$ N <sub>g</sub>            | 0.026 | 12  | 0.002 | 0.939  | 0.52 | 0.1              |
| Correlation $\times$ ICC $\times$ N <sub>i</sub>            | 0.009 | 12  | 0.001 | 0.337  | 0.98 | 0.0              |
| Correlation $\times$ N <sub>g</sub> $\times$ N <sub>i</sub> | 0.032 | 16  | 0.002 | 0.878  | 0.60 | 0.2              |
| $ICC \times N_g \times Ni$                                  | 0.106 | 48  | 0.002 | 0.975  | 0.53 | 0.4              |
| Error (BS)  | 0.109 | 48  | 0.002 |        |      |                  |
| Within subjects   |       |     |       |        |      |                  |
| Method  | 0.025 | 1   | 0.025 | 12.243 | 0.00 | 0.0              |
| Method $\times$ correlation                                 | 0.019 | 1   | 0.019 | 9.339  | 0.00 | 0.0              |
| Method $\times$ ICC   | 0.001 | 3   | 0.000 | 0.231  | 0.87 | 0.0              |
| Method $\times$ N <sub>g</sub>                              | 0.009 | 4   | 0.002 | 1.124  | 0.35 | 0.0              |
| Method $\times$ N <sub>i</sub>                              | 0.010 | 4   | 0.003 | 1.245  | 0.30 | 0.0              |
| Method $\times$ correlation $\times$ ICC                    | 0.004 | 3   | 0.001 | 0.643  | 0.59 | 0.0              |
| Method × correlation × $N_g$                                | 0.006 | 4   | 0.001 | 0.68   | 0.61 | 0.0              |
| Method $\times$ correlation $\times$ N <sub>i</sub>         | 0.005 | 4   | 0.001 | 0.673  | 0.61 | 0.0              |
| Method $\times$ ICC $\times$ N <sub>g</sub>                 | 0.036 | 12  | 0.003 | 1.465  | 0.14 | 0.1              |
| Method $\times$ ICC $\times$ N <sub>i</sub>                 | 0.015 | 12  | 0.001 | 0.628  | 0.82 | 0.0              |
| $Method \times N_g \times N_i$                              | 0.033 | 16  | 0.002 | 1.022  | 0.44 | 0.1              |
| Error (WS)  | 0.277 | 136 | 0.002 |        |      |                  |



*Figure 3.* Interaction plot. An illustration of the main effect of method on the bias for the estimate of the  $\beta_g$  parameter.

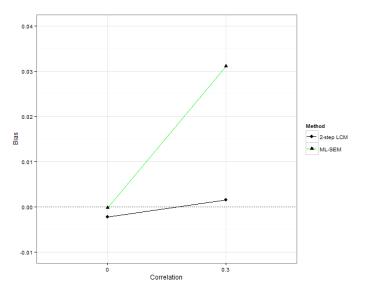


Figure 4. Interaction plot. An illustration of the two-way interaction between method and correlation on the bias for the estimate of the  $\beta_g$  parameter.

### Effects That Influence Coverage of the 95% Confidence Interval of the Parameter Estimates

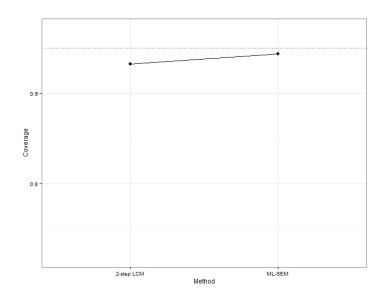
**Coverage for the estimate of the intercept.** Table 5 summarizes the results of the ANOVA on the coverage for the estimate of the intercept. There was a large effect (partial  $\eta^2 = .65$ ) of method, F(1,136) = 255.33, p > .001, where the coverage values for the 95% CI of the intercept obtained with the ML-SEM method were closer to the desired value of .95 than the coverage values for the 2-step LCM method (figure 5). There was also a large effect (partial  $\eta^2 = .33$ ) of the two-way interaction

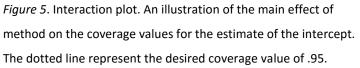
between method and number of groups, F(2,136) = 17.09, p > .001. When only 20 groups are included in the simulation, the coverage values for both methods are low, and there is no difference between the methods. Increasing the number of groups to 50 results in an improvement of the coverage values for both methods, but the improvement is larger for the ML-SEM method. The coverage values for both methods continue to improve as the number of groups is increased further, and the ML-SEM method continues to perform better than the 2-step LCM method (figure 6).

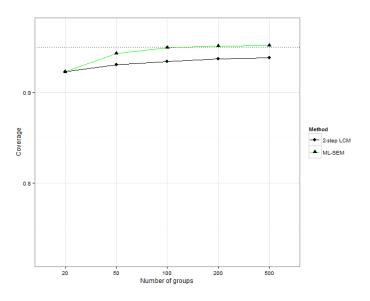
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*Results of a mixed between-within subjects ANOVA on the coverage values for the estimate of the intercept* 

| Source  | SS    | df  | MS    | F       | р    | partial $\eta^2$ |
|---|-------|-----|-------|---------|------|------------------|
| Between subjects                                    |       |     |       |         |      |                  |
| Correlation   | 0.000 | 1   | 0.000 | 3.232   | 0.08 | 0.06             |
| ICC   | 0.008 | 3   | 0.003 | 36.324  | 0.00 | 0.69             |
| N <sub>g</sub>                                      | 0.026 | 4   | 0.007 | 94.892  | 0.00 | 0.89             |
| N <sub>i</sub>                                      | 0.007 | 4   | 0.002 | 25.384  | 0.00 | 0.68             |
| Correlation × ICC                                   | 0.000 | 3   | 0.000 | 2.341   | 0.08 | 0.13             |
| Correlation $\times$ N <sub>g</sub>                 | 0.000 | 4   | 0.000 | 1.32    | 0.28 | 0.10             |
| Correlation $\times$ N <sub>i</sub>                 | 0.000 | 4   | 0.000 | 1.14    | 0.35 | 0.09             |
| $ICC \times N_g$                                    | 0.003 | 12  | 0.000 | 3.36    | 0.00 | 0.46             |
| $ICC \times N_i$                                    | 0.003 | 12  | 0.000 | 4.096   | 0.00 | 0.51             |
| $N_g \times N_i$                                    | 0.006 | 16  | 0.000 | 5.702   | 0.00 | 0.66             |
| Correlation $\times$ ICC $\times$ N <sub>g</sub>    | 0.002 | 12  | 0.000 | 1.818   | 0.07 | 0.31             |
| Correlation $\times$ ICC $\times$ N <sub>i</sub>    | 0.001 | 12  | 0.000 | 1.423   | 0.19 | 0.26             |
| Correlation $\times N_g \times N_i$                 | 0.001 | 16  | 0.000 | 1.124   | 0.36 | 0.27             |
| $ICC \times N_g \times Ni$                          | 0.007 | 48  | 0.000 | 2.005   | 0.01 | 0.67             |
| Error (BS)  | 0.003 | 48  | 0.000 |         |      |                  |
| Within subjects                                     |       |     |       |         |      |                  |
| Method  | 0.013 | 1   | 0.013 | 255.329 | 0.00 | 0.65             |
| Method $	imes$ correlation                          | 0.000 | 1   | 0.000 | 0.997   | 0.32 | 0.01             |
| Method $\times$ ICC                                 | 0.030 | 3   | 0.010 | 202.909 | 0.00 | 0.82             |
| Method $	imes$ N <sub>g</sub>                       | 0.003 | 4   | 0.001 | 17.09   | 0.00 | 0.33             |
| Method $\times$ N <sub>i</sub>                      | 0.027 | 4   | 0.007 | 135.275 | 0.00 | 0.80             |
| Method $\times$ correlation $\times$ ICC            | 0.000 | 3   | 0.000 | 0.142   | 0.93 | 0.00             |
| Method $\times$ correlation $\times$ N <sub>g</sub> | 0.000 | 4   | 0.000 | 0.196   | 0.94 | 0.01             |
| Method $\times$ correlation $\times$ N <sub>i</sub> | 0.000 | 4   | 0.000 | 0.196   | 0.94 | 0.01             |
| Method $\times$ ICC $\times$ N <sub>g</sub>         | 0.001 | 12  | 0.000 | 0.969   | 0.48 | 0.08             |
| Method $\times$ ICC $\times$ N <sub>i</sub>         | 0.019 | 12  | 0.002 | 31.73   | 0.00 | 0.74             |
| $Method \times N_g \times N_i$                      | 0.001 | 16  | 0.000 | 1.529   | 0.10 | 0.15             |
| Error (WS)  | 0.007 | 136 | 0.000 |         |      |                  |

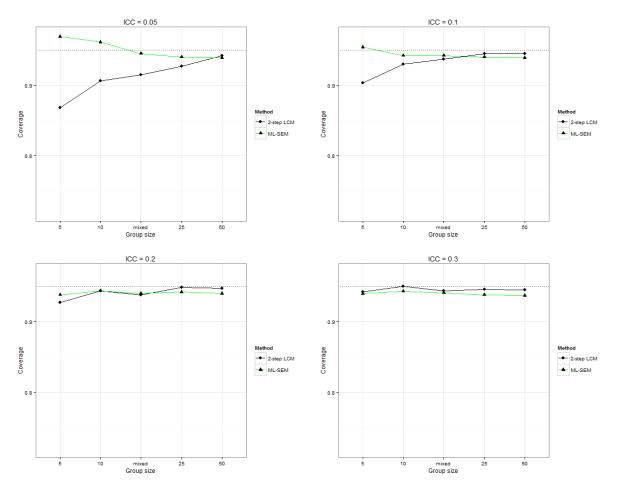






*Figure 6*. Interaction plot. An illustration of the two-way interaction between method and number of groups on the coverage values for the estimate of the intercept. The dotted line represent the desired coverage value of .95.

Additionally, there was a large effect (partial  $\eta^2 = .74$ ) of the three-way interaction between method, intraclass correlation and group size, F(12,136) = 31.73, *p* < .001. When the ICC is 0.2 or 0.3, there is little difference between the coverage values obtained with both methods, and the size of the groups seems to have little effect (figure 7, bottom). A lower ICC of 0.1 seems to mainly effect the 2-step LCM method, which produces a lower coverage value when the groups consist of only 5 individuals. Larger groups result in better coverage values for the 2-step LCM method (figure 7, top right). When the ICC is as low as 0.05, the difference between both methods becomes more pronounced. When the groups consist of only 5 individuals, the coverage value obtained with 2-step LCM is very low, while the coverage obtained with ML-SEM seems to be a bit higher than 0.95. Increasing the size of the groups improves the coverage values obtained with 2-step LCM, and both methods perform about the same when each group contains 50 individuals (figure 7, top left).



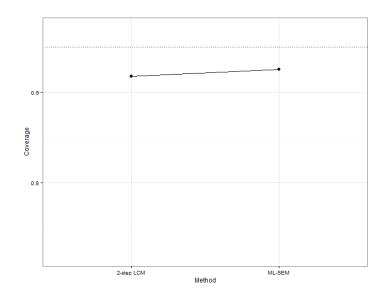
*Figure 7.* Interaction plot. An illustration of the three-way interaction between method, ICC and group size on the coverage values for the estimate of the intercept. The dotted line represent the desired coverage value of .95.

**Coverage for the estimate of**  $\beta_i$ **.** Table 6 summarizes the results of the ANOVA on the coverage for the estimate of the  $\beta_i$  parameter. There was a large effect (partial  $\eta^2 = .46$ ) of method, F(1,136) = 115.44, p > .001. In general, the coverage values for the 95% CI of  $\beta_i$  obtained with the ML-SEM method were closer to the desired value of .95 than the coverage values for the 2-step LCM method (figure 8). There were three significant three-way interactions that include the effect of method, and we will discuss each in turn.

Table 6

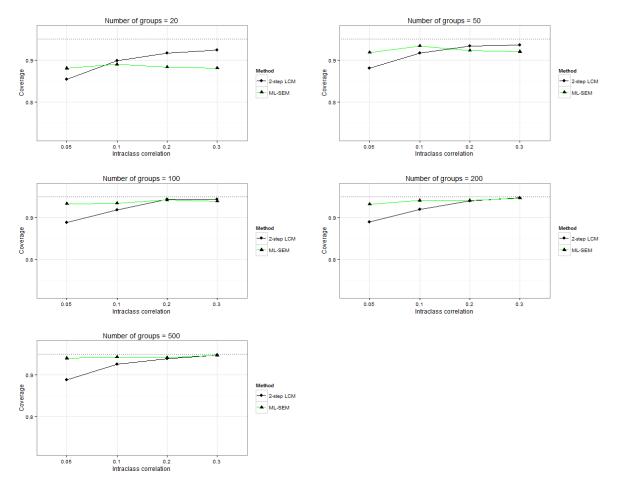
Results of a mixed between-within subjects ANOVA on the coverage values for the estimate of the  $\beta_i$  parameter

| Source  | SS    | df  | MS    | F       | р    | partial $\eta^2$ |
|---|-------|-----|-------|---------|------|------------------|
| Between subjects                                    |       |     |       |         |      |                  |
| Correlation   | 0.002 | 1   | 0.002 | 15.462  | 0.00 | 0.24             |
| ICC   | 0.067 | 3   | 0.022 | 142.945 | 0.00 | 0.90             |
| N <sub>g</sub>                                      | 0.103 | 4   | 0.026 | 165.117 | 0.00 | 0.93             |
| Ni  | 0.083 | 4   | 0.021 | 132.902 | 0.00 | 0.92             |
| Correlation $\times$ ICC                            | 0.001 | 3   | 0.000 | 1.333   | 0.27 | 0.08             |
| Correlation $\times N_{g}$                          | 0.000 | 4   | 0.000 | 0.236   | 0.92 | 0.02             |
| Correlation $\times N_i$                            | 0.001 | 4   | 0.000 | 1.532   | 0.21 | 0.11             |
| $ICC \times N_g$                                    | 0.002 | 12  | 0.000 | 1.158   | 0.34 | 0.22             |
| $ICC \times N_i$                                    | 0.099 | 12  | 0.008 | 53.251  | 0.00 | 0.93             |
| $N_g \times N_i$                                    | 0.003 | 16  | 0.000 | 1.005   | 0.47 | 0.25             |
| Correlation $\times$ ICC $\times$ N <sub>g</sub>    | 0.002 | 12  | 0.000 | 1.214   | 0.30 | 0.23             |
| Correlation $\times$ ICC $\times$ N <sub>i</sub>    | 0.002 | 12  | 0.000 | 1.275   | 0.26 | 0.24             |
| Correlation $\times N_g \times N_i$                 | 0.002 | 16  | 0.000 | 0.961   | 0.51 | 0.24             |
| $ICC \times N_g \times Ni$                          | 0.008 | 48  | 0.000 | 1.038   | 0.45 | 0.51             |
| Error (BS)  | 0.007 | 48  | 0.000 |         |      |                  |
| Within subjects                                     |       |     |       |         |      |                  |
| Method  | 0.007 | 1   | 0.007 | 115.44  | 0.00 | 0.46             |
| Method $\times$ correlation                         | 0.000 | 1   | 0.000 | 4.504   | 0.04 | 0.03             |
| Method $\times$ ICC                                 | 0.045 | 3   | 0.015 | 264.258 | 0.00 | 0.85             |
| Method $\times$ N <sub>g</sub>                      | 0.014 | 4   | 0.004 | 62.711  | 0.00 | 0.65             |
| $Method 	imes N_i$                                  | 0.054 | 4   | 0.013 | 237.472 | 0.00 | 0.87             |
| Method $\times$ correlation $\times$ ICC            | 0.000 | 3   | 0.000 | 0.387   | 0.76 | 0.01             |
| Method $\times$ correlation $\times$ N <sub>g</sub> | 0.000 | 4   | 0.000 | 1.744   | 0.14 | 0.05             |
| Method $\times$ correlation $\times$ N <sub>i</sub> | 0.000 | 4   | 0.000 | 0.49    | 0.74 | 0.01             |
| Method $\times$ ICC $\times$ N <sub>g</sub>         | 0.001 | 12  | 0.000 | 2.105   | 0.02 | 0.16             |
| Method $\times$ ICC $\times$ N <sub>i</sub>         | 0.023 | 12  | 0.002 | 33.116  | 0.00 | 0.75             |
| Method $\times N_g \times N_i$                      | 0.003 | 16  | 0.000 | 2.924   | 0.00 | 0.26             |
| Error (WS)  | 0.008 | 136 | 0.000 |         |      |                  |



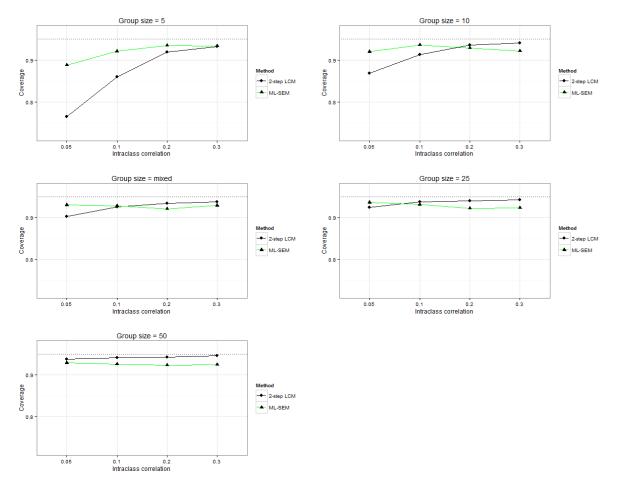
*Figure 8*. Interaction plot. An illustration of the main effect of method on the coverage values for the estimate of the  $\beta_i$  parameter. The dotted line represent the desired coverage value of .95.

First, there was a large effect (partial  $\eta^2 = .16$ ) of the three-way interaction between method, ICC, and number of groups, F(12,136) = 2.11, p < .05. When only 20 groups are included in the simulation, the coverage values are consistently low when the ML-SEM method is used, regardless of the ICC (figure 9, top left). Increasing the number of groups improves the performance of the ML-SEM method, which continues to be unaffected by the ICC (figure 9). The 2-step LCM method, on the other hand, shows a consistent pattern of performing poorly when the ICC is 0.05, with the coverage values improving as the ICC increases. This results in the 2-step LCM method outperforming the ML-SEM method when the number of groups is small and the ICC is high (figure 9, top), and the ML-SEM method outperforming the 2-step LCM method when the ICC is low.



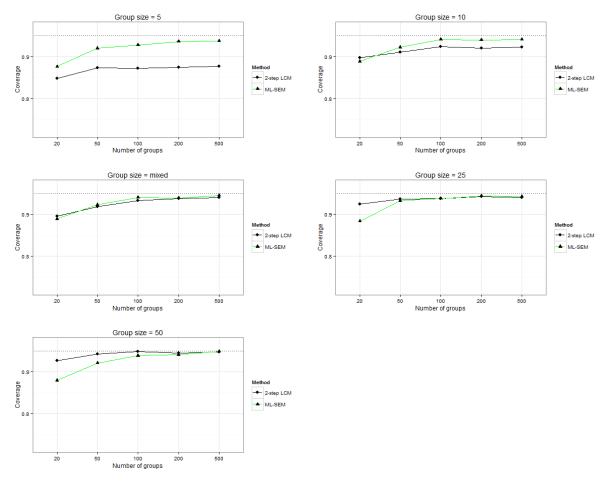
*Figure 9.* Interaction plot. An illustration of the three-way interaction between method, ICC and number of groups on the coverage values for the estimate of the  $\beta_i$  parameter. The dotted line represent the desired coverage value of .95.

Second, there was large effect (partial  $\eta^2 = .75$ ) of the three-way interaction between method, ICC, and group size, F(12,136) = 33.12, p < .001. When the groups sizes are large (mixed, 25, or 50 individuals), the coverage values for both methods are mostly good, and not influenced by the ICC (figure 10, middle and bottom). With 10 individuals in each group, the 2-step LCM performs poorly when the ICC is 0.05 (figure 10, top right). With only 5 individuals in each group, the coverage value obtained with 2-step LCM is even lower when the ICC is 0.05, and the ML-SEM method seems to perform a little less well also (though the effect is much smaller). The performance of the 2-step LCM method increases as the ICC increases, and when the ICC is 0.3 both methods perform about equally well (figure 10, top left).



*Figure 10.* Interaction plot. An illustration of the three-way interaction between method, ICC and group size on the coverage values for the estimate of the  $\beta_i$  parameter. The dotted line represent the desired coverage value of .95.

Last, there was a large effect (partial  $n^2 = .26$ ) of the three-way interaction between method, number of groups and group size, F(16,136) = 2.92, *p* < .001. The ML-SEM method shows a consistent pattern of performing poorly when only 20 groups are included in the simulation, and the coverage values improve as the number of groups increases. This effect does not seem to be influenced by group size (figure 11). The 2-step LCM method performs poorly when the group size is 5, and slightly worse when only 20 groups of 5 individuals each are included in the simulation, compared to 50, 100, 200, or 500 groups of 5 individuals each (figure 11, top left). When the group size is this small, the ML-SEM method always performs better than the 2-step LCM. The difference between both methods is much smaller when the group size is increased to 10, and both methods perform equally well in the case of mixed group sizes (figure 11, top right and middle left). When the size of the groups is increased further to 25 or 50, the 2-step LCM method performs well regardless of the number of groups, resulting in the 2-step LCM method outperforming the ML-SEM method when only a small number of large groups is included in the simulation (figure 11, middle right and bottom).



*Figure 11*. Interaction plot. An illustration of the three-way interaction between method, number of groups and group size on the coverage values for the estimate of the  $\beta_i$  parameter. The dotted line represent the desired coverage value of .95.

**Coverage for the estimate of**  $\beta_g$ . Table 7 summarizes the results of the ANOVA on the coverage for the estimate of the  $\beta_g$  parameter. There was a large (partial  $\eta^2 = .52$ ) effect of method, F(1,136) = 148.95, p > .001. In general, the coverage values for the 95% CI of  $\beta_g$  obtained with the 2-step LCM method were closer to the desired value of .95 than the coverage values for the ML-SEM method (figure 12). There were five significant three-way interactions that include the effect of method, and we will discuss each in turn.

| Source  | SS    | df  | MS    | F       | р    | partial $\eta^2$ |
|---|-------|-----|-------|---------|------|------------------|
| Between subjects  |       |     |       |         |      |                  |
| Correlation   | 0.130 | 1   | 0.130 | 500.81  | 0.00 | 0.91             |
| ICC   | 0.137 | 3   | 0.046 | 176.235 | 0.00 | 0.92             |
| N <sub>g</sub>  | 0.069 | 4   | 0.017 | 66.369  | 0.00 | 0.85             |
| N <sub>i</sub>  | 0.116 | 4   | 0.029 | 111.663 | 0.00 | 0.90             |
| Correlation $\times$ ICC                                    | 0.056 | 3   | 0.019 | 71.926  | 0.00 | 0.82             |
| Correlation $\times$ N <sub>g</sub>                         | 0.086 | 4   | 0.022 | 83.202  | 0.00 | 0.87             |
| Correlation $\times$ N <sub>i</sub>                         | 0.060 | 4   | 0.015 | 57.633  | 0.00 | 0.83             |
| $\rm ICC 	imes N_g$   | 0.055 | 12  | 0.005 | 17.734  | 0.00 | 0.82             |
| $ICC \times N_i$  | 0.051 | 12  | 0.004 | 16.525  | 0.00 | 0.81             |
| $N_g 	imes N_i$   | 0.064 | 16  | 0.004 | 15.36   | 0.00 | 0.84             |
| Correlation $\times$ ICC $\times$ N <sub>g</sub>            | 0.047 | 12  | 0.004 | 15.243  | 0.00 | 0.79             |
| Correlation $\times$ ICC $\times$ N <sub>i</sub>            | 0.021 | 12  | 0.002 | 6.768   | 0.00 | 0.63             |
| Correlation $\times$ N <sub>g</sub> $\times$ N <sub>i</sub> | 0.045 | 16  | 0.003 | 10.919  | 0.00 | 0.78             |
| $ICC \times N_g \times Ni$                                  | 0.026 | 48  | 0.001 | 2.111   | 0.01 | 0.68             |
| Error (BS)  | 0.012 | 48  | 0.000 |         |      |                  |
| Within subjects   |       |     |       |         |      |                  |
| Method  | 0.140 | 1   | 0.140 | 148.954 | 0.00 | 0.52             |
| Method $\times$ correlation                                 | 0.096 | 1   | 0.096 | 101.735 | 0.00 | 0.43             |
| Method $\times$ ICC   | 0.010 | 3   | 0.003 | 3.634   | 0.01 | 0.07             |
| Method $\times$ N <sub>g</sub>                              | 0.058 | 4   | 0.015 | 15.434  | 0.00 | 0.31             |
| Method $\times N_i$   | 0.009 | 4   | 0.002 | 2.491   | 0.05 | 0.07             |
| Method $\times$ correlation $\times$ ICC                    | 0.043 | 3   | 0.014 | 15.287  | 0.00 | 0.25             |
| Method × correlation × $N_g$                                | 0.101 | 4   | 0.025 | 26.83   | 0.00 | 0.44             |
| Method $\times$ correlation $\times$ N <sub>i</sub>         | 0.043 | 4   | 0.011 | 11.454  | 0.00 | 0.25             |
| Method $\times$ ICC $\times$ N <sub>g</sub>                 | 0.037 | 12  | 0.003 | 3.273   | 0.00 | 0.22             |
| Method $\times$ ICC $\times$ N <sub>i</sub>                 | 0.005 | 12  | 0.000 | 0.419   | 0.95 | 0.04             |
| $Method \times N_g \times N_i$                              | 0.043 | 16  | 0.003 | 2.827   | 0.00 | 0.25             |
| Error (WS)  | 0.128 | 136 | 0.001 |         |      |                  |

Results of a mixed between-within subjects ANOVA on the coverage values for the estimate of the  $\beta_a$  parameter

Table 7

First, there was a large effect (partial  $\eta^2 = .25$ ) of the three-way interaction between method, correlation, and intraclass correlation, F(3,136) = 15.29, p < .001. When the independent variables are uncorrelated, the coverage values are about equal for both methods, regardless of ICC (figure 13, top). When the independent variables are correlated, the ML-SEM method performs worse than the 2-step LCM method when the ICC is 0.05. Increasing the ICC, mainly increases the performance of the ML-SEM method, but when the ICC is 0.3 ML-SEM still performs somewhat worse than the 2-step LCM method (figure 13, bottom).

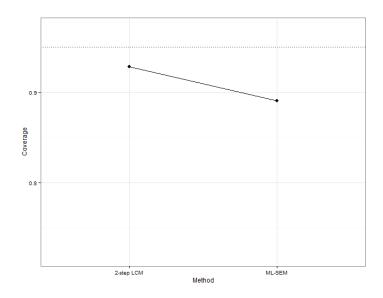


Figure 12. Interaction plot. An illustration of the main effect of method on the coverage values for the estimate of the  $\beta_g$  parameter. The dotted line represent the desired coverage value of .95.

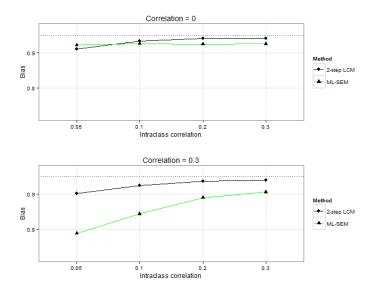


Figure 13. Interaction plot. An illustration of the three-way interaction between method, correlation and ICC on the coverage values for the estimate of the  $\beta_g$  parameter. The dotted line represent the desired coverage value of .95.

Second, there was a large effect (partial  $\eta^2 = .44$ ) of the three-way interaction between method, correlation, and number of groups, F(4,136) = 26.83, *p* < .001. When the independent variables are uncorrelated, the coverage values are about equal for both methods, except when only 20 groups are included in the simulation, in which case the ML-SEM method performs poorly (figure 14, top).

When the independent variables are correlated, something interesting happens. The 2-step LCM method still performs well, regardless of the number of groups, and the ML-SEM method still performs poorly when only 20 groups are included in the simulation. When the number of groups is increased to 50 or 100, the performance of ML-SEM increases somewhat, but is decreases again when the number of groups is increased to 200 or 500. At 500 groups, the coverage values obtained with ML-SEM are even worse than they were at 20 groups (figure 14, bottom).

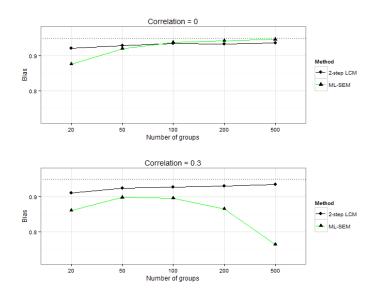


Figure 14. Interaction plot. An illustration of the three-way interaction between method, correlation and number of groups on the coverage values for the estimate of the  $\beta_g$  parameter. The dotted line represent the desired coverage value of .95.

The third large effect (partial  $\eta^2 = .25$ ) was that of the three-way interaction between method, correlation, and group size, F(4,136) = 11.45, p < .001. When the independent variables are uncorrelated, the coverage values are about equal for both methods, and they don't seem to be influenced much by the size of the groups (figure 15, top). When the independent variables are correlated, the 2-step LCM method consistently outperforms the ML-SEM method. The biggest difference between both methods occurs when each group contains only 5 individuals, and this difference decreases somewhat as the size of the groups increases (figure 15, bottom).

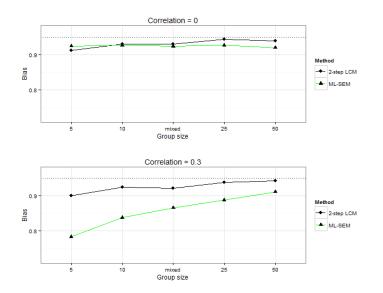
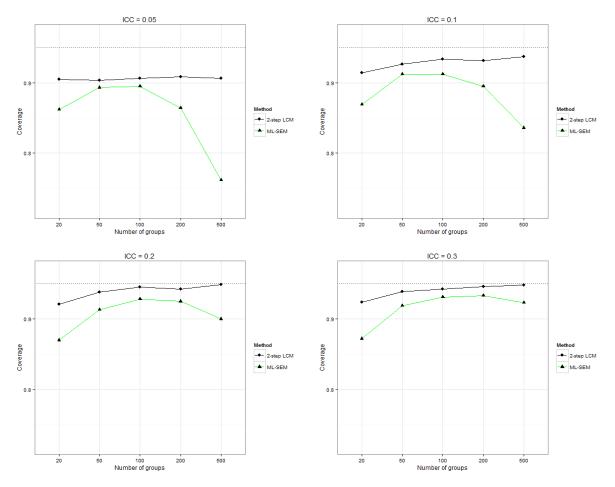


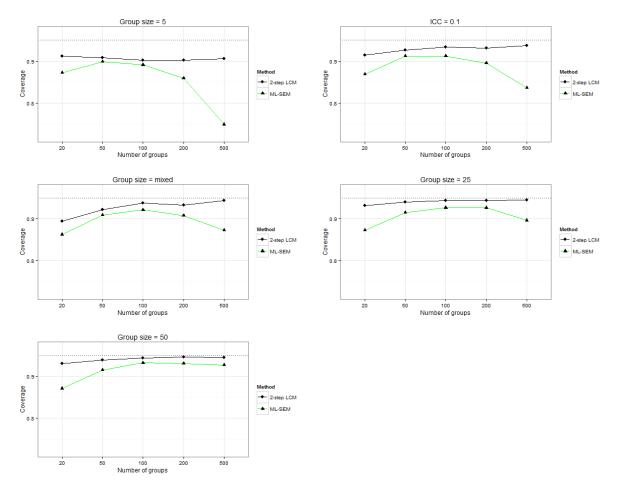
Figure 15. Interaction plot. An illustration of the three-way interaction between method, correlation and group size on the coverage values for the estimate of the  $\beta_g$  parameter. The dotted line represent the desired coverage value of .95.

The fourth large effect (partial  $n^2 = .22$ ) was the three-way interaction between method, ICC, and number of groups, F(12,136) = 3.27, p < .001. The coverage value obtained with the 2-step LCM method is somewhat low when the ICC is 0.05, regardless of the number of groups (figure 16, top left). At the higher ICC's, the coverage values for the 2-step LCM method seem to increase slightly as more groups are used in the simulation. This pattern is the same, whether the ICC is 0.1, 0.2, or 0.3 (figure 16, top right and bottom). The ML-SEM method shows a different pattern, and its coverage values are never better than those obtained with the 2-step LCM method. The coverage value obtained with ML-SEM is always low when only 20 groups are included in the simulation, and the value increases when 50 or 100 groups are used. When the ICC is 0.05, the coverage value decreases again when the number of groups is increased to 200 or 500, and at 500 groups the coverage is even worse than it was at 20 groups (figure 16, top right). When the ICC is 0.1 or 0.2, we also see this decrease in coverage when the number of groups in increased to 200 or 500, but to a much lesser extend (figure 16, top right and middle left). When the ICC is 0.3, the coverage stays pretty much what is was when the number of groups is increased from 100 to 200 and 500 (figure 16, bottom right).



*Figure 16*. Interaction plot. An illustration of the three-way interaction between method, ICC and number of groups on the coverage values for the estimate of the  $\beta_g$  parameter. The dotted line represent the desired coverage value of .95.

Finally, there was a large effect (partial  $n^2 = .25$ ) of the three-way interaction between method, number of groups, and group size, F(16,136) = 2.83, p < .001. The 2-step LCM method performs well in terms of coverage when each group is 25 or 50 individuals large, regardless of the number of groups included in the simulation (figure 17, middle right and bottom). When each group contains only 5 individuals the method performs slightly worse, and is again unaffected by the number of groups (figure 17, top left). In the conditions with 10, or a mixed number of individuals per group the coverage values are lower when only 20 groups are included in the simulation, and the coverage value increases as the number of groups increases (figure 17, top right and middle left). The ML-SEM method shows a pattern similar to the one described for the interaction between method, ICC, and number of groups. The coverage values are always low when only 20 groups are included in the simulation, and the value increases when the number of groups is increased to 50 or 100. When the size of each group is only 5, the coverage value decreases again when the number of groups is increased to 200 or 500, and at 500 groups the coverage is lower than it was at 20 groups (figure 17, top left). The same pattern can be seen when each group contains 10 or a mixed number of individuals, but the drop in coverage at 200 and 500 groups is less extreme than it was when the groups were small (figure 17, top left and middle right). When each group contains 20 or 50 individuals, there seems to be little to no drop in coverage going from 100 to 200 and 500 groups (figure 17, middle left and bottom).



*Figure 17.* Interaction plot. An illustration of the three-way interaction between method, number of groups and group size on the coverage values for the estimate of the  $\beta_g$  parameter. The dotted line represent the desired coverage value of .95.

#### Conclusion

Overall, we have not seen one of the methods surface as clearly better than the other, much depends on the examined conditions and which parameter estimate we are interested in. Looking at the bias for the estimates of both  $\beta_i$  and  $\beta_g$ , we can see that the 2-step LCM method almost always performed better than ML-SEM in the presence of correlated independent variables. When the independent variables are uncorrelated, both methods perform about the same. The coverage values for the 95% Cl's provide a more varied picture. Looking at the coverage for the 95% Cl of the

estimate of the intercept, ML-SEM outperforms the 2-step LCM method in conditions with 50 or more groups, and in conditions with a low ICC combined with a small group size. In other conditions, both methods perform equally well. Looking at the coverage values for the 95% CI of the estimate of  $\beta_i$ , ML-SEM generally performs better, especially when the group size is small. Only in conditions with a high ICC combined with a small number of groups does the 2-step LCM perform better. Lastly, we looked at the coverage for the 95% CI of the estimate of  $\beta_g$ , where the 2-step LCM generally performs better, especially in conditions where the independent variables are correlated.

These results lead us to conclude that there is currently, for the conditions we have examined, no clear preference as to which method should be used when analysing micro-macro data. Even though the 2-step LCM seems to perform better in terms of bias, the coverage values (for the intercept and  $\beta_i$ ) indicate that standard errors are not estimated optimally. We conclude that both methods have their strengths, and the choice for either method should be guided by the design of the study and which parameter estimate is of most interest.

#### Discussion

Some of the conclusions outlined in the previous chapter hinge on the results of ANOVAs that did not meet the required assumptions of normally distributed residuals and homogeneous variances. Failure to meet these assumptions may lead to an increased chance of committing a type-I error (see Glass, Peckham, and Sanders (1972) for an overview of the consequences of not meeting the assumptions for ANOVA). Though ANOVA is generally robust against moderate violations of the underlying assumptions, especially when the sample size is large (note that our sample size was not very large because we were forced to used cell-averages), a large number of groups actually increases the sensitivity of the ANOVA and the risk of finding false-positive results (Glass, Peckham, & Sanders, 1972). Unfortunately, there is no viable non-parametric alternative for the type of design we have investigated (a mixed design with more than two factors), and an extended search for a better alternative was beyond the scope of the current study. We decided that the ANOVA was our best option to summarize the results and compare the performance of the 2-step LCM and the ML-SEM method. Though this certainly does not mean all (or even most) of our conclusions are invalid, we should be cautious about interpreting the results. The plots we have shown to illustrate the interaction effects give a good indication that there are differences between methods for a number of the conditions we have examined. In the future, we could take some interesting results from the current study, and use those in a simulation with a simpler design to further investigate the performance of 2-step LCM and ML-SEM.

At the same time, we need to be careful not to dismiss ML-SEM for designs with only a small number of groups. For the current study, we used the default accelerated EM-algorithm in Mplus for estimation, but Mplus also offers a Bayesian estimation method that may perform better when the number of groups is small (Meuleman & Billiet, 2009). The exact behaviour of the Bayesian estimation method for the analysis of micro-macro data with a small number of groups would be an interesting topic to investigate in a separate study.

We should also keep in mind that the model we investigated was fairly simple, as our intention was both to illustrate that there are viable options for the analysis of multilevel models with grouplevel outcomes, and to initiate the systematic comparison of different methods for the analysis of such models. In the future, the research could be expanded to more complex models with multiple level-1 and/or level-2 independent variables. We could also investigate the performance of the methods in the presence of missing data, interactions between the independent variables, and mediation effects.

Lastly, we should consider the fact that not all researchers will have access to specialized software such as Mplus. The 2-step LCM method only requires the use of the freely available R software, which can certainly be a reason to opt for the 2-step method (provided it performs well, which we found it did). It does require the researcher to have a little knowledge of programming, which may make the method less appealing for some..

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Table 1

|              |          |           |             | Bias $\widehat{oldsymbol{eta}}_0$ | $\widehat{\beta}_0$   | Coverage $\widehat{eta_0}$ | age $\widehat{eta_0}$ | Bias $\widehat{oldsymbol{eta}}_t$ | $\hat{B}_{t}$ | Coverage $\widehat{m{eta}}_l$ | age $\widehat{m{eta}}_l$ | Bias $\widehat{eta}_g$ | $\widehat{\beta}_g$ | Coverage $\widehat{eta}_g$ | ge $\widehat{eta}_g$ |
|--------------|----------|-----------|-------------|-----------------------------------|---|----------------------------|-----------------------|-----------------------------------|---------------|-------------------------------|--------------------------|------------------------|---------------------|----------------------------|----------------------|
| $ ho_{z\xi}$ | ICCX     | 2<br>B    | ż           | LCM                               | SEM   | LCM                        | SEM                   | LCM                               | SEM           | LCM                           | SEM                      | LCM                    | SEM                 | LCM                        | SEM                  |
| 0            | 0.05     | 20        | ъ           | -0.04                             | 9.00  | 0.91                       | 0.96                  | -0.24                             | 0.01          | 0.74                          | 0.83                     | -0.01                  | 0.00                | 0.92                       | 06.0                 |
| 0            | 0.05     | 20        | 10          | 0.02                              | 00.0  | 0.91                       | 0.95                  | -0.07                             | 0.09          | 0.85                          | 0.88                     | 0.03                   | 0.00                | 0.91                       | 0.87                 |
| 0            | 0.05     | 20        | 25          | -0.01                             | 00.0  | 0.92                       | 0.93                  | 0.05                              | 0.08          | 0.89                          | 0.89                     | 0.00                   | 0.00                | 0.93                       | 0.89                 |
| 0            | 0.05     | 20        | 50          | 0.00                              | 00.0  | 0.92                       | 0.91                  | 0.02                              | 0.02          | 0.92                          | 0.89                     | 0.00                   | 0.00                | 0.92                       | 0.86                 |
| 0            | 0.05     | 20        | mixed       | -0.02                             | 00.0  | 0.89                       | 0.93                  | 0.07                              | 0.08          | 0.87                          | 0.91                     | 0.00                   | -0.01               | 0.88                       | 0.86                 |
| 0            | 0.05     | 50        | ъ           | -0.01                             | 0.01  | 0.89                       | 0.98                  | -0.56                             | 0.08          | 0.79                          | 0.89                     | -0.08                  | 0.00                | 06.0                       | 0.92                 |
| 0            | 0.05     | 50        | 10          | -0.07                             | 00.0  | 06.0                       | 0.96                  | -0.53                             | 0.13          | 0.86                          | 0.91                     | -0.04                  | 0.00                | 0.89                       | 0.92                 |
| 0            | 0.05     | 50        | 25          | 0.00                              | 00.0  | 0.93                       | 0.94                  | 0.04                              | 0.03          | 0.92                          | 0.94                     | 0.00                   | 0.00                | 0.91                       | 0.91                 |
| 0            | 0.05     | 50        | 50          | 0.00                              | 00.0  | 0.94                       | 0.94                  | 0.01                              | 0.01          | 0.95                          | 0.93                     | 0.00                   | 0.00                | 0.92                       | 06.0                 |
| 0            | 0.05     | 50        | mixed       | 0.00                              | 0.00  | 0.91                       | 0.95                  | 0.02                              | 0.03          | 06.0                          | 0.93                     | 0.00                   | 0.00                | 0.91                       | 0.92                 |
| 0            | 0.05     | 100       | ъ           | -0.01                             | 00.0  | 0.85                       | 0.97                  | 0.07                              | 0.12          | 0.77                          | 06.0                     | 0.00                   | 0.00                | 0.84                       | 0.93                 |
| 0            | 0.05     | 100       | 10          | 0.00                              | 00.0  | 06.0                       | 0.97                  | 0.09                              | 0.06          | 0.88                          | 0.95                     | 0.00                   | 0.00                | 0.91                       | 0.94                 |
| 0            | 0.05     | 100       | 25          | 0.00                              | 0.00  | 0.93                       | 0.95                  | 0.02                              | 0.01          | 0.92                          | 0.95                     | 0.00                   | 0.00                | 0.94                       | 0.93                 |
| 0            | 0.05     | 100       | 50          | 0.00                              | 0.00  | 0.94                       | 0.94                  | 0.00                              | 0.00          | 0.95                          | 0.94                     | 0.00                   | 0.00                | 0.94                       | 0.93                 |
| 0            | 0.05     | 100       | mixed       | 0.00                              | 00.0  | 0.92                       | 0.95                  | 0.02                              | 0.01          | 0.93                          | 0.95                     | 0.00                   | 0.00                | 0.92                       | 0.92                 |
| 0            | 0.05     | 200       | ы           | -0.02                             | 0.00  | 0.85                       | 0.98                  | 0.18                              | 0.07          | 0.79                          | 0.92                     | -0.01                  | 0.00                | 0.84                       | 0.94                 |
| 0            | 0.05     | 200       | 10          | 0.00                              | 00.0  | 0.92                       | 0.97                  | 0.03                              | 0.03          | 0.87                          | 0.94                     | 0.00                   | 0.00                | 0.91                       | 0.95                 |
| 0            | 0.05     | 200       | 25          | 0.00                              | 0.00  | 0.94                       | 0.95                  | 0.00                              | 0.00          | 0.92                          | 0.94                     | 0.00                   | 0.00                | 0.94                       | 0.95                 |
| 0            | 0.05     | 200       | 50          | 0.00                              | 0.00  | 0.95                       | 0.95                  | 0.00                              | 0.00          | 0.94                          | 0.94                     | 0.00                   | 0.00                | 0.93                       | 0.92                 |
| 0            | 0.05     | 200       | mixed       | 0.00                              | 00.0  | 0.94                       | 0.95                  | 0.00                              | 0.00          | 0.92                          | 0.94                     | 0.00                   | 0.00                | 0.93                       | 0.96                 |
| Note. I      | LCM= 2-: | step late | ent covaria | te methoc                         | Note. LCM= 2-step latent covariate method, SEM = multilevel SEM | ultilevel Sı               | EM                    |                                   |               |                               |                          |                        |                     |                            |                      |

## Appendix I: Results simulations

(table continues)

|               |             | 5   |       |       | Blac $\widehat{R}_{i}$ Coverage $\widehat{R}_{i}$ Blac $\widehat{R}_{i}$ | COVER | Coverage $\widehat{R_n}$ | Bias <u>R</u> . | s Â. | Coverage | age <u>R</u> . | Bias $\widehat{R_{-}}$ | Ś     | Cover | Coverage $\widehat{R}_{-}$ |
|---------------|-------------|-----|-------|-------|--|-------|--------------------------|-----------------|------|----------|----------------|------------------------|-------|-------|----------------------------|
|               |             |     |       | 22    | 04   |       | 0 d 2 0 0                | 2               | 140  |          | Id san         | 20                     | рч    |       | ubu pg                     |
| $\rho_{z\xi}$ | CCX<br>ICCX | 2°° | ž     | LCM   | SEM  | LCM   | SEM                      | LCM             | SEM  | LCM      | SEM            | LCM                    | SEM   | LCM   | SEM                        |
| 0             | 0.05        | 500 | ъ     | 0.00  | 0.00   | 0.85  | 0.97                     | 0.08            | 0.04 | 0.74     | 0.93           | 0.00                   | 0.00  | 0.83  | 0.94                       |
| 0             | 0.05        | 500 | 10    | 0.00  | 0.00   | 0.92  | 0.96                     | 0.01            | 0.01 | 06.0     | 0.95           | 0.00                   | 0.00  | 0.92  | 0.95                       |
| 0             | 0.05        | 500 | 25    | 0.00  | 0.00   | 0.93  | 0.94                     | 0.00            | 0.00 | 0.94     | 0.96           | 0.00                   | 0.00  | 0.94  | 0.95                       |
| 0             | 0.05        | 500 | 50    | 0.00  | 0.00   | 0.95  | 0.95                     | 0.00            | 0.00 | 0.96     | 0.96           | 0.00                   | 0.00  | 0.94  | 0.94                       |
| 0             | 0.05        | 500 | mixed | 0.00  | 0.00   | 0.94  | 0.96                     | 0.00            | 0.00 | 0.91     | 0.94           | 0.00                   | 0.00  | 0.92  | 0.94                       |
| 0             | 0.1         | 20  | ъ     | 0.08  | 0.02   | 0.91  | 0.95                     | 0.07            | 0.09 | 0.84     | 0.87           | 0.14                   | -0.01 | 0.91  | 0.86                       |
| 0             | 0.1         | 20  | 10    | 0.03  | 0.01   | 0.91  | 0.92                     | -0.01           | 0.11 | 06.0     | 0.92           | 0.00                   | 0.00  | 06.0  | 0.89                       |
| 0             | 0.1         | 20  | 25    | 0.00  | 0.00   | 0.93  | 0.91                     | 0.03            | 0.03 | 0.93     | 0.90           | 0.01                   | 0.01  | 0.93  | 0.87                       |
| 0             | 0.1         | 20  | 50    | 0.01  | 0.01   | 0.94  | 0.91                     | 0.02            | 0.01 | 0.94     | 0.89           | 0.01                   | 0.01  | 0.95  | 0.89                       |
| 0             | 0.1         | 20  | mixed | 0.01  | -0.01  | 0.93  | 0.93                     | 0.02            | 0.06 | 0.92     | 0.91           | -0.02                  | -0.01 | 0.91  | 0.87                       |
| 0             | 0.1         | 50  | ы     | 0.07  | 0.00   | 0.91  | 0.97                     | -0.61           | 0.13 | 0.88     | 0.94           | -0.04                  | 0.00  | 06.0  | 0.92                       |
| 0             | 0.1         | 50  | 10    | 0.00  | 0.00   | 0.92  | 0.95                     | 0.03            | 0.05 | 0.91     | 0.95           | -0.01                  | 0.00  | 0.92  | 0.93                       |
| 0             | 0.1         | 50  | 25    | 0.00  | 0.00   | 0.94  | 0.93                     | 0.01            | 0.01 | 0.94     | 0.94           | 0.00                   | 0.00  | 0.96  | 0.94                       |
| 0             | 0.1         | 50  | 50    | -0.01 | -0.01  | 0.96  | 0.95                     | 00.0            | 0.00 | 0.94     | 0.92           | 00.0                   | 0.00  | 0.95  | 0.93                       |
| 0             | 0.1         | 50  | mixed | 0.00  | 0.00   | 0.94  | 0.94                     | 0.01            | 0.01 | 0.93     | 0.93           | 0.00                   | 0.00  | 0.93  | 0.92                       |
| 0             | 0.1         | 100 | ъ     | -0.01 | 0.00   | 06.0  | 0.96                     | 0.02            | 0.04 | 0.86     | 0.93           | 0.01                   | 0.00  | 06.0  | 0.92                       |
| 0             | 0.1         | 100 | 10    | 0.00  | 0.00   | 0.94  | 0.94                     | 0.01            | 0.01 | 0.93     | 0.96           | 0.00                   | 0.00  | 0.94  | 0.94                       |
| 0             | 0.1         | 100 | 25    | 0.00  | 0.00   | 0.95  | 0.95                     | 0.01            | 0.00 | 0.95     | 0.94           | 0.00                   | 0.00  | 0.95  | 0.94                       |
| 0             | 0.1         | 100 | 50    | 0.00  | 0.00   | 0.94  | 0.94                     | 0.00            | 0.00 | 0.94     | 0.93           | 0.00                   | 0.00  | 0.94  | 0.93                       |
| 0             | 0.1         | 100 | mixed | 0.00  | 0.00   | 0.95  | 0.95                     | 0.00            | 0.00 | 0.93     | 0.94           | 00.0                   | 00.0  | 0.96  | 0.96                       |
|               |             |     |       |       |  |       |                          |                 |      |          |                |                        |       |       |                            |

Table 1 (cont.)

(table continues)

|                             |        | ņ   |       |      | Bias $\widehat{eta_0}$ | Cover | Coverage $\widehat{eta}_0$ | Bia  | Coverage $\widehat{\beta_0}$ Bias $\widehat{\beta_1}$ | Coverage | age $\widehat{eta}_l$ | Bias  | Bias $\widehat{eta_g}$ | Cover | Coverage $\widehat{eta}_g$ |
|-----------------------------|--------|-----|-------|------|------------------------|-------|----------------------------|------|---|----------|-----------------------|-------|------------------------|-------|----------------------------|
| $D_{\sigma \tilde{\Sigma}}$ | ů<br>Ľ | z   | z     | LCM  | SEM                    | LCM   | SEM                        | LCM  | SEM   | LCM      | SEM                   | LCM   | SEM                    | LCM   | SEM                        |
| 0                           | 0.1    | 200 | ъ     | 0.00 | 0.00                   | 0.91  | 0.95                       | 0.02 | 0.03  | 0.87     | 0.95                  | 0.00  | 00.0                   | 0.91  | 0.95                       |
| 0                           | 0.1    | 200 | 10    | 0.00 | 0.00                   | 0.94  | 0.95                       | 0.01 | 0.01  | 0.91     | 0.94                  | 00.0  | 00.0                   | 0.93  | 0.94                       |
| 0                           | 0.1    | 200 | 25    | 0.00 | 0.00                   | 0.95  | 0.95                       | 0.01 | 0.00  | 0.95     | 0.96                  | 0.00  | 00.0                   | 0.95  | 0.94                       |
| 0                           | 0.1    | 200 | 50    | 0.00 | 0.00                   | 0.95  | 0.95                       | 0.00 | 0.00  | 0.95     | 0.95                  | 0.00  | 00.0                   | 0.94  | 0.93                       |
| 0                           | 0.1    | 200 | mixed | 0.00 | 0.00                   | 0.94  | 0.95                       | 0.01 | 0.00  | 0.94     | 0.95                  | 0.00  | 00.0                   | 0.94  | 0.94                       |
| 0                           | 0.1    | 500 | ъ     | 0.00 | 0.00                   | 0.92  | 0.95                       | 0.01 | 0.01  | 0.88     | 0.96                  | 0.00  | 00.0                   | 0.91  | 0.94                       |
| 0                           | 0.1    | 500 | 10    | 0.00 | 0.00                   | 0.94  | 0.95                       | 0.00 | 0.00  | 0.92     | 0.94                  | 0.00  | 00.0                   | 0.94  | 0.95                       |
| 0                           | 0.1    | 500 | 25    | 0.00 | 0.00                   | 0.95  | 0.96                       | 0.00 | 0.00  | 0.94     | 0.94                  | 0.00  | 00.0                   | 0.95  | 0.95                       |
| 0                           | 0.1    | 500 | 50    | 0.00 | 0.00                   | 0.94  | 0.95                       | 0.00 | 0.00  | 0.95     | 0.95                  | 0.00  | 00.0                   | 0.95  | 0.95                       |
| 0                           | 0.1    | 500 | mixed | 0.00 | 0.00                   | 0.95  | 0.95                       | 0.00 | 0.00  | 0.92     | 0.94                  | 0.00  | 00.0                   | 0.94  | 0.94                       |
| 0                           | 0.2    | 20  | ы     | 0.01 | -0.01                  | 0.91  | 0.91                       | 0.11 | 0.10  | 0.91     | 06.0                  | -0.01 | 00.0                   | 0.92  | 0.88                       |
| 0                           | 0.2    | 20  | 10    | 0.00 | 0.00                   | 0.92  | 0.92                       | 0.06 | 0.05  | 0.92     | 06.0                  | -0.01 | -0.01                  | 0.93  | 0.86                       |
| 0                           | 0.2    | 20  | 25    | 0.01 | 0.01                   | 0.94  | 0.91                       | 0.01 | 0.01  | 0.94     | 0.88                  | 0.00  | 00.0                   | 0.94  | 0.88                       |
| 0                           | 0.2    | 20  | 50    | 0.00 | 0.00                   | 0.94  | 0.91                       | 0.00 | 0.00  | 0.93     | 0.89                  | 0.00  | 00.0                   | 0.93  | 0.87                       |
| 0                           | 0.2    | 20  | mixed | 0.00 | 0.00                   | 0.92  | 0.91                       | 0.03 | 0.03  | 0.91     | 0.88                  | 0.01  | 00.0                   | 06.0  | 0.87                       |
| 0                           | 0.2    | 50  | ы     | 0.00 | 0.00                   | 0.94  | 0.95                       | 0.03 | 0.01  | 0.92     | 0.94                  | 00.0  | 00.0                   | 0.94  | 0.92                       |
| 0                           | 0.2    | 50  | 10    | 0.00 | 0.00                   | 0.94  | 0.94                       | 0.00 | 00.0  | 0.93     | 0.91                  | 0.00  | 00.0                   | 0.94  | 0.91                       |
| 0                           | 0.2    | 50  | 25    | 0.00 | 0.00                   | 0.94  | 0.93                       | 0.01 | 00.0  | 0.94     | 0.92                  | 00.0  | 00.0                   | 0.95  | 0.93                       |
| 0                           | 0.2    | 50  | 50    | 0.00 | 0.00                   | 0.93  | 0.93                       | 0.00 | 00.0  | 0.94     | 0.92                  | 0.00  | 00.0                   | 0.95  | 0.92                       |
| 0                           | 0.2    | 50  | mixed | 0.00 | 0.00                   | 0.93  | 0.94                       | 0.01 | 0.00  | 0.93     | 0.91                  | 00.0  | 0.00                   | 0.93  | 0.91                       |
|                             |        |     |       |      |                        |       |                            |      |   |          |                       |       |                        |       |                            |

Table 1 (cont.)

(table continues)

|              |            | nverug | המוע שומא מווע בטעבועקב עמועבא לטו |                         |           | י רטבולינים |                  | - arch FCI |                  |          | (               |       | (                      |          | (                     |
|--------------|------------|--------|------------------------------------|-------------------------|-----------|-------------|------------------|------------|------------------|----------|-----------------|-------|------------------------|----------|-----------------------|
|              |            |        | ı                                  | Bias $\overline{eta}_0$ | $\beta_0$ | Cover       | Coverage $eta_0$ | Bia        | Bias $ar{eta}_l$ | Coverage | age $ar{eta}_l$ | Bia   | Bias $\widehat{eta}_g$ | Coverage | age $\widehat{eta}_g$ |
| $ ho_{z\xi}$ | CCX<br>ICC | ۳<br>ع | ž                                  | LCM                     | SEM       | LCM         | SEM              | LCM        | SEM              | LCM      | SEM             | LCM   | SEM                    | LCM      | SEM                   |
| 0            | 0.2        | 100    | ъ                                  | 0.00                    | 0.00      | 0.93        | 0.95             | 0.01       | 0.01             | 0.93     | 0.96            | 0.00  | 00.0                   | 0.95     | 0.95                  |
| 0            | 0.2        | 100    | 10                                 | 0.00                    | 0.00      | 0.96        | 0.96             | 0.01       | 0.01             | 0.94     | 0.94            | 0.00  | 00.0                   | 0.94     | 0.94                  |
| 0            | 0.2        | 100    | 25                                 | 0.00                    | 0.00      | 0.95        | 0.95             | 0.00       | 00.0             | 0.95     | 0.94            | 0.00  | 00.0                   | 0.95     | 0.94                  |
| 0            | 0.2        | 100    | 50                                 | 0.00                    | 0.00      | 0.94        | 0.94             | 0.00       | 00.0             | 0.95     | 0.94            | 0.00  | 00.0                   | 0.95     | 0.94                  |
| 0            | 0.2        | 100    | mixed                              | 0.00                    | 0.00      | 0.95        | 0.95             | 0.00       | 00.0             | 0.94     | 0.93            | 0.00  | 00.0                   | 0.94     | 0.93                  |
| 0            | 0.2        | 200    | ъ                                  | 0.00                    | 0.00      | 0.94        | 0.94             | 0.01       | 0.01             | 0.92     | 0.95            | 0.00  | 00.0                   | 0.93     | 0.94                  |
| 0            | 0.2        | 200    | 10                                 | 0.00                    | 0.00      | 0.95        | 0.95             | 0.00       | 00.0             | 0.95     | 0.94            | 0.00  | 00.0                   | 0.94     | 0.94                  |
| 0            | 0.2        | 200    | 25                                 | 0.00                    | 0.00      | 0.96        | 0.96             | 0.00       | 00.0             | 0.94     | 0.94            | 0.00  | 00.0                   | 0.96     | 0.95                  |
| 0            | 0.2        | 200    | 50                                 | 0.00                    | 0.00      | 0.95        | 0.95             | 0.00       | 00.0             | 0.95     | 0.94            | 0.00  | 00.0                   | 0.94     | 0.94                  |
| 0            | 0.2        | 200    | mixed                              | 0.00                    | 0.00      | 0.94        | 0.94             | 0.00       | 00.0             | 0.95     | 0.94            | 0.00  | 00.0                   | 0.94     | 0.94                  |
| 0            | 0.2        | 500    | ъ                                  | 0.00                    | 0.00      | 0.93        | 0.94             | 0.00       | 00.0             | 0.92     | 0.95            | 0.00  | 00.0                   | 0.94     | 0.94                  |
| 0            | 0.2        | 500    | 10                                 | 0.00                    | 0.00      | 0.95        | 0.96             | 0.00       | 00.0             | 0.94     | 0.94            | 0.00  | 00.0                   | 0.95     | 0.96                  |
| 0            | 0.2        | 500    | 25                                 | 0.00                    | 0.00      | 0.95        | 0.95             | 0.00       | 00.0             | 0.94     | 0.94            | 0.00  | 00.0                   | 0.95     | 0.94                  |
| 0            | 0.2        | 500    | 50                                 | 0.00                    | 0.00      | 0.96        | 0.96             | 0.00       | 00.0             | 0.95     | 0.96            | 0.00  | 00.0                   | 0.95     | 0.95                  |
| 0            | 0.2        | 500    | mixed                              | 0.00                    | 0.00      | 0.95        | 0.96             | 0.00       | 00.0             | 0.95     | 0.95            | 0.00  | 00.0                   | 0.96     | 0.96                  |
| 0            | 0.3        | 20     | ы                                  | -0.11                   | 0.00      | 0.93        | 0.92             | 0.93       | 0.06             | 0.92     | 0.91            | -0.20 | 00.0                   | 0.93     | 0.89                  |
| 0            | 0.3        | 20     | 10                                 | 0.01                    | 0.01      | 0.94        | 0.91             | 0.02       | 0.02             | 0.93     | 0.88            | 0.00  | 00.0                   | 0.92     | 0.87                  |
| 0            | 0.3        | 20     | 25                                 | 0.00                    | 0.00      | 0.95        | 0.93             | 0.01       | 0.01             | 0.93     | 0.87            | 00.0  | 00.0                   | 0.93     | 0.88                  |
| 0            | 0.3        | 20     | 50                                 | 0.00                    | 0.00      | 0.93        | 06.0             | 0.01       | 0.01             | 0.92     | 0.87            | 0.00  | 00.0                   | 0.93     | 0.87                  |
| 0            | 0.3        | 20     | mixed                              | 0.00                    | 0.00      | 0.92        | 0.91             | 0.01       | 0.01             | 0.91     | 0.88            | 00.0  | 00.0                   | 0.92     | 0.88                  |
|              |            |        |                                    |                         |           |             |                  |            |                  |          |                 |       |                        |          |                       |

Table 1 (cont.)

(table continues)

|               |         |     | 1     | 200  | $\rho_0$ |      | cuverage p0 |       | $p_l d$ spig | LOVELAGE PI | age p <sub>l</sub> | Da   | $_{gd}$ spig | LOVEL AGE $p_g$ | age pg |
|---------------|---------|-----|-------|------|----------|------|-------------|-------|--------------|-------------|--------------------|------|--------------|-----------------|--------|
| $\rho_{z\xi}$ | IC<br>C | å   | ž     | LCM  | SEM      | LCM  | SEM         | LCM   | SEM          | LCM         | SEM                | LCM  | SEM          | LCM             | SEM    |
| 0             | 0.3     | 50  | S     | 0.00 | 0.00     | 0.95 | 0.94        | 0.01  | 0.01         | 0.95        | 0.95               | 0.00 | 00.0         | 0.93            | 0.92   |
| 0             | 0.3     | 50  | 10    | 0.00 | 0.00     | 0.95 | 0.94        | 0.01  | 0.01         | 0.94        | 0.91               | 0.00 | 00.0         | 0.94            | 0.93   |
| 0             | 0.3     | 50  | 25    | 0.00 | 0.00     | 0.94 | 0.93        | 0.00  | 00.0         | 0.94        | 0.92               | 0.00 | 00.0         | 0.95            | 0.92   |
| 0             | 0.3     | 50  | 50    | 0.00 | 0.00     | 0.94 | 0.93        | -0.01 | -0.01        | 0.95        | 0.92               | 0.00 | 00.0         | 0.93            | 0.91   |
| 0             | 0.3     | 50  | mixed | 0.00 | 0.00     | 0.94 | 0.94        | 0.01  | 0.00         | 0.92        | 0.92               | 0.00 | 00.0         | 0.93            | 0.92   |
| 0             | 0.3     | 100 | ы     | 0.00 | 0.00     | 0.93 | 0.93        | 0.01  | 0.01         | 0.94        | 0.95               | 0.00 | 00.0         | 0.94            | 0.94   |
| 0             | 0.3     | 100 | 10    | 0.00 | 0.00     | 0.95 | 0.95        | 0.00  | 00.0         | 0.96        | 0.94               | 0.00 | 00.0         | 0.95            | 0.94   |
| 0             | 0.3     | 100 | 25    | 0.00 | 0.00     | 0.96 | 0.95        | 0.00  | 00.0         | 0.94        | 0.94               | 0.00 | 00.0         | 0.96            | 0.95   |
| 0             | 0.3     | 100 | 50    | 0.00 | 0.00     | 0.95 | 0.94        | 0.00  | 0.00         | 0.95        | 0.94               | 0.00 | 00.0         | 0.94            | 0.93   |
| 0             | 0.3     | 100 | mixed | 0.00 | 0.00     | 0.94 | 0.94        | 0.01  | 00.0         | 0.94        | 0.94               | 0.00 | 00.0         | 0.94            | 0.94   |
| 0             | 0.3     | 200 | ъ     | 0.00 | 0.00     | 0.94 | 0.95        | 0.00  | 00.0         | 0.94        | 0.95               | 0.00 | 00.0         | 0.94            | 0.94   |
| 0             | 0.3     | 200 | 10    | 0.00 | 0.00     | 0.95 | 0.95        | 0.00  | 00.0         | 0.95        | 0.95               | 0.00 | 00.0         | 0.96            | 0.95   |
| 0             | 0.3     | 200 | 25    | 0.00 | 0.00     | 0.95 | 0.95        | 0.00  | 00.0         | 0.96        | 0.95               | 0.00 | 00.0         | 0.95            | 0.94   |
| 0             | 0.3     | 200 | 50    | 0.00 | 0.00     | 0.95 | 0.95        | 0.00  | 00.0         | 0.94        | 0.93               | 0.00 | 00.0         | 0.95            | 0.95   |
| 0             | 0.3     | 200 | mixed | 0.00 | 0.00     | 0.95 | 0.95        | 0.00  | 00.0         | 0.95        | 0.95               | 0.00 | 00.0         | 0.94            | 0.94   |
| 0             | 0.3     | 500 | ъ     | 0.00 | 0.00     | 0.95 | 0.95        | 0.00  | 00.0         | 0.94        | 0.95               | 0.00 | 00.0         | 0.95            | 0.95   |
| 0             | 0.3     | 500 | 10    | 0.00 | 0.00     | 0.95 | 0.95        | 0.00  | 00.0         | 0.95        | 0.95               | 0.00 | 00.0         | 0.95            | 0.95   |
| 0             | 0.3     | 500 | 25    | 0.00 | 0.00     | 0.95 | 0.95        | 0.00  | 00.0         | 0.94        | 0.94               | 0.00 | 00.0         | 0.93            | 0.93   |
| 0             | 0.3     | 500 | 50    | 0.00 | 0.00     | 0.96 | 0.96        | 0.00  | 00.0         | 0.94        | 0.94               | 0.00 | 00.0         | 0.94            | 0.94   |
| 0             | 0.3     | 500 | mixed | 0.00 | 0.00     | 0.96 | 0.96        | 0.00  | 00.0         | 0.96        | 0.95               | 0.00 | 00.0         | 0.95            | 0.95   |

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| • | aw has and coverage values for the three regression coefficients using | 2 22          |   |

Table 1 (cont.)

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| KaW D         | ias ana c | nhniano | kuw pias aria coverage values for | _                      | נווב נווו בב ובאובאוחוו      | ו רחב))ורוב | <ul> <li>Anne anne</li> </ul> | בשובה ברוא             | נטבווים מאוווע ביובא באבא געווע ואוב-אבואו | DEIVI    |                         |                        |                     |          |                       |
|---------------|-----------|---------|-----------------------------------|------------------------|------------------------------|-------------|-------------------------------|------------------------|--|----------|-------------------------|------------------------|---------------------|----------|-----------------------|
|               |           |         | I                                 | Bias $\widehat{eta}_0$ | $\widehat{oldsymbol{eta}}_0$ | Cover       | Coverage $\widehat{eta}_0$    | Bias $\widehat{eta}_t$ | $s \widehat{\beta}_l$                      | Coverage | age $\widehat{eta}_{l}$ | Bias $\widehat{eta}_g$ | $\widehat{\beta_g}$ | Coverage | age $\widehat{eta}_g$ |
| $\rho_{z\xi}$ | ICC       | z       | ź                                 | LCM                    | SEM                          | LCM         | SEM                           | LCM                    | SEM  | LCM      | SEM                     | LCM                    | SEM                 | LCM      | SEM                   |
| 0.3           | 0.05      | 20      | ъ                                 | -0.03                  | -0.01                        | 06.0        | 0.95                          | -0.17                  | 0.02                                       | 0.74     | 0.84                    | -0.03                  | 0.07                | 0.87     | 0.84                  |
| 0.3           | 0.05      | 20      | 10                                | -0.04                  | 0.01                         | 0.91        | 0.95                          | 0.00                   | 0.06                                       | 0.85     | 0.87                    | 0.01                   | 0.07                | 0.89     | 0.82                  |
| 0.3           | 0.05      | 20      | 25                                | 0.00                   | 0.00                         | 0.91        | 0.92                          | 0.07                   | 0.06                                       | 0.92     | 0.91                    | -0.01                  | 0.04                | 0.93     | 0.85                  |
| 0.3           | 0.05      | 20      | 50                                | 0.00                   | 0.00                         | 0.93        | 0.92                          | 0.05                   | 0.03                                       | 0.92     | 0.89                    | -0.01                  | 0.03                | 0.94     | 0.88                  |
| 0.3           | 0.05      | 20      | mixed                             | 0.37                   | 0.01                         | 0.89        | 0.92                          | -1.02                  | 0.09                                       | 0.84     | 0.89                    | 0.73                   | 0.05                | 0.86     | 0.85                  |
| 0.3           | 0.05      | 50      | ъ                                 | 0.47                   | 0.00                         | 0.89        | 0.98                          | 0.86                   | 0.08                                       | 0.77     | 0.87                    | -0.18                  | 0.07                | 0.87     | 0.84                  |
| 0.3           | 0.05      | 50      | 10                                | -0.04                  | 0.00                         | 0.89        | 0.97                          | -0.02                  | 0.10                                       | 0.85     | 0.91                    | 0.01                   | 0.06                | 0.89     | 0.86                  |
| 0.3           | 0.05      | 50      | 25                                | 0.00                   | 0.00                         | 0.93        | 0.95                          | 0.05                   | 0.02                                       | 0.93     | 0.95                    | -0.01                  | 0.04                | 0.92     | 0.88                  |
| 0.3           | 0.05      | 50      | 50                                | 0.00                   | 0.00                         | 0.95        | 0.94                          | 0.01                   | 0.00                                       | 0.93     | 0.92                    | 0.00                   | 0.03                | 0.93     | 06.0                  |
| 0.3           | 0.05      | 50      | mixed                             | -0.02                  | 0.00                         | 0.89        | 0.94                          | 0.53                   | 0.03                                       | 06.0     | 0.93                    | -0.27                  | 0.05                | 0.89     | 0.88                  |
| 0.3           | 0.05      | 100     | Ŋ                                 | 0.10                   | 0.00                         | 0.85        | 0.97                          | -0.36                  | 0.10                                       | 0.75     | 0.87                    | 0.18                   | 0.07                | 0.82     | 0.78                  |
| 0.3           | 0.05      | 100     | 10                                | 0.00                   | 0.00                         | 06.0        | 0.97                          | 0.13                   | 0.03                                       | 0.89     | 0.93                    | -0.04                  | 0.06                | 0.89     | 0.84                  |
| 0.3           | 0.05      | 100     | 25                                | 0.00                   | 0.00                         | 0.92        | 0.93                          | 0.01                   | 0.00                                       | 0.93     | 0.94                    | -0.01                  | 0.04                | 0.93     | 0.89                  |
| 0.3           | 0.05      | 100     | 50                                | 0.00                   | 0.00                         | 0.95        | 0.95                          | 0.00                   | -0.01                                      | 0.95     | 0.95                    | 00.0                   | 0.03                | 0.95     | 0.92                  |
| 0.3           | 0.05      | 100     | mixed                             | 0.00                   | 00.0                         | 0.92        | 0.95                          | 0.02                   | 0.00                                       | 0.91     | 0.95                    | 00.0                   | 0.05                | 0.92     | 0.87                  |
| 0.3           | 0.05      | 200     | ъ                                 | 0.00                   | 0.00                         | 0.84        | 0.97                          | 0.07                   | 0.08                                       | 0.77     | 0.91                    | -0.03                  | 0.07                | 0.82     | 0.65                  |
| 0.3           | 0.05      | 200     | 10                                | 0.00                   | 0.00                         | 0.91        | 0.96                          | 0.02                   | 0.00                                       | 0.88     | 0.93                    | -0.01                  | 0.06                | 0.91     | 0.74                  |
| 0.3           | 0.05      | 200     | 25                                | 0.00                   | 0.00                         | 0.94        | 0.95                          | 0.00                   | -0.01                                      | 0.94     | 0.94                    | 00.0                   | 0.04                | 0.94     | 0.85                  |
| 0.3           | 0.05      | 200     | 50                                | 0.00                   | 00.0                         | 0.95        | 0.95                          | 0.00                   | -0.01                                      | 0.93     | 0.93                    | 00.0                   | 0.03                | 0.95     | 06.0                  |
| 0.3           | 0.05      | 200     | mixed                             | 0.00                   | 0.00                         | 0.93        | 0.95                          | 0.01                   | -0.01                                      | 0.93     | 0.93                    | 00.0                   | 0.05                | 0.91     | 0.78                  |
|               |           |         |                                   |                        |                              |             |                               |                        |  |          |                         |                        |                     |          |                       |

Table 1 (cont.)

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(table continues)

|               |                  |     |       | 200   | $\rho_0$ pld | Lover | coverage $p_0$ |       | bids $p_l$ | Cover | Coverage $\beta_l$ | Bia   | Bias $\widehat{eta}_g$ | Coverage $\widehat{eta}_g$ | age $\widehat{eta}_g$ |
|---------------|------------------|-----|-------|-------|--------------|-------|----------------|-------|------------|-------|--------------------|-------|------------------------|----------------------------|-----------------------|
| $\rho_{z\xi}$ | ICC <sub>X</sub> | z   | ž     | LCM   | SEM          | LCM   | SEM            | LCM   | SEM        | LCM   | SEM                | LCM   | SEM                    | LCM                        | SEM                   |
| 0.3           | 0.05             | 500 | S     | 0.01  | 0.00         | 0.85  | 0.96           | 0.13  | 0.02       | 0.79  | 0.92               | -0.05 | 0.07                   | 0.82                       | 0.31                  |
| 0.3           | 0.05             | 500 | 10    | 0.00  | 0.00         | 06.0  | 0.95           | 0.01  | -0.01      | 0.86  | 0.93               | 0.00  | 0.06                   | 0.89                       | 0.43                  |
| 0.3           | 0.05             | 500 | 25    | 0.00  | 0.00         | 0.92  | 0.94           | 0.00  | 0.00       | 0.93  | 0.94               | 0.00  | 0.04                   | 0.94                       | 0.69                  |
| 0.3           | 0.05             | 500 | 50    | 0.00  | 0.00         | 0.94  | 0.94           | 00.0  | -0.01      | 0.93  | 0.94               | 0.00  | 0.03                   | 0.94                       | 0.84                  |
| 0.3           | 0.05             | 500 | mixed | 0.00  | 0.00         | 0.92  | 0.95           | 0.00  | -0.01      | 0.92  | 0.93               | 0.00  | 0.05                   | 0.92                       | 0.63                  |
| 0.3           | 0.1              | 20  | ы     | 0.05  | 0.00         | 06.0  | 0.95           | -0.29 | 0.10       | 0.82  | 0.87               | 0.07  | 0.06                   | 06.0                       | 0.86                  |
| 0.3           | 0.1              | 20  | 10    | 0.08  | -0.01        | 0.91  | 0.92           | 0.23  | 0.08       | 06.0  | 06.0               | -0.08 | 0.05                   | 06.0                       | 0.86                  |
| 0.3           | 0.1              | 20  | 25    | 0.00  | 0.00         | 0.93  | 0.91           | 0.02  | 0.02       | 0.92  | 0.89               | 0.00  | 0.03                   | 0.93                       | 0.87                  |
| 0.3           | 0.1              | 20  | 50    | -0.01 | -0.01        | 0.95  | 0.93           | 0.03  | 0.02       | 0.92  | 0.87               | -0.01 | 0.01                   | 0.93                       | 0.87                  |
| 0.3           | 0.1              | 20  | mixed | 0.02  | 0.00         | 06.0  | 0.92           | 0.09  | 0.03       | 0.89  | 0.88               | -0.01 | 0.04                   | 0.88                       | 0.85                  |
| 0.3           | 0.1              | 50  | ъ     | 0.01  | 0.00         | 0.87  | 0.95           | 0.02  | 0.08       | 0.85  | 0.92               | 0.07  | 0.06                   | 0.87                       | 0.86                  |
| 0.3           | 0.1              | 50  | 10    | 0.00  | 0.01         | 0.92  | 0.94           | 0.13  | 0.04       | 0.91  | 0.94               | -0.04 | 0.05                   | 0.91                       | 0.88                  |
| 0.3           | 0.1              | 50  | 25    | 0.00  | 0.00         | 0.95  | 0.94           | 0.01  | 0.00       | 0.93  | 0.94               | -0.01 | 0.02                   | 0.95                       | 0.91                  |
| 0.3           | 0.1              | 50  | 50    | 0.00  | 0.00         | 0.93  | 0.92           | 0.01  | 0.00       | 0.95  | 0.93               | 0.00  | 0.02                   | 0.94                       | 0.92                  |
| 0.3           | 0.1              | 50  | mixed | 0.00  | 0.00         | 0.93  | 0.93           | 0.03  | 0.01       | 0.92  | 0.93               | 0.00  | 0.04                   | 0.93                       | 0.91                  |
| 0.3           | 0.1              | 100 | ы     | -0.01 | 0.00         | 06.0  | 0.96           | -0.12 | 0.05       | 0.87  | 0.93               | 0.03  | 0.06                   | 0.89                       | 0.82                  |
| 0.3           | 0.1              | 100 | 10    | 0.00  | 0.00         | 0.94  | 0.95           | 0.02  | 0.00       | 0.91  | 0.93               | 0.00  | 0.05                   | 0.94                       | 0.86                  |
| 0.3           | 0.1              | 100 | 25    | 0.00  | 0.00         | 0.95  | 0.95           | 0.01  | 0.00       | 0.93  | 0.93               | 0.00  | 0.02                   | 0.93                       | 0.91                  |
| 0.3           | 0.1              | 100 | 50    | 0.00  | 0.00         | 0.94  | 0.94           | 00.0  | 00.0       | 0.94  | 0.93               | 0.00  | 0.02                   | 0.93                       | 0.93                  |
| 0.3           | 0.1              | 100 | mixed | 0.00  | 0.00         | 0.94  | 0.95           | 0.01  | -0.01      | 0.92  | 0.92               | 0.00  | 0.04                   | 0.95                       | 0.91                  |

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Table 1 (cont.)

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|              |                  | 200    | יומי ביום מות בסגבו מאר גמומבה לה |       |           |       | с<br>2) (с       |      |              |          | ¢             | i     | (0           | G     | (0               |
|--------------|------------------|--------|-----------------------------------|-------|-----------|-------|------------------|------|--------------|----------|---------------|-------|--------------|-------|------------------|
|              |                  |        | ·                                 | Bias  | $\beta_0$ | Cover | Coverage $eta_0$ | Bia  | Bias $eta_l$ | Coverage | age $\beta_l$ | Bias  | Bias $eta_g$ | Cover | Coverage $eta_g$ |
| $ ho_{z\xi}$ | ICC <sub>X</sub> | ۳<br>ع | Ž                                 | LCM   | SEM       | LCM   | SEM              | LCM  | SEM          | LCM      | SEM           | LCM   | SEM          | LCM   | SEM              |
| 0.3          | 0.1              | 200    | ъ                                 | 0.00  | 0.00      | 06.0  | 0.95             | 0.04 | 0.01         | 0.85     | 0.91          | -0.01 | 0.06         | 06.0  | 0.71             |
| 0.3          | 0.1              | 200    | 10                                | 0.00  | 0.00      | 0.94  | 0.95             | 0.01 | -0.01        | 0.92     | 0.94          | 00.0  | 0.05         | 0.92  | 0.83             |
| 0.3          | 0.1              | 200    | 25                                | 0.00  | 0.00      | 0.95  | 0.95             | 0.00 | -0.01        | 0.95     | 0.94          | 00.0  | 0.03         | 0.93  | 06.0             |
| 0.3          | 0.1              | 200    | 50                                | 0.00  | 0.00      | 0.95  | 0.95             | 0.00 | 00.0         | 0.94     | 0.94          | 00.0  | 0.02         | 0.96  | 0.93             |
| 0.3          | 0.1              | 200    | mixed                             | 0.00  | 0.00      | 0.94  | 0.95             | 0.01 | -0.01        | 0.92     | 0.93          | 00.0  | 0.03         | 0.93  | 0.88             |
| 0.3          | 0.1              | 500    | ы                                 | 0.00  | 0.00      | 0.91  | 0.95             | 0.01 | -0.01        | 0.88     | 0.93          | 00.0  | 0.06         | 0.92  | 0.44             |
| 0.3          | 0.1              | 500    | 10                                | 0.00  | 0.00      | 0.94  | 0.95             | 0.01 | -0.01        | 0.92     | 0.94          | 0.00  | 0.04         | 0.93  | 0.67             |
| 0.3          | 0.1              | 500    | 25                                | 0.00  | 0.00      | 0.95  | 0.95             | 0.00 | -0.01        | 0.94     | 0.93          | 00.0  | 0.03         | 0.94  | 0.84             |
| 0.3          | 0.1              | 500    | 50                                | 0.00  | 0.00      | 0.95  | 0.95             | 0.00 | -0.01        | 0.95     | 0.95          | 00.0  | 0.02         | 0.95  | 0.92             |
| 0.3          | 0.1              | 500    | mixed                             | 0.00  | 0.00      | 0.95  | 0.95             | 0.00 | -0.01        | 0.96     | 0.95          | 00.0  | 0.03         | 0.94  | 0.76             |
| 0.3          | 0.2              | 20     | ы                                 | -0.01 | 0.00      | 0.92  | 0.94             | 0.14 | 0.09         | 06.0     | 06.0          | -0.03 | 0.04         | 0.91  | 0.87             |
| 0.3          | 0.2              | 20     | 10                                | 0.01  | 0.00      | 0.93  | 0.91             | 0.05 | 0.02         | 0.92     | 0.90          | -0.01 | 0.03         | 0.93  | 0.87             |
| 0.3          | 0.2              | 20     | 25                                | 0.00  | 0.00      | 0.95  | 0.93             | 0.02 | 0.01         | 0.92     | 0.86          | -0.01 | 0.01         | 0.93  | 0.88             |
| 0.3          | 0.2              | 20     | 50                                | 0.00  | 0.00      | 0.94  | 0.92             | 0.00 | -0.01        | 0.92     | 0.85          | 00.0  | 0.01         | 0.92  | 0.86             |
| 0.3          | 0.2              | 20     | mixed                             | 0.00  | 0.00      | 0.92  | 0.91             | 0.04 | 0.02         | 06.0     | 0.87          | -0.02 | 0.01         | 06.0  | 0.86             |
| 0.3          | 0.2              | 50     | ы                                 | 0.00  | 0.00      | 0.93  | 0.94             | 0.06 | 0.02         | 0.91     | 0.93          | -0.02 | 0.04         | 0.92  | 0.89             |
| 0.3          | 0.2              | 50     | 10                                | 0.00  | 0.00      | 0.94  | 0.94             | 0.01 | 00.0         | 0.94     | 0.93          | 00.0  | 0.03         | 0.94  | 0.91             |
| 0.3          | 0.2              | 50     | 25                                | 0.00  | 0.00      | 0.95  | 0.94             | 0.00 | 00.0         | 0.95     | 0.93          | 00.0  | 0.02         | 0.95  | 0.92             |
| 0.3          | 0.2              | 50     | 50                                | 0.00  | 0.00      | 0.95  | 0.94             | 0.01 | 0.00         | 0.94     | 0.92          | -0.01 | 0.00         | 0.95  | 0.93             |
| 0.3          | 0.2              | 50     | mixed                             | 0.00  | 0.00      | 0.94  | 0.94             | 0.01 | 0.00         | 0.93     | 0.92          | 0.00  | 0.02         | 0.91  | 0.89             |
|              |                  |        |                                   |       |           |       |                  |      |              |          |               |       |              |       |                  |

Table 1 (cont.)

(table continues)

| KaW D        | kaw bias ana coverage values | coverag | e values for | r the three              | the three regression coefficients using z-step LCM and ML-SEM | i coejjicie.               | urs using -           | VNT dais-2 | יו מנומ ואור-ר           | DEIVI    |                       |               |                     |          |                       |
|--------------|------------------------------|---------|--------------|--------------------------|---|----------------------------|-----------------------|------------|--------------------------|----------|-----------------------|---------------|---------------------|----------|-----------------------|
|              |                              |         | I            | Bias $\widehat{\beta_0}$ | $\widehat{\beta}_0$   | Coverage $\widehat{eta}_0$ | age $\widehat{eta_0}$ | Bias       | s $\widehat{m{eta}}_{l}$ | Coverage | age $\widehat{eta}_l$ | Bias <i>j</i> | s $\widehat{eta}_g$ | Coverage | age $\widehat{eta}_g$ |
| $ ho_{z\xi}$ | ICC <sup>X</sup>             | z       | Ż            | LCM                      | SEM   | LCM                        | SEM                   | LCM        | SEM                      | LCM      | SEM                   | LCM           | SEM                 | LCM      | SEM                   |
| 0.3          | 0.2                          | 100     | ъ            | 0.00                     | 0.00  | 0.92                       | 0.93                  | 0.02       | 0.00                     | 0.93     | 0.95                  | 0.00          | 0.04                | 0.94     | 0.88                  |
| 0.3          | 0.2                          | 100     | 10           | 0.00                     | 00.0  | 0.95                       | 0.95                  | 0.01       | 0.00                     | 0.95     | 0.95                  | 00.0          | 0.03                | 0.93     | 0.91                  |
| 0.3          | 0.2                          | 100     | 25           | 0.00                     | 00.00   | 0.95                       | 0.95                  | 0.00       | 0.00                     | 0.94     | 0.93                  | 00.0          | 0.01                | 0.95     | 0.93                  |
| 0.3          | 0.2                          | 100     | 50           | 0.00                     | 00.00   | 0.95                       | 0.94                  | 0.00       | 0.00                     | 0.96     | 0.94                  | 0.00          | 0.01                | 96.0     | 0.94                  |
| 0.3          | 0.2                          | 100     | mixed        | 0.00                     | 00.0  | 0.94                       | 0.95                  | 0.01       | 0.00                     | 0.94     | 0.94                  | 00.0          | 0.03                | 0.94     | 0.92                  |
| 0.3          | 0.2                          | 200     | ъ            | 0.00                     | 00.0  | 0.92                       | 0.94                  | 0.01       | -0.01                    | 0.93     | 0.95                  | 00.0          | 0.04                | 0.93     | 0.85                  |
| 0.3          | 0.2                          | 200     | 10           | 0.00                     | 00.0  | 0.95                       | 0.95                  | 00.0       | -0.01                    | 0.93     | 0.94                  | 0.00          | 0.03                | 0.95     | 0.91                  |
| 0.3          | 0.2                          | 200     | 25           | 0.00                     | 00.0  | 0.95                       | 0.95                  | 00.0       | 0.00                     | 0.94     | 0.94                  | 00.0          | 0.01                | 0.94     | 0.94                  |
| 0.3          | 0.2                          | 200     | 50           | 0.00                     | 0.00  | 0.96                       | 0.96                  | 00.0       | 0.00                     | 0.95     | 0.95                  | 00.0          | 0.01                | 0.95     | 0.94                  |
| 0.3          | 0.2                          | 200     | mixed        | 0.00                     | 00.0  | 0.94                       | 0.95                  | 00.0       | -0.01                    | 0.94     | 0.92                  | 00.0          | 0.02                | 0.94     | 06.0                  |
| 0.3          | 0.2                          | 500     | ъ            | 0.00                     | 00.00   | 0.93                       | 0.94                  | 0.00       | -0.01                    | 0.92     | 0.92                  | 00.0          | 0.04                | 0.93     | 0.66                  |
| 0.3          | 0.2                          | 500     | 10           | 0.00                     | 00.0  | 0.95                       | 0.96                  | 0.00       | -0.01                    | 0.94     | 0.94                  | 00.0          | 0.03                | 0.95     | 0.84                  |
| 0.3          | 0.2                          | 500     | 25           | 0.00                     | 00.0  | 0.95                       | 0.95                  | 00.0       | -0.01                    | 0.94     | 0.94                  | 00.0          | 0.01                | 0.95     | 0.93                  |
| 0.3          | 0.2                          | 500     | 50           | 0.00                     | 00.0  | 0.95                       | 0.95                  | 00.0       | 0.00                     | 0.94     | 0.93                  | 00.0          | 0.01                | 0.95     | 0.94                  |
| 0.3          | 0.2                          | 500     | mixed        | 0.00                     | 00.0  | 0.95                       | 0.95                  | 00.0       | -0.01                    | 0.95     | 0.95                  | 00.0          | 0.02                | 0.96     | 0.88                  |
| 0.3          | 0.3                          | 20      | ъ            | 0.00                     | 00.00   | 0.92                       | 0.92                  | 0.06       | 0.03                     | 0.91     | 0.89                  | 00.0          | 0.04                | 0.93     | 0.88                  |
| 0.3          | 0.3                          | 20      | 10           | 0.00                     | 00.00   | 0.94                       | 0.92                  | 0.03       | 0.02                     | 0.91     | 0.86                  | -0.01         | 0.02                | 0.92     | 0.86                  |
| 0.3          | 0.3                          | 20      | 25           | 0.00                     | 00.00   | 0.94                       | 0.91                  | 0.01       | 0.01                     | 0.94     | 0.87                  | -0.01         | 0.00                | 0.93     | 0.86                  |
| 0.3          | 0.3                          | 20      | 50           | -0.01                    | -0.01   | 0.93                       | 0.91                  | 0.00       | 0.00                     | 0.95     | 0.89                  | -0.01         | 0.00                | 0.93     | 0.87                  |
| 0.3          | 0.3                          | 20      | mixed        | 0.00                     | 0.00  | 0.93                       | 0.92                  | 0.02       | 0.01                     | 0.92     | 0.89                  | 0.00          | 0.01                | 06.0     | 0.86                  |
|              |                              |         |              |                          |   |                            |                       |            |                          |          |                       |               |                     |          |                       |

Table 1 (cont.)

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(table continues)

| ICC <sub>x</sub> N <sub>s</sub> N <sub>1</sub> LCM         SEM         LCM         S  |     |        | I     | Bias $\widehat{eta}_0$ | $\widehat{\beta}_0$ | Cover | Coverage $\widehat{eta_0}$ | Bia: | Bias $\widehat{oldsymbol{eta}}_l$ | Coverage | age $\widehat{m{eta}}_{l}$ | Bia:  | Bias $\widehat{eta_g}$ | Cover | Coverage $\widehat{\beta_g}$ |
|---|-----|--------|-------|------------------------|---------------------|-------|----------------------------|------|-----------------------------------|----------|----------------------------|-------|------------------------|-------|------------------------------|
| 0.3 $50$ $5$ $0.00$ $0.95$ $0.3$ $50$ $10$ $0.00$ $0.96$ $0.3$ $50$ $25$ $0.00$ $0.96$ $0.3$ $50$ $25$ $0.00$ $0.94$ $0.3$ $50$ $50$ $0.00$ $0.94$ $0.3$ $100$ $5$ $0.00$ $0.94$ $0.3$ $100$ $5$ $0.00$ $0.96$ $0.3$ $100$ $5$ $0.00$ $0.95$ $0.3$ $100$ $25$ $0.00$ $0.96$ $0.3$ $200$ $10$ $0.00$ $0.96$ $0.3$ $200$ $10$ $0.00$ $0.96$ $0.3$ $200$ $10$ $0.00$ $0.96$ $0.3$ $200$ $10$ $0.00$ $0.96$ $0.3$ $200$ $10$ $0.00$ $0.96$ $0.3$ $200$ $10$ $0.00$ $0.96$ $0.3$ <   |     | ۳<br>۳ | ź     | LCM                    | SEM                 | LCM   | SEM                        | LCM  | SEM                               | LCM      | SEM                        | LCM   | SEM                    | LCM   | SEM                          |
| 0.3         50         10         0.00         0.96           0.3         50         25         0.00         0.93           0.3         50         50         0.00         0.93           0.3         50         50         0.00         0.94           0.3         50         mixed         0.00         0.94           0.3         100         5         0.00         0.90         0.94           0.3         100         5         0.00         0.90         0.94           0.3         100         5         0.00         0.90         0.94           0.3         100         5         0.00         0.90         0.95           0.3         100         50         0.00         0.90         0.95           0.3         200         5         0.00         0.90         0.95           0.3         200         10         0.90         0.96         9.96           0.3         200         10         0.90         9.96         9.96           0.3         200         10         0.90         9.96         9.96           0.3         200         10         0.90 <t< td=""><td>0.3</td><td>50</td><td>ъ</td><td>0.00</td><td>00.0</td><td>0.95</td><td>0.94</td><td>0.02</td><td>00.0</td><td>0.92</td><td>0.92</td><td>-0.01</td><td>0.03</td><td>0.94</td><td>0.92</td></t<> | 0.3 | 50     | ъ     | 0.00                   | 00.0                | 0.95  | 0.94                       | 0.02 | 00.0                              | 0.92     | 0.92                       | -0.01 | 0.03                   | 0.94  | 0.92                         |
| 0.3         50         25         0.00         0.03           0.3         50         50         0.00         0.04           0.3         50         70         0.00         0.94           0.3         100         5         0.00         0.94           0.3         100         5         0.00         0.94           0.3         100         5         0.00         0.93           0.3         100         25         0.00         0.93           0.3         100         25         0.00         0.95           0.3         100         7         0.90         0.95           0.3         200         7         0.90         0.90           0.3         200         7         0.90         0.95           0.3         200         7         0.90         0.96           0.3         200         7         0.90         0.96           0.3         200         7         0.90         0.96           0.3         200         9         0.90         0.96           0.3         500         7         0.90         0.96           0.3         500   | 0.3 | 50     | 10    | 0.00                   | 00.0                | 0.96  | 0.95                       | 0.01 | 0.01                              | 0.94     | 0.92                       | 0.00  | 0.02                   | 0.96  | 0.93                         |
| 0.3         50         50         0.00         0.04           0.3         50         mixed         0.00         0.94           0.3         100         5         0.00         0.90         0.94           0.3         100         5         0.00         0.00         0.94           0.3         100         5         0.00         0.90         0.95           0.3         100         25         0.00         0.00         0.95           0.3         100         50         0.00         0.90         0.95           0.3         200         50         0.00         0.90         0.95           0.3         200         50         0.00         0.90         0.96           0.3         200         10         0.90         0.96         9.96           0.3         200         10         0.90         9.96         9.96           0.3         200         10         0.90         9.96         9.96           0.3         200         10         0.90         9.96         9.96           0.3         500         10         0.90         9.96         9.96           0.3   | 0.3 | 50     | 25    | 0.00                   | 00.0                | 0.93  | 0.93                       | 00.0 | 00.0                              | 0.94     | 0.92                       | 0.00  | 0.01                   | 0.93  | 0.91                         |
| 0.3 $50$ mixed $0.00$ $0.00$ $0.94$ $0.3$ $100$ $5$ $0.00$ $0.95$ $0.95$ $0.3$ $100$ $5$ $0.00$ $0.90$ $0.95$ $0.3$ $100$ $25$ $0.00$ $0.95$ $0.95$ $0.3$ $100$ $25$ $0.00$ $0.90$ $0.95$ $0.3$ $100$ $7$ $0.00$ $0.90$ $0.95$ $0.3$ $200$ $7$ $0.00$ $0.90$ $0.96$ $0.3$ $200$ $10$ $0.00$ $0.90$ $0.96$ $0.3$ $200$ $10$ $0.00$ $0.90$ $0.96$ $0.3$ $200$ $10$ $0.00$ $0.96$ $0.96$ $0.3$ $200$ $100$ $0.00$ $0.96$ $0.96$ $0.3$ $200$ $100$ $0.00$ $0.96$ $0.96$ $0.3$ $200$ $100$ $0.00$ $0.96$ <td< td=""><td>0.3</td><td>50</td><td>50</td><td>0.00</td><td>0.00</td><td>0.94</td><td>0.93</td><td>00.0</td><td>00.0</td><td>0.94</td><td>0.91</td><td>0.00</td><td>0.00</td><td>0.94</td><td>0.91</td></td<>   | 0.3 | 50     | 50    | 0.00                   | 0.00                | 0.94  | 0.93                       | 00.0 | 00.0                              | 0.94     | 0.91                       | 0.00  | 0.00                   | 0.94  | 0.91                         |
| 0.3         100         5         0.00         0.95           0.3         100         10         0.00         0.95           0.3         100         25         0.00         0.95           0.3         100         25         0.00         0.95           0.3         100         25         0.00         0.95           0.3         100         70         0.90         0.95           0.3         100         70         0.00         0.95           0.3         200         7         0.00         0.90           0.3         200         10         0.00         0.95           0.3         200         10         0.00         0.95           0.3         200         200         0.00         0.90           0.3         200         10         0.90         0.95           0.3         200         0.00         0.90         0.95           0.3         500         10         0.90         0.95           0.3         500         10         0.90         0.95           0.3         500         200         0.90         0.90           0.3   | 0.3 | 50     | mixed | 0.00                   | 00.0                | 0.94  | 0.93                       | 0.01 | 00.0                              | 0.92     | 0.91                       | 0.00  | 0.02                   | 0.94  | 0.92                         |
| 0.3         100         10         0.00         0.95           0.3         100         25         0.00         0.00         0.95           0.3         100         50         0.00         0.00         0.95           0.3         100         50         0.00         0.00         0.95           0.3         100         mixed         0.00         0.00         0.96           0.3         200         50         0.00         0.00         0.96           0.3         200         50         0.00         0.00         0.96           0.3         200         50         0.00         0.00         0.95           0.3         200         50         0.00         0.00         0.94           0.3         200         50         0.00         0.90         0.94           0.3         500         50         0.00         0.99         9.94           0.3         500         10         0.00         0.99         9.94           0.3         500         10         0.00         0.99         9.96           0.3         500         25         0.00         0.90         9.96      <  | 0.3 | 100    | ъ     | 0.00                   | 00.0                | 0.95  | 0.95                       | 00.0 | -0.01                             | 0.93     | 0.93                       | 0.00  | 0.03                   | 0.94  | 0.91                         |
| 0.3         100         25         0.00         0.95           0.3         100         50         0.00         0.96           0.3         100         50         0.00         0.96           0.3         100         mixed         0.00         0.96           0.3         200         7         0.00         0.96           0.3         200         7         0.00         0.96           0.3         200         7         0.00         0.90           0.3         200         7         0.00         0.96           0.3         200         25         0.00         0.90         9.94           0.3         200         50         0.00         0.90         9.94           0.3         200         7         0.90         0.94         9.94           0.3         200         7         0.90         0.94         9.94           0.3         500         7         0.90         0.96         9.94           0.3         500         7         0.90         0.96         9.95           0.3         500         7         0.90         0.96         9.95   | 0.3 | 100    | 10    | 0.00                   | 00.0                | 0.95  | 0.95                       | 00.0 | 00.0                              | 0.93     | 0.93                       | 0.00  | 0.02                   | 0.94  | 0.92                         |
| 0.3         100         50         0.00         0.96           0.3         100         mixed         0.00         0.96           0.3         200         5         0.00         0.96           0.3         200         5         0.00         0.96           0.3         200         5         0.00         0.96           0.3         200         5         0.00         0.96           0.3         200         5         0.00         0.90           0.3         200         5         0.00         0.90           0.3         200         5         0.90         0.90           0.3         200         5         0.90         0.90           0.3         5         0.90         9         9           0.3         5         0.90         9         9           0.3         5         0.90         9         9           0.3         5         0.90         9         9           0.3         5         9         9         9         9           0.3         5         9         9         9         9           0.3         5  | 0.3 | 100    | 25    | 0.00                   | 00.0                | 0.95  | 0.94                       | 00.0 | 00.0                              | 0.94     | 0.93                       | 0.00  | 0.01                   | 0.94  | 0.92                         |
| 0.3         100         mixed         0.00         0.00         0.96           0.3         200         5         0.00         0.90         0.95           0.3         200         10         0.00         0.90         0.95           0.3         200         10         0.00         0.95         0.95           0.3         200         25         0.00         0.96         0.94           0.3         200         50         0.00         0.90         0.94           0.3         200         mixed         0.00         0.90         0.94           0.3         200         mixed         0.00         0.90         0.94           0.3         500         mixed         0.90         0.90         0.95           0.3         500         10         0.00         0.90         0.95           0.3         500         25         0.00         0.90         0.95           0.3         500         500         0.00         0.90         0.95           0.3         500         500         0.90         0.90         0.95  | 0.3 | 100    | 50    | 0.00                   | 00.0                | 0.96  | 0.96                       | 00.0 | 00.0                              | 0.95     | 0.94                       | 0.00  | 00.0                   | 0.94  | 0.94                         |
| 0.3         200         5         0.00         0.95           0.3         200         10         0.00         0.95           0.3         200         10         0.00         0.95           0.3         200         25         0.00         0.95           0.3         200         25         0.00         0.95           0.3         200         50         0.00         0.90         0.94           0.3         200         mixed         0.00         0.90         0.94           0.3         200         mixed         0.00         0.90         0.94           0.3         500         10         0.00         0.90         0.95           0.3         500         25         0.00         0.90         0.95           0.3         500         25         0.00         0.90         0.95           0.3         500         50         0.00         0.90         0.95   | 0.3 | 100    | mixed | 0.00                   | 00.0                | 0.96  | 0.96                       | 0.00 | 00.0                              | 0.95     | 0.95                       | 0.00  | 0.01                   | 0.93  | 0.92                         |
| 0.3         200         10         0.00         0.00         0.95           0.3         200         25         0.00         0.04         0.94           0.3         200         50         0.00         0.00         0.94           0.3         200         50         0.00         0.00         0.94           0.3         200         mixed         0.00         0.00         0.94           0.3         500         5         0.00         0.00         0.95           0.3         500         5         0.00         0.00         0.96           0.3         500         25         0.00         0.00         0.95           0.3         500         50         0.00         0.00         0.95           0.3         500         50         0.00         0.00         0.95  | 0.3 | 200    | ъ     | 0.00                   | 00.0                | 0.95  | 0.95                       | 00.0 | -0.01                             | 0.93     | 0.95                       | 0.00  | 0.03                   | 0.95  | 0.89                         |
| 0.3       200       25       0.00       0.94         0.3       200       50       0.00       0.94         0.3       200       50       0.00       0.94         0.3       200       mixed       0.00       0.94         0.3       500       7       0.00       0.95         0.3       500       5       0.00       0.90       0.95         0.3       500       10       0.00       0.90       0.95         0.3       500       25       0.00       0.00       0.96         0.3       500       50       0.00       0.90       0.95         0.3       500       50       0.00       0.00       0.95   | 0.3 | 200    | 10    | 0.00                   | 00.0                | 0.95  | 0.95                       | 0.00 | -0.01                             | 0.95     | 0.94                       | 0.00  | 0.02                   | 0.95  | 0.92                         |
| 0.3       200       50       0.00       0.94         0.3       200       mixed       0.00       0.00       0.95         0.3       500       5       0.00       0.00       0.95         0.3       500       5       0.00       0.00       0.95         0.3       500       10       0.00       0.00       0.95         0.3       500       25       0.00       0.00       0.95         0.3       500       75       0.00       0.00       0.95         0.3       500       50       0.00       0.00       0.95   | 0.3 | 200    | 25    | 0.00                   | 00.0                | 0.94  | 0.94                       | 00.0 | 00.0                              | 0.94     | 0.94                       | 0.00  | 0.01                   | 0.94  | 0.94                         |
| 0.3         200         mixed         0.00         0.00         0.95           0.3         500         5         0.00         0.00         0.95           0.3         500         10         0.00         0.00         0.95           0.3         500         10         0.00         0.00         0.95           0.3         500         25         0.00         0.00         0.95           0.3         500         50         0.00         0.00         0.95           0.3         500         50         0.00         0.00         0.95   | 0.3 | 200    | 50    | 0.00                   | 00.0                | 0.94  | 0.94                       | 00.0 | 00.0                              | 0.96     | 0.95                       | 0.00  | 0.00                   | 0.95  | 0.94                         |
| 0.3     500     5     0.00     0.95       0.3     500     10     0.00     0.96       0.3     500     25     0.00     0.96       0.3     500     25     0.00     0.95       0.3     500     50     0.00     0.95       0.3     500     70     0.00     0.95  | 0.3 | 200    | mixed | 0.00                   | 00.0                | 0.95  | 0.95                       | 00.0 | 00.0                              | 0.95     | 0.95                       | 0.00  | 0.01                   | 0.93  | 0.92                         |
| 0.3     500     10     0.00     0.96       0.3     500     25     0.00     0.00     0.95       0.3     500     50     0.00     0.00     0.95       0.3     500     50     0.00     0.00     0.95  | 0.3 | 500    | ъ     | 0.00                   | 00.0                | 0.95  | 0.95                       | 00.0 | -0.01                             | 0.94     | 0.94                       | 0.00  | 0.03                   | 0.95  | 0.81                         |
| 0.3 500 25 0.00 0.00 0.95<br>0.3 500 50 0.00 0.00 0.95  | 0.3 | 500    | 10    | 0.00                   | 00.0                | 0.96  | 0.96                       | 00.0 | 00.0                              | 0.95     | 0.94                       | 0.00  | 0.02                   | 0.95  | 06.0                         |
| 0.3 500 50 0.00 0.00 0.95   |     | 500    | 25    | 0.00                   | 00.0                | 0.95  | 0.95                       | 00.0 | 00.0                              | 0.95     | 0.95                       | 0.00  | 0.01                   | 0.96  | 0.94                         |
| ۍ دي mixed 0.00 0.00 0.95   | 0.3 | 500    | 50    | 0.00                   | 00.0                | 0.95  | 0.95                       | 00.0 | 00.0                              | 0.96     | 0.96                       | 0.00  | 0.00                   | 0.94  | 0.94                         |
| 3 500   | 0.3 | 500    | mixed | 0.00                   | 00.00               | 0.95  | 0.95                       | 0.00 | 00.0                              | 0.95     | 0.95                       | 0.00  | 0.01                   | 0.96  | 0.92                         |

Table 1 (cont.)

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## Appendix II: Syntax 2-step latent covariate model

Original S-plus syntax taken from Croon & Van Veldhoven (2007). Only a few minor alterations were needed to make the script run in R, some comments were added to make the script easier to read.

```
# script file calls a user defined function called: varnames
varnames <- function(name, m) {</pre>
  test <- is.character(name)</pre>
  if(!test)
    stop("Variable names should be alphanumeric!")
  v <- paste(name, 1:m, sep = "")</pre>
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}
# fmt is a formula specifying the regression model for y on the
# group explanatory variables x and the corrected group means xtilde
# example: fmt <- as.formula (y~z1+z2+z3+xtilde1+xtilde2)</pre>
fmt <- as.formula(y~z1+xtilde1)</pre>
# data1 contains the group-level variables
# data2 contains the individual-level variables
data1 <- my data ind
data2 <- my data group
# number of groups
ng <- dim(data1)[1]
# number of individuals
nt <- dim(data2)[1]</pre>
# group id numbers
q <- data2[,1]</pre>
# number of individuals per group
ns <- tapply(rep(1,nt),g,sum)</pre>
# number of group level independent variables
mgroup <- dim(data1)[2]-1</pre>
# number of individual level independent variables
mind <- \dim(data2)[2]-1
# matrix of individual level independent variables
x <- as.matrix(data2[,1+(1:mind)])</pre>
# matrix of group level independent variables
z <- as.matrix(data1[,1:mgroup])</pre>
# matrix of group level dependent variable
y <- data1[,1+mgroup]</pre>
# mean x
mux <- apply(x,2,mean)</pre>
# mean z
muz <- apply(z,2,mean)</pre>
# observed scores on z minus mean z
dz <- z - matrix(rep(muz,ng),ncol=mgroup,byrow=T)</pre>
# placeholder matrix
xmean <- matrix(0,ng,mind)</pre>
```

```
# group-mean for each variable in x
for (k in 1:mind) {
  xmean[,k]<- tapply(x[,k],g,mean)</pre>
}
# variance matrix
vv <- var(cbind(z,xmean))</pre>
# number(ed) group level independent variables (1, 2, 3,..., mgroup)
ind1 <- 1:mgroup</pre>
# number of group level independent variables + number(ed) individual
# level independent variables (1+mgroup, 2+mgroup, 3+mgroup,..., mind+mgroup)
ind2 <- mgroup + (1:mind)</pre>
# variance z group
vzz <- vv[ind1, ind1, drop=F]</pre>
# covariance z group and x ind (?)
vzxi <- vv[ind1, ind2, drop=F]</pre>
# individual deviations from group mean
mm <- matrix(rep(c(xmean), rep(ns, mind)), ncol=mind)</pre>
d <- x -mm
# MSE (SSE/G-1)
mse <- t(na.omit(d)) %*\% na.omit(d)/(nt-ng)
vu <- mse
# group level deviations from grand mean
d <- mm - matrix(rep(mux,nt),ncol=mind,byrow=T)</pre>
# MSA (SSA/N-G)
msa <- t(d) %*% d/(ng-1)
# estimation between group covariance matrix
cc <- (nt - sum(ns^2)/nt)/(ng-1)
vxi <- (msa-mse)/cc
# computation of adjusted group means
xtilde <- matrix(0,ng,mind)</pre>
r2 <- solve(vzz,vzxi)
r1 <- vxi - t(vzxi) %*% r2
id <- diag(mind)
# adjusted group means
for (l in 1:ng) {
  p <- solve(r1 +vu/ns[1],r1)</pre>
  q <- r2 %*% (id-p)
  xtilde[1,] <- xmean[1,] %*% p +mux %*% (id-p) + dz[1,] %*% q</pre>
}
# combine z_group, group means x_ind, adjusted group means x_ind, and y_group in df
daf <- data.frame(z,xmean,xtilde,y)</pre>
# set column names
dimnames(daf)[[2]]<-
c(varnames("z",mgroup),varnames("xmean",mind),varnames("xtilde",mind),"y")
# regression analysis according to model specified by fmt
res <- lm(fmt,daf)</pre>
# White-Davidson-MacKinnon correction for heteroscedasticity
# formula, independent variables
tm <- unlist(fmt[[3]])</pre>
```

```
# unlist variable names
var <- unlist(strsplit(as.character(tm), " "))</pre>
lab <- var[!var=="+"]</pre>
u <- cbind(rep(1,ng),as.matrix(daf[lab]))</pre>
e <- res$residuals
p <- solve(t(u) %*% u)
h <- diag(u %*% p %*% t(u))
d <- e^2/(1-h)
v <- p %*% t(u) %*% diag(d) %*% u %*% p
se <- sqrt(diag(v))</pre>
# parameter estimates
estim <- summary(res)$coefficients[,1:2]</pre>
# combine with corrected se
estim <- cbind(estim,se)</pre>
# set column names
dimnames(estim)[[2]] <- c("b","uncorrected se","corrected se")</pre>
```

## Appendix III: Syntax multilevel structural equations model

This Mplus syntax is copied from Preacher, Zhang & Zyphur (2010) and adapted for use with the micro-macro model described in this paper.

TITLE: Micro-macro ML-SEM DATA: FILE IS data.csv; ! file containing raw data in long format VARIABLE: NAMES ARE group x\_ind z\_group y\_group; USEVARIABLES ARE group x\_ind z\_group y\_group; CLUSTER IS group; ! Level-2 grouping identifier BETWEEN ARE z group y group; ! identify variables with only between variance; ANALYSIS: TYPE IS TWOLEVEL RANDOM; MODEL: ! model specification follows %WITHIN% ! model for within effects follows x ind; ! estimate level-1 (residual) variance for x ind %BETWEEN% ! model for between effects follows z group y group; ! estimate level-2 (residual) variances for z group and y group y\_group ON z\_group; ! regress y\_group on z\_group y\_group ON x\_ind; ! regress y\_group on x\_ind OUTPUT: TECH1 TECH8; ! request parameter specifications, starting values, ! and optimization history for all effects