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“A New Situation”

Ludwig Wittgenstein on

the First Incompleteness Theorem

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Dieser Geist ... äußert sich in einem Fortschritt, in einem Bauen immer größerer und komplizierterer Strukturen, jener andere in einem Streben nach Klarheit und Durchsichtigkeit welcher Strukturen immer.

– Ludwig Wittgenstein, *Philosophische Bemerkungen*, Vorwort

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References to primary literature

In citing from Wittgenstein's writings in German I will use the original language, giving the English translations in a footnote. However, when treating the texts from Wittgenstein's *Nachlass*, I will cite only the English translations. All translations are done by Timothy Pope (University of Lethbridge) under the authority of Victor Rodych (University of Lethbridge), and can be found in (Rodych 2002) and (Rodych 2003). In passages where Wittgenstein uses symbolic notation, I have used modern notation, instead of Wittgenstein's own.

PG *Philosophische Grammatik. Werkausgabe Band 4*, ed. Rush Rhees (Frankfurt: Suhrkamp, 1984; first edn. 1969).

Philosophical Grammar, tr. A. J. P. Kenny (Oxford: Blackwell, 1974). References are to page numbers.

PR *Philosophische Bemerkungen. Werkausgabe Band 2*, ed. Rush Rhees (Frankfurt: Suhrkamp, 1984; first edn. 1964).

Philosophical Remarks, tr. R. Hargreaves and R. White (Oxford: Blackwell, 1975). References are to numbered paragraphs.

PU	*Philosophische Untersuchungen* in: *Werkausgabe Band 1*, ed. G. E. M. Anscombe and R. Rhees (Frankfurt: Suhrkamp, 1984; 1st edn 1953). *Philosophical Investigations*, tr. G. E. M. Anscombe (Oxford: Basil Blackwell, 1978). References are to the sections of Part I (except the footnotes), and to the page numbers of the footnotes and of part II.
LFM	*Wittgenstein's Lectures on the Foundations of Mathematics, Cambridge 1939* from the notes of R. G. Bosanquet, N. Malcolm, R. Rhees and Y. Smythies, ed. C. Diamond (Chicago: Chicago University Press, 1989; 1st edn. 1976). References are to page numbers.
RFM	*Bemerkungen über die Grundlagen der Mathematik. Werkausgabe Band 6* ed. G. H. von Wright, R. Rhees and G.E.M. Anscombe (Frankfurt: Suhrkamp, 1984; 1st edn. 1956). *Remarks on the Foundations of Mathematics*, tr. G. E. M. Anscombe (Oxford: Basil Blackwell, 1978). References are to sections and paragraphs.
TLP	*Tractatus Logico-Philosophicus* in: *Werkausgabe Band 1*, (Frankfurt: Suhrkamp, 1984; 1st edn. 1922). References are to numbered paragraphs.
WVC	*Wittgenstein und der Wiener Kreis. Werkausgabe Band 3*, shorthand notes recorded by Friedrich Waismann, ed. B. F. McGuinness (Frankfurt: Suhrkamp, 1984; first edn. 1967). *Ludwig Wittgenstein and the Vienna Circle*, tr. B. F. McGuinness (Oxford: Blackwell, 1979). References are to page numbers.

Chapter 1

Introduction

It may safely be said that among the topics Wittgenstein wrote on during his lifetime, mathematics has attracted the least interest from commentators, at least relative to the extent of the writings Wittgenstein produced on the topic during his lifetime. From 1929 to 1944, around half of his writings where devoted to the subject. The reason for this lack of interest is probably that mathematics as a topic is only discussed in a handful of paragraphs in the *Tractatus Logico-Philosophics* and takes a very minor position in the *Philosophische Untersuchungen*, featuring only in examples of rule-following. Therefore it is largely absent from the two most extensive works on philosophy which Wittgenstein prepared for publication during his lifetime. As a result the importance of the topic is easily overlooked. The publication of the *Bemerkungen über die Grundlagen der Mathematik* in 1956 did not much to change matters. A large part of the writings on mathematics written between 1937 and 1944 now became available, but in a heavily edited form, which failed to make the impression it deserves. Early reviewers, like Paul Bernays, Georg

Kreisel and Michael Dummett were dismissive of what they perceived as a '(strictly) finitist' account of mathematics, with Kreisel even calling it "a surprisingly insignificant product".[1]

Among the criticized portions of the book were the 1937 remarks about Gödel's First Incompleteness Theorem (FIT). These were dismissed by Kreisel, Bernays[2] and Dummett[3] on the grounds that Wittgenstein appeared to have had no proper understanding of the result by Gödel. Interest in the comments have since then soared, mainly after the publication of a thorough examination of the comments by Stuart Shanker. The debate was later fueled by Juliet Floyd and Victor Rodych who have both contributed to the discussion with several articles (with Floyd collaborating with Hilary Putnam on one of them). Others, namely Mark Steiner, Wolfgang Kienzler and Sebastian Sunday Grève have (the latter two jointly) contributed one each.

In this thesis I will evaluate and make use of the commentaries written from 1988 onwards. In the next chapter I will provide a summary and brief discussion of the position each commentator has taken in the debate. As we will see, the positions taken by each of those commentators differ in a remarkably high degree. However, it is possible to distinguish several lines of conflict, with each side of the line representing a different view on Wittgenstein's philosophy of mathematics as a whole. Therefore it seems worthwhile to evaluate Wittgenstein's view on such topics as Platonism in mathematics, the role of mathematical propositions within and outside of mathematics and the concepts 'truth' and 'proof'. Hereby I will take into account the fact that Wittgenstein's thoughts on the subject changed significantly between 1933 and 1937. In the third chapter I will discuss this, and defend an approach to Wittgenstein which is not commonly adopted. This

1. Kreisel 1958.
2. Bernays 1959.
3. Dummett 1959.

means I will explain that Wittgenstein should not be seen as a revisionist or radical constructivist.

With the picture of Wittgenstein's general philosophy of mathematics clear, we can look at his remarks on FIT. These are scattered throughout his published works and the *Nachlass*, the latter of which is only completely made public on CD-ROM[4] and online.[5] The most substantial part of those remarks is contained in *Anhang* III of *Teil* I of the *Bemerkungen* (RFM I A.III). This part can be viewed as a thoroughly worked out essay on the topic, was written in 1937, and was intended to be published as part of the *Philosophische Untersuchungen.* Wittgenstein later abandoned the idea of publication, and after 1944 abandoned the philosophy of mathematics altogether. Apart from this appendix, Wittgenstein mentions Gödel's result explicitly, although briefly, in (RFM VII, 19), and discusses the topic more thoroughly in (RFM VII, 21–22). Wittgenstein also discussed the topic during the lectures he held on the foundations of mathematics in Cambridge in 1939. These later comments, together with the unpublished one in the *Nachlass*, give further insight in the significance Gödel's result had for Wittgenstein. Therefore I will discuss them here as well.

Finally, in my conclusion, I will answer my research questions. The main question I want to answer is:

> What were the origins of Wittgenstein's remarks on FIT?

In order to answer this question, I will try to answer the following sub-questions:

1. What did Wittgenstein think about such notions as '(mathematical) truth', 'proof', 'proposition' and 'meaning'?

4. This was done during the period 1998–2000 as part of the Bergen project.
5. www.wittgensteinsource.org

2. How does FIT challenge the meaning of these notions?

3. Did Wittgenstein have a correct understanding of Gödel's theorem and proof?

Chapter 2

Reading Wittgenstein on Gödel

From 1988 onwards, up to five approaches have been offered to interpret the remarks made by Wittgenstein. In what follows, I will name those: 1. The anti-platonist approach, offered by Stuart Shanker; 2. The compatibility approach, offered by Juliet Floyd and Hillary Putnam; 3. The revisionist approach, offered by Victor Rodych; 4. The enhanced compatibility approach,[1] offered by Mark Steiner; and 5. The inconclusive approach; offered by Wolfgang Kienzler and Sebastian Sunday Grève. I will summarize those approaches here and evaluate them afterwards, bringing to light the main lines of division. In the last part of this chapter, I will give an outline of my own approach, which I will develop further in the rest of my thesis.

1. On naming this approach, I draw on Rodych (2006).

2.1 Five approaches to Wittgenstein's remarks

2.1.1 Shanker: the anti-platonist approach

The first thorough examination of Wittgenstein's remarks on Gödel was offered in a lengthy discussion by Stuart Shanker.[2] Shanker follows Wittgenstein in questioning Gödel's own interpretation of his theorem. It was Wittgenstein's objective to show that "the nature of mathematics forces a reinterpretation of Gödel's theorem."[3] As Shanker makes clear, Gödel's attitude toward mathematics was "thoroughly platonist" and that he saw "no obstacle to the notion of 'true but unprovable propositions'."[4] What makes Wittgenstein's critique of FIT so unlike conventional treatments of the issues, i.e. those by scholars in mathematical logic, is that it asks the question: "what if the issues raised by the framework conditions inspiring Gödel's interpretation of his theorem are *philosophical*, not mathematical; how then do we fix the boundaries of Gödel's problem?"[5]

According to Shanker the problem must be placed within the metamathematical framework established by Hilbert. This view sees Wittgenstein as regarding the Theorem a closure on Hilbert's program, whereas the conventional interpretation treats the Theorem as some kind of transitional construction. This transitional view blurs the distinction between philosophy and mathematics, a blurring Wittgenstein opposed on all counts. It was this blending, which was demanded by the Hilbert Program, which made Wittgenstein reject the concept of metamathematics. On Shanker's reading, Wittgenstein's critique of Gödel's Theorem has its origins already

2. Shanker 1988.
3. 236.
4. 176.
5. 178–179.

in 1931, when Wittgenstein launched his attack on the Hilbert Program. According to Wittgenstein, we cannot talk *about* a system, only *within* a system. It is the "Hilbertinian premise which underpins Gödel's interpretation"[6] which is attacked by Wittgenstein, and in which he sees an attempt at revitalizing platonism.

2.1.2 Floyd (and Putnam): the compatibility approach

I In her first paper on the topic, Floyd argues that Wittgenstein does not want to refute Gödel's result, but merely wanted to "deflate [its] *apparent* significance."[7] According to Floyd, Wittgenstein likens FIT with that of the impossibility proofs in geometry, more specifically that of the impossibility to construct the trisection of a given angle by ruler and compass. Like the 1837 proof of this by Pierre Wantzel, the proof of FIT shows a certain formal construction to be impossible. Furthermore, Floyd evaluates Wittgenstein's attitude to the notion of 'mathematical proposition', the "protean (if not illusory) character" made sure that "Wittgenstein could not but have treated Gödel's theorem in the way he did."[8]

In her second paper,[9] Floyd offers a reasonable amount of historical-anecdotal evidence to support the fact that Wittgenstein *did* understand the result obtained by Gödel. She also notes that the main objective of Wittgenstein was to deflate philosophical talk about the Theorem, which did not require him to question the mathematical consequences of it. This includes talk about 'true but unprovable propositions'. The proof was not important to him, as "it is Gödel's metaphysical realism that breads superstition and scepticism, not Gödel's proofs."[10]

6. Shanker 1988, 233.
7. Floyd 1995, 375.
8. 395.
9. Floyd 2001.
10. 298.

In between those two papers Floyd also published one jointly with Hilary Putnam.[11] In this paper Floyd and Putnam attribute a genuine understanding of the Theorem to Wittgenstein, going as far as claiming that Wittgenstein grasped the notion of ω-consistency, which is involved in the second part of FIT. Furthermore they argue that Wittgenstein makes a "philosophical claim of great interest", namely "if one assumes (...) that $\neg P$ is provable in Russell's system one should (...) give up the the "translation" of P by the English sentence 'P is not provable'" as a consequence of the ω-consistency of PM.[12] Again it is remarked that Wittgenstein did not want to refute the proof, but only wanted to "by-pass" it.[13].

2.1.3 Rodych: the revisionist approach

Rodych has discussed the topic on several occasions, presenting a vision completely at odds with that of Shanker, Floyd and Putnam. In his first paper,[14] Rodych argues that on Wittgenstein's *own* terms a true but unprovable proposition can not exist, for if there could be something as a true but unprovable proposition in the system PM, what then does it mean for a proposition to be true in PM? Furthermore, Rodych states that Wittgenstein thinks that the natural language interpretation of the Gödel sentence P should be given up if we accept the proof of FIT. According to Rodych, Wittgenstein makes a mistake when he thinks that this interpretation is involved in the proof, whereas in fact the only relevant interpretation is the standard number-theoretic interpretation of P.

In a sense, Rodych is right to hold that only this interpretation matters, as Gödel only used

11. Floyd and Putnam 2000.
12. 624–625.
13. (626). I believe there another interpretation is possible for this passages then the one given by Floyd and Putnam, but I will discuss this in section 4.2.2
14. Rodych 1999.

this interpretation, seeing the natural-language interpretation as a mere corollary of the construction of the Gödel number G. Nevertheless, John Myhill has given three interpretations of FIT, none of which is number-theoretic in nature,[15] whereas John Findlay, with whom Wittgenstein was acquainted, argued that the Theorem "raises the issue of undecidability in the arithmetical as well as in the linguistic realm."[16] Therefore it seems at least possible to legitimately question FIT from the linguistic point of view.

In his second paper[17] Rodych discusses the remarks on Gödel published as part of the complete publication of the *Nachlass*, which he also invokes in his third paper.[18] His take on Wittgenstein is here more negative, accusing him of a lack of understanding of the topic. He still maintains the view that Wittgenstein's aim was to refute the mathematical proof, an aim he sees as consistent with the "genuine radicality of Wittgenstein's philosophy of mathematics".[19] The term 'radical' is here meant to apply to the seemingly constructivist and revisionist nature of Wittgenstein's philosophy of mathematics, as well as the common interpretation of Wittgenstein's position that mathematics can only get meaning through application.

2.1.4 Steiner: the enhanced incompatibility approach

Steiner characterizes Wittgenstein's remarks about the First Incompleteness Theorem as "indefensible" and "a quixotic and ill-informed attempt to refute Gödel's proof."[20] According to Steiner, Wittgenstein had no business writing them, as he should have treated the Theorem as

15. Myhill 1960, 461.
16. Findlay 1942.
17. Rodych 2002.
18. Rodych 2003.
19. 282.
20. Steiner 2001, 258.

he treated any other mathematical theorem: as a valid one. Contrary Rodych's view, Steiner holds that Wittgenstein's philosophy of mathematics is "non-revisionist"[21] and that according to him philosophy and mathematics had nothing to say to each other. But in treating FIT as he did, Wittgenstein made the Theorem a part of philosophy, going against his own principles. Furthermore, Steiner argues that Wittgenstein tries to refute an "informal version of a semantical version of Gödel's Theorem.[22] What Wittgenstein should have done, after being confronted with the Theorem, is to argue that "Gödel's Theorem had made it impossible to identify mathematical truth with provability, which should have encouraged the conclusion that mathematical truth is multicolored."[23] Steiner also argues that Wittgenstein is concerned with showing that truth is a family-resemblance concept,[24] a view FIT supports. But, as Rodych correctly notes in his discussion of Steiner's paper,[25] nowhere does Steiner cite textual evidence that Wittgenstein really saw 'mathematical truth' as a family resemblance concept, which makes this approach actually quite weak.

2.1.5 Kienzler & Grève: the inconclusive approach

A different approach is offered by Wolfgang Kienzler and Sebastian Sunday Grève.[26] What they offer is a thorough examination of RFM I A.III, and they conclude that Wittgenstein makes several attempts to make sense of the dilemma that is raised by FIT. The dilemma is this:

> Given our elementary assumption that mathematics is a practice which consists en-

21. Steiner 2001, 258.
22. 263.
23. 273.
24. 260.
25. Rodych 2006.
26. Kienzler and Grève 2016.

> tirely of proofs, in order for P to be a proper part of mathematics, we will have to actually prove it; however, once P has been proved, the statement that it was 'unprovable' becomes problematic.[27]

Wittgenstein's treatment of the Theorem is conceptual, rather than technical. He endeavors unfruitfully to make a definite sense of the idea of an 'unprovable sentence'. He begins with asking what it means for a proposition to be true in PM. The first attempt in answering the question is to define 'true proposition' either as axiom or as a proven theorem, which means that the answer to the posed question is negative. But as truth is seen to be dependent on the system, one might also ask whether it is possible for a proposition independently from the system in which symbolism it is written. But to assert that P is true in another system, won't satisfy the Gödelian interlocutor. Later on in the text, Wittgenstein shifts to answering the question "what might be the implications of such a statement for the *mathematical practice* that it purports to be addressing?"[28] Kienzler and Grève acknowledge that Wittgenstein tries to forge an analogy with the proof of the impossibility of the trisection, but that he sees that such an analogy "never gets of the ground."[29] The rest of the discussion turns around the question what kind of language-game may be played with the sentence P, which he concludes is questionable.

2.2 Evaluation

Having described the several approaches adopted towards explaining Wittgenstein's remarks about Gödel we can see that several questions are raised. The first is how Wittgenstein sees

27. Kienzler and Grève 2016, 88.
28. 114.
29. 104.

the relation between philosophy on the one hand and mathematics on the other. Notwithstanding passages in both the *Tractatus* and the *Philosophische Untersuchungen* which indicate the contrary, Wittgenstein is commonly seen as a revisionist about mathematics. This means he advocates a position towards mathematics which discards parts of mathematics, for example cases in which the infinite is involved. This position is adopted by for instance Dummett,[30] Frascolla[31] and Marion,[32] and in the discussion about FIT it is taken by Rodych. The main argument for this view is that Wittgenstein's philosophy of mathematics exhibits strong intuitionist traits, with Wittgenstein maintaining views which are even more radical than those of Brouwer and Poincaré. On the other hand there are those who see Wittgenstein as proposing a non-revisionary account, with Wittgenstein taking a more 'anthropological' view towards mathematics. This view is defended by Dawson,[33] who argues that Wittgenstein is neither a constructivist nor a revisionist. The same position is taken by Floyd, Shanker and Steiner, with the latter accusing Wittgenstein of relinquishing his position when discussing Gödel's Theorem.

The second question that is raised concerns the role and the meaning of mathematical propositions. Again there is a popular stance, which in this case holds that mathematical proposition derives its meaning from the application the proposition has. This application is than supposed to be 'extra-mathematical' (e.g. physical), which is why set theory is excluded as a branch of mathematics.[34]. Dawson also challenges this view, arguing that from Wittgenstein's perspective pure mathematics is a legitimate branch of mathematics, albeit a fringe one.[35]

30. Dummett 1959.
31. Frascolla 1994.
32. Marion 1998.
33. Dawson 2016.
34. Rodych 2000.
35. Dawson 2014.

The third question that is raised is whether Wittgenstein understood the meaning and the proof of FIT. As Rodych correctly asserts, the Theorem is nothing more than a result in finite number theory and the proof involves nothing more than standard number theoretic permutations. The 'natural-language interpretation' of the Gödelian sentence P is never used in the proof, and is only given in the introduction of the original paper.[36]. As Wittgenstein seems concerned with the natural language interpretation it is understandable for Rodych to think that Wittgenstein thought that FIT is only about this interpretation.

What is sometimes noted by Wittgenstein's commentators, is that there is a transition in Wittgenstein's thinking on mathematics. Not only is there a shift in his thinking between his Tractarian period to his later (post-1929) period, there is also in this later period a profound shift on Wittgenstein's philosophy of mathematics.[37] The two periods range from approximately 1929 to 1933 for the first phase and from 1934 to 1944 for the later. Gerrard notes that in both phases Wittgenstein tried to forge a counterargument to the 'Hardyian Picture',[38] a term based on the mathematician G.H. Hardy, whose views on mathematical proof, espoused in the eponymous article,[39] were strongly opposed by Wittgenstein. This opposition to Hardy is noted by Shanker, but he seems to be unaware of the change in thought from 1933 to 1937, the year Wittgenstein first wrote on FIT. Rodych is aware of the later shift, but sometimes seems to think that the notes from 1937 can be seen in the light of Wittgenstein's writings in 1931-1933, which I believe is indefensible. The stance of Steiner, Floyd and Kienzler and Grève on this point is not clear.

36. Gödel 1931, 1992.
37. See for instance (Gerrard 1991; Frascolla 1994; Marion 1998; Floyd 2005).
38. Gerrard 1991, 126.
39. Hardy 1929.

2.3 My approach

In the remainder of this thesis, I will offer my own approach to the question of the origins of the remarks within Wittgenstein's general philosophy of mathematics. As I see it, Wittgenstein does not propose a radically new method for doing mathematics, but indeed offers a description of the the role of certain concepts within the practice of mathematics. The most important concepts in this regard are that of truth and provability.

My premise will be that FIT confronted Wittgenstein with a redefining of those two terms within mathematics, which was at odds with the way those concepts were used before. In a sense, my approach comes close to that of Floyd and Kienzler and Grève, albeit with a few differences. It is for instance notable that the latter's discussion of the topic is confined to RFM I A.III. This is in a sense understandable, as this the only text in which Wittgenstein offers a complete treatment of FIT. However, as their conclusion is that Wittgenstein ultimately showed there could not be made any sense of the natural language interpretation, it maybe worthwhile to look at the other, later texts by Wittgenstein on the subject. Therefore I will include in my discussion also the paragraphs 21 and 22 from RFM VII and the discussion of true but unprovable sentences from the lectures Wittgenstein gave in Cambridge in 1939. Furthermore, it is noteworthy that Rodych is the only one to use the unpublished texts from the *Nachlass*. It seems that some of those texts support my theory, so I will discuss them as well.

Chapter 3

Wittgenstein's Philosophy of Mathematics

Within the wider field of philosophy, the philosophy of mathematics was one of Wittgenstein's principal interests. He was driven towards philosophy during his studies in aeronautical engineering in Manchester, wishing to find out why the mathematical formulas he learned pertained to reality. It is therefore rather surprising to find out that the subject is largely absent from his two most well-known publications. In the *Tractatus*, only two passages are devoted to mathematics (TLP 6.02–6.2031, 6.2–241), and *Philosophishe Untersuchungen* treats the subject only in examples of rule-following. Nevertheless, Wittgenstein devoted half of his writings between 1929 and 1944 on the subject, eventually claiming his work on mathematics as his "chief contribution" in philosophy.[1] Most parts of his writings on mathematics have been published posthumously, with

1. Glock 1996, 231.

the *Philosophische Bemerkungen* and *Philosophische Grammatik* covering the period roughly from 1929 to 1935, and the *Bemerkungen über die Grundlagen der Mathematik* covering the period 1937–1944. Furthermore, parts of his lectures held in Cambridge (including most notably those published in *Lectures on the Foundations of Mathematics*) contain discussions of his ideas on mathematics, as well as his conversations with members of the Vienna Circle, which were published by Waismann.

Wittgenstein covered a broad range of topics in his discussions of mathematics, with some – such as Platonism, the relation between logic and mathematics and the problem of consistency – being under constant scrutiny, whereas others – such as the topic of this thesis – are only piecemeal addressed. In order to understand his writings on Gödel we will have to address Wittgenstein's opinions on other topics which are possibly related to this topic, which will be the subject of this chapter.

3.1 Frome calculi to language-games

Over the years Wittgenstein's ideas on mathematics, like the rest of his philosophy, changed considerably. This development can be divided in three stages: 1. The early period of the *Tractatus*; 2. The intermediate period, ranging from 1929 to 1935; and 3. The later period, ranging from 1935 onwards. For this thesis, the early period is irrelevant, so I will not discuss this here. Concerning the intermediate period and the later period, it must be mentioned that the distinction between those was in the beginning largely overlooked, until Gerrard drew attention to the significant differences between Wittgenstein's 'philosophies of mathematics'[2], after which

2. Gerrard 1991.

this development has become the subject of academic scrutiny. Gerrard argues that in both periods Wittgenstein espouses different conceptions, which he labels the *calculus* conception and the *language-game* conception.

The former holds that mathematics essentially consists of independent calculi. Gerrard lists four characteristics of this conception. The first concerns the autonomy of the calculus. According to Wittgenstein (mathematical) propositions acquire their meaning from the calculus to which they belongs. This makes such a calculus autonomous, in the sense that it is closed and self-contained. The fact that it is closed precludes the possibility of critique from the outside of the calculus. The second characteristic of the calculus conception concerns the relation between the calculus and the application of the calculus. At this point we have to recognize that the calculus consists of rules, and that there is an *"unüberbrückbare Kluft zwischen Regel und Anwendung oder Gesetz und Spezialfall."*[3] (PR 164) This means there is a sharp separation between the calculus and its application. The third characteristic of the calculus conception is that every calculus is strictly defined by its rules, which means that whenever a new rule is added to the calculus, a completely new calculus is defined. The fourth characteristic is that we can only talk about individual calculi. It is not possible to talk about general conditions for calculi. This last characteristic stresses the sharp separation induced by the third characteristic even further.

An important aspect of this intermediate period in Wittgenstein's thinking is the strong verificationism which it exhibits. This means that a mathematical proposition is only meaningful when there is a method – a decision procedure – by which we can determine the truth or falsity

3. "unbridgeable gulf between between rule and application, or law and special case."

of a proposition. According to Rodych there are two reasons why the intermediate Wittgenstein would reject FIT.[4] The first is that Wittgenstein denies the possibility of quantification over an infinite domain. For instance, a proposition such as $(\exists n) 4 + n = 7$ cannot be replaced by a finite logical sum and is therefore not a logical sum (PR 127). The only way we can properly speak of statements about all numbers is by invoking natural induction.[5] FIT can be reduced to a number-theoretic expression, which quantifies over an infinite domain as well, which would prompt Wittgenstein into rejecting it.

The second reason Wittgenstein would reject the Theorem involves the criterion of decidability. As we have seen, a statement can only be a (meaningful) proposition when there is some decision procedure. But this would mean that an "undecidable proposition" such as the Gödel sentence P, would be a contradiction-in-terms. Undecidable propositions would have no sense and are neither true nor false. However, P might indeed be unprovable or undecidable within the calculus PM, its very construction makes it true (and decidable) *outside* PM. But speaking of a proposition which is true outside the calculus of which it is a part conflicts with the autonomy principle for calculi as stated above. This means that Wittgenstein, rather than rejecting the FIT, would have been forced to abandon the calculus conception, which he eventually did. Noteworthy, Gerrard also sees the result by Gödel as proving that the calculus conception fails.[6]

Whatever it was that invoked the transition, from the middle of the 1930s Wittgenstein's views began to shift towards the language-game conception of mathematics. This was the view

4. See (Rodych 1999, 174–176)

5. Rodych seems to believe that Wittgenstein does not think that Fermat's Last Theorem, when proved by induction, yields a proposition of arithmetic. He bases this on PR 189, but overlooks the fact that Wittgenstein *does* give a criterion for FLT to be a proposition of arithmetic.

6. A more extreme variant of this argument sees Wittgenstein abandoning the philosophy of mathematics completely because of FIT. "For what Gödel's Theorem demonstrates, on its standard interpretation, is that conventional mathematics has a richness which [Wittgenstein] cannot explain." (Potter 2011, 136) This last conclusion possibly is too far-fetched, but it shows nevertheless how central FIT could have been to Wittgenstein.

he increasingly held when he wrote his remarks on Gödel and is therefore of primary interest to my thesis. The language game conception sees mathematics as "*eine Familie von Tätigkeiten zu einer Familie von Zwecken*"[7] (RFM V, 15). One of the most important changes is that it can account for the growth of mathematical knowledge. This is because under the language-game conception the addition of new rules no longer makes mathematics into a completely different game, but only extends it. In fact, a proven theorem only serves as another linguistic rule. As Wittgenstein writes:

> Das *ist wahr daran, daß Mathematik Logik ist: sie bewegt sich in den Regeln unserer Sprache. Und das gibt ihr ihre besondere Festigkeit; ihre abgesonderte und unangreifbare Stellung.*[8] (RFM I, 165)

Frascolla notes a further change in Wittgenstein's position. Under the calculus conception the notion of mathematical proposition (as contrasted with empirical proposition) is indispensable, and Wittgenstein unduly tries to save the notion. In the later writings, however, there appears a significant decline in its value:

> [The notion of mathematical proposition] no longer has any attraction for Wittgenstein. Connected to the notion remain only the misleading suggestions that lead to matching mathematics and empirical science (and thus, almost inevitably, to platonism).[9]

The language-game conception has several ramifications with respect to FIT. This will be dis-

7. "a family of activities with a family of purposes"
8. *So* much is true when it's said that mathematics is logic: it moves are from rules of our language to other rules of our language. And this gives it its peculiar solidity, its unassailable position, set apart.
9. Frascolla 1994, 127.

cussed in more detail in chapter 4.

3.2 Wittgenstein on Platonism

Although Wittgenstein's thinking on mathematics went through several transformations throughout his career, over the years he remained consistent on several points. One of those was his opposition to the often-held doctrine known as 'Platonism' or '(mathematical) realism'. Gerrard, however, does not think this term is in this case "a happy choice",[10] as this doctrine holds that mathematical propositions are about things (mathematical objects) which exist in a mathematical reality. As this is not exactly what Wittgenstein objected to, Gerrard offers the term 'the Hardyian Picture' to label the view Wittgenstein rejected. This term he derived from Hardy, who taught at Cambridge at the same time as Wittgenstein, and whose views as espoused in his article on mathematical proof[11] were explicitly attacked by Wittgenstein on several occasions. The Hardyian Picture is described as a conception of mathematical reality independent from the practice in which we use it and from the language we use to describe it. On the contrary Wittgenstein held that there is no such thing as a mathematical reality outside our dealings with it. Hardy, however, held that mathematical reality *does* exist independently, and that it is the mathematician's job to discover truths about it:

> It seems to me that no philosophy can possibly be sympathetic to a mathematician which does not admit, in one manner or another, the immutable and unconditional validity of mathematical truth. Mathematical theorems are true or false; their truth

10. Gerrard 1991, 127.
11. 'Mathematical proof', see (Hardy 1929)

> or falsity is absolute and independent of our knowledge of them.[12]

To this the notion of proof is essentially tied. In the Hardyian picture, a proof is not necessary for a proposition to be true, as truth is warranted by this mathematical reality.

Wittgenstein's resistance to this Hardyian picture served as a common thread throughout his work on mathematics. As Gerrard notes,

> ... as different as the calculus and language-game conceptions are, they are both motivated by an opposition to the Hardyian Picture's view that there is an underlying mathematical reality which our language and practice must mirror or be responsible to.[13]

Wittgenstein's main objection to the Hardyian picture is that a mathematical realm existing independent from us cannot warrant the truth of our propositions. To understand this, we must consider what Wittgenstein holds to be the meaning of mathematical propositions. The meaning of propositions in general is determined by the way we use it. But in order for a sentence like $2+2=4$ to have a use in our practice, we must be able to translate symbols like 2, 4, + and = to our ordinary language such that the equation contained in the sentence remains true. Therefore it can only be our language and practice which serve as a criterion for truth, rather than the Platonist realm of mathematical objects. Wittgenstein states repeatedly that mathematics is not discovered, but invented:

> *Der Mathematiker ist ein Erfinder, kein Entdecker.*[14] (RFM I, 168)

12. Hardy 1929, 4.
13. Gerrard 1991, 131.
14. "The mathematician is an inventor, not a discoverer."

> I shall try again and again to show what is called a mathematical discovery had much better be called a mathematical invention. (LFM 22)

> We know as much as God does in mathematics. (LFM 104)

Tied to his objection to Platonism is his view on the notion of 'proof'. For him, a proof does not serve to decide the truth of a given conjecture – such as Goldbach's or, at the time, Fermat's – but rather to give a a sense or a meaning to it. Once a proposition is proven it serves as a new rule within the language-game.

His opposition to Platonism or the Hardyian picture brought Wittgenstein at considerable odds with Gödel. The latter's writings exhibit strongly Platonist views:

> ... the assumption of [mathematical objects existing independently of our definitions and constructions] is quite as legitimate as the assumption of physical bodies and there is quite as much reason to believe in their existence. They are in the same sense necessary to obtain a satisfactory system of mathematics as physical bodies are necessary for a satisfactory theory of our sense perceptions.[15]

Like Hardy, Gödel believed that mathematicians only try to discover the truth of certain mathematical propositions, which is laid down in some mathematical realm. This becomes apparent in his writings on the continuum problem:

> ... a proof of the undecidability of Cantor's conjecture from the accepted axioms of set theory (...) would by no means solve the problem. For (...) the set-theoretical concepts and theorems describe some well-determined reality, in which Cantor's con-

15. Gödel 1990a, 128.

> jecture must be either true or false. Hence its undecidability from the axioms being assumed today can only mean that these axioms do not contain a complete description of that reality.[16]

In other words, if we cannot decide the truth or falsity of a given proposition by the use of certain axioms, we need to amend those axioms.

In the case of FIT, it is especially noteworthy that Gödel remarks that "[under the assumption that modern mathematics is consistent] the solution of certain arithmetical problems requires the use of assumptions essentially transcending arithmetic".[17] It is therefore not implausible to envision Wittgenstein's remarks on the Theorem to be actually directed against Gödel's platonism, as Shanker has done.

3.3 Mathematical propositions, meaning and applicability

For Wittgenstein, the meaning of a sentence is decided by its 'grammar', i.e. the use it has in the language game in which it is employed. To this rule propositions of mathematics are no exception. But when it comes to the meaning of a mathematical proposition, there is a difference with other propositions, which lies in its relation to its proof. As we saw, a proof does not establish the truth of a mathematical proposition, but rather connects it with the other propositions within the framework of the mathematical language-game at hand:

> *Die Beweise ordnen die Sätze.*
>
> *Sie geben ihnen Zusammenhang.*[18] (RFM VI, 1)

16. Gödel 1990b, 260.
17. Gödel 1990a, 121.
18. Proofs give propositions an order. They organize them.

But there is also a strong relation between the proof of the proposition and its application.

> *Der Beweis steht im Hintergrund des Satzes, wie die Anwendung. Er hängt auch mit die Anwendung zusammen.*[19] (RFM VI, 2)

One might also say that the proof serves to give the proposition meaning. An important aspect of Wittgenstein's thinking about proofs is therefore that we have to look at the proof in order to see what exactly has been proved:

> What I am out to show you could be expressed very crudely as "If you want to know what has been proved, look at the proof" or "You can't know what has been proved until you know what is called a proof of it." (LFM 39)

This last remark is essential as it involves the relation between a proof and the prose surrounding the proof. Before I proceed, I want to say a few things about this latter relation.

Wittgenstein mentions on several occasions that we should make a sharp distinction between the symbolic notation of a mathematical proposition and the prose which we use to assert the proposition. As Marion notes, Wittgenstein saw that the appearance of prose was necessary as "a mathematical proof shows us something that it can not say by itself."[20] But there is a danger in focusing on this prose, because, as Wittgenstein mentions with regard to a proof of continuity of a function:

> *(...) der Wortausdruck des Angeblich bewiesenen Satzes ist meist irreführend, denn er verschleiert das eigentliche Ziel des Beweises, das in diesem mit voller Klarheit zu*

19. The proof, like the application, lies in the background of the proposition. And it hangs together with the application.
20. Marion 1998, 5.

> *sehen ist.*[21] (PG 369)

To drive his point home, Wittgenstein describes the relation between mathematical propositions and language-games as follows:

> *Man möchte sagen, das Verständnis eines mathematischen Satzes sei nicht durch seine Wortform garantiert, wie im Fall der meisten nicht-mathematischen Sätze. Das heißt – so scheint es – daß der Wortlaut das* Sprachspiel *nicht bestimmt, in welchen der Satz funktioniert.*[22] (RFM V, 25)

Summarizing, this considerations leads to the conclusion that Wittgenstein wants the proofs of the propositions to speak for themselves, as our prose interpretations, if they are not inessential, may be utterly misleading. Floyd links this perspective to FIT arguing that

> 'There are true but unprovable propositions in mathematics' is misleading prose for the philosopher, according to Wittgenstein. It fools people into thinking that they understand Gödel's theorem simply in virtue of their grasp of the notions of *mathematical proof* and *mathematical truth*. And it fools them into thinking that Gödel's theorem supports or requires a particular metaphysical view.[23]

There is much to disagree with in Floyd's interpretation (as I will do in chapter 4) but it certainly is one of the better interpretations of Wittgenstein's attitude towards Gödel.

Now back to the issue of application of propositions. In the context of mathematics, we must be aware that 'application' can mean two separate things. The first is application *of* the calculus

21. The verbal expression of the allegedly proved proposition is in most cases misleading, because it conceals the real purport of the proof, which can be seen with full clarity in the proof itself.

22. One would like to say that the understanding of a mathematical propositions is not guaranteed by its verbal form, as is the case with most non-mathematical propositions. This means–so it appears–that the words don't determine the *language-game* in which the proposition functions.

23. Floyd 2001, 299.

or language game. By this we mean the application of mathematical propositions to something outside of mathematics, such as physics. The second is application *of a rule* of the calculus or the language game. By this we mean for instance the application of the rule '+2' to get the series $1000, 1002, 1004, \ldots$. Gerrard asserts that this distinction is not drawn by Wittgenstein during the calculus phase, where application is only taken to be of the first kind. Furthermore, Wittgenstein seems to think that a calculus is something which is separated from reality, but also from practice. This was another reason Wittgenstein had to abandon the calculus conception.

It is often maintained that Wittgenstein in his later period did not consider the second kind as a legitimate form of application:

> *Ich will sagen: Es ist der Mathematik wesentlich, daß ihre Zeichen auch im* Zivil *gebraucht werden.*
>
> *Es ist der Gebrauch außerhalb der Mathematik, also die* Bedeutung *der Zeichen, was das Zeichenspiel zur Mathematik macht.* [24] (RFM V, 2)

Such passages have commonly been interpreted as meaning that the only thing that counts as mathematics are *applied* mathematics. This has been advanced as a reason for Wittgenstein to reject pure mathematics in general, and more specifically the number-theoretic FIT. In a fragment of the *Nachlass*,[25] Wittgenstein states that Gödel does not understand the relation between mathematics and application. I will discuss this passage in more detail in section 4.2.3.

Dawson, in contrast with the more common reading of Wittgenstein, argues that "Wittgenstein's view (...) does make a case that mathematics with direct applications should be seen

24. I want to say: it is essential to mathematics that its signs are also employed in *mufti*.
It is the use outside mathematics, and so the *meaning* of the signs that makes the sign-game into mathematics.
25. MS 124, 115r; March 5, 1944

as more central to our concept of mathematics" but that "the legitimacy of pure mathematics (including foundational systems) is not called into question."[26] On the quotation given above, he says that we should see this not as a final verdict on the character of mathematics, but rather as a suggestion that the signs of mathematics are employed *in mufti* is a characteristic of "certain key exemplars of the family-resemblance term 'mathematics"'.[27] There certainly are branches of mathematics in which the signs acquire meaning trough their use outside mathematics, such as arithmetic, and it is arithmetic which is the subject of RFM V, 2. Moreover, in the series of lectures held in 1939, Wittgenstein makes the following remark:

> The calculus (system of calculations) [of professor Littlewood] is what it is. It has a use or it hasn't. But its use consists either in the mathematical use–(a) in the calculus which Littlewood gives, or (b) in other calculi to which it may be applied–or in a use outside mathematics. (LFM 254)

This makes clear that, 'application' is not confined to 'extra-mathematical application' but definitely has a broader meaning. Furthermore, it must be observed that Wittgenstein talks about pure mathematics in several places. At one point,[28] Wittgenstein imagines a people who only have applied mathematics and to whom the concept of pure mathematics is completely foreign. In such a community there are only rules to move from empirical statements to empirical statements, without ever writing such a rule down. However:

> *Diese Leute sollen nicht zu der Auffassung kommen, daß sie mathematische Entdeckungen machen, – sondern* nur *physikalische Entdeckungen.*[29] (RFM IV, 16)

26. Dawson 2014, 4132.
27. 4140.
28. RFM IV, 15-19
29. These people are not supposed to arrive at the conception of making mathematical discoveries – but *only* of

It seems to me that Wittgenstein is making a case that applied mathematics is not possible when there is no concept of pure mathematics.

On the account of Wittgenstein's opinion about pure mathematics given so far, it seems improbable that Wittgenstein does reject this notion, and therefore there is no reason to assume he would reject FIT on the grounds that pure mathematics is meaningless in general. An additional reason for this is that, were it the case that Wittgenstein rejected pure mathematics, the question might be raised why Wittgenstein devoted this much attention to the subject. However, this does not preclude that Wittgenstein rejected Gödel's result on other grounds. The possibility of Wittgenstein being a revisionist in mathematics will therefore be examined in the next section.

3.4 Wittgenstein's 'quasi-revisionism'

Most commentators of Wittgenstein's philosophy of mathematics have answered the question about the revisionist nature of his thoughts positively. They do so in spite of several remarks in which he stresses he does not want to interfere with the actual work of mathematicians:

> *Der Philosoph notiert eigentlich nur das, was der Mathematiker so gelegentlich über seine Tätigkeit hinwirft.*[30] (PG 369)

Usually, Wittgenstein is considered not to keep his own promise when discussing certain areas. In this section, I want to consider two of these: 1. The validity of the Law of Excluded Middle (LEM); and 2. Set theory.

making physical discoveries.

30. The philosopher only marks what the mathematician casually throws off about his activities.

With respect to the first of these, it must be noted that Wittgenstein is often accused of either rejecting the use of logical laws such as LEM in specific instances or even outright rejecting the validity of such laws altogether. His true position, however, seems more nuanced than it is often taken to be. This is perhaps best illustrated by his remarks on propositions of the form 'the sequence φ (i.e. 777) occurs somewhere in the expansion of π' in RFM V. The background for this discussion is provided by two articles by Alice Ambrose on finitism,[31] which were inspired by lectures delivered by Wittgenstein at Cambridge in the period 1932-1935. Wittgenstein seems to distance himself from Ambrose's position in this discussion, of which I will give a short outline.

The basic assumption Wittgenstein makes is that we can not say in advance that LEM will hold in specific cases. We have to ask ourselves how the rule '$p \vee \neg p$' is applied, and therefore it is necessary to know what p means (RFM V, 17-18). As I pointed out before, propositions acquire meaning through their proofs. Now let p be the proposition '777 occurs in the expansion of π'. First of all, we must be aware that there does not really exist something like *the* expansion of π, as this is infinite, but only a *method* for expanding π (RFM V, 9). Therefore it is even more important to look at the proof of p to find out what is meant by this expression. We can imagine several proofs of p. The first one is that we find the sequence 777 by sheer luck after several iterations of our method for expanding π. It can also be that 777 is proven to occur somewhere in the expansion by some ingenious existence proof. However, Wittgenstein remarks it is unclear whether such existence proofs really prove the existence of some object. This is because it is rather doubtful that such a proposition has any meaning, as we are rather uncertain about how to use it (RFM V, 46). And as we have seen, understanding a proposition follows

31. Ambrose 1935a, 1935b.

from our knowing its application.

The requirement that propositions have meaning is the fundamental difference between finitism and formalism. Some commentators, such as Frascolla, Marion and Rodych have labeled Wittgenstein as a finitist (or even a strict finitist) because of his remarks in RFM V. It is true that Wittgenstein outlines the finitist case very well, but it must also be noted that Wittgenstein is critical of both ways of doing mathematics:

> *'Einen mathematischen Satz verstehen' - das ist ein sehr vager Begriff.*
>
> *Sagst du aber "Aufs Verstehen kommt's überhaupt nicht an. Die mathematischen Sätze sind nur Stellungen in einem Spiel", so ist das auch Unsinn! 'Mathematik' ist eben kein scharf umgezogener Begriff.*[32] (RFM V, 46)

At another point, Wittgenstein even explicitly criticizes finitism while liking it to behaviorism:

> *Finitismus and Behaviourismus sind ganz ähnliche Richtungen. Beide sagen: hier ist doch nur... Beide Leugnen die Existenz von etwas, beide zu dem Zweck, um aus einer Verwirrung zu entkommen.*[33] (RFM II, 61)

Furthermore, in the 1939 lectures, Wittgenstein counters the claim that he is trying to refute any results in mathematics. After acknowledging that it is sometimes held that the rejection of Platonism leads to finitism he says:

> There is a *muddle* at present, an unclarity. But this doesn't mean that certain mathematical propositions are *wrong*, but that we think their interest lies in something in

32. 'Understanding a mathematical proposition'–that is a very vague concept.
But if you say "The point isn't understanding at all. Mathematical propositions are only positions in a game" that too is nonsense! 'Mathematics' is *not* a sharply delimited concept.

33. Finitism and behaviorism are quite similar trends. Both say, but surely, all we have here is... Both deny the existence of something, both with a view of escaping from a confusion.

> which it does not lie. I am *not* saying transfinite propositions are *false*, but that the wrong pictures go with them. (LFM 141)

With respect to set theory, Wittgenstein has made several criticizing remarks. Rodych, in another article on Wittgenstein's philosophy of mathematics, has discussed these remarks and concludes that Wittgenstein sees set theory "as a *mathematical* "sign game," which is only mathematical in that it is a formal calculus with a somewhat tenuous connection to the solid core of fully meaningful, mathematical "language-games."" [34] Rodych insists that Wittgenstein demands that mathematics has a extra-systemic application and therefore the "*later* Wittgenstein regards [set theory] as something *less than* a full mathematical calculus *because* it does not have an extra-systemic application." [35] However, as I explained in section 3.3, it is not the case that Wittgenstein makes such a demand, and Rodych's arguments with respect to this point are unconvincing. Indeed, Rodych's arguments seem very strange, as he only affirms that Wittgenstein considers set theory as something which is not a *full* mathematical calculus, which is not the same as saying that set theory is not a mathematical calculus *at all*. So until here, Rodych's argument is very much the same as the argument I gave on pure mathematics in the previous section, albeit with a different conclusion.

This means that set theory is not being 'dropped out' of mathematics as a result of the lack of an extra-systemic application, but it does so, as Wittgenstein hopes, for another reason. His problem with set theory, is not its validity as a branch of mathematics, but rather as a foundational system. Glock argues that Wittgenstein is critical at *any* attempt of providing secure

34. Rodych 2000, 283.
35. 305.

foundations for mathematics, for two reasons.[36] First of all, the belief that contradictions like those of Russell can lead to scepticism, which led to the search for foundations, is a superstition only to be overcome by philosophical clarification rather than providing a foundation of mathematics on first principles. The second reason is that foundational systems, such as Hilbert's meta-mathematics but also set theory, only yields new mathematical calculi, which leads to an infinite hierarchy. Concerning set theory, Wittgenstein therefore notes:

> Hilbert: "No one is going to turn us out of the paradise which Cantor has created." I would say, "I wouldn't dream of trying to drive anyone out of this paradise." I would try to do something quite different: I would try to show you that it is not a paradise–so that you'll leave of your own accord. (LFM 103)

So it is in the sense that Wittgenstein hopes that the metaphysical claims about certain branches of mathematics will be dropped – and as a result the corresponding enterprise will be discontinued – that he can be considered a 'revisionist' or better, following a suggestion by Frascolla, as 'quasi-revisionist'.[37] Considering this position, it seems not likely that Wittgenstein would try to refute the proof of FIT. But it is still possible that Wittgenstein wanted to do away with the metaphysical and epistemological convictions derived from it.

36. Glock 1996.
37. Frascolla 1994, 160.

Chapter 4

The Remarks on Gödel

Having discussed Wittgenstein's general opinions on mathematics, it is time to turn to the specific topic of Gödel's FIT. The most substantial discussion of this in the writings of Wittgenstein is contained in (RFM VII A.III), so I will discuss this part in the first section. As the section is completely devoted to the Theorem, I will discuss the whole of it, while dividing the twenty numbered paragraphs in different groupings to capture the line of the argument (or: arguments) Wittgenstein tries to make.

In the second section of this chapter, I will evaluate Wittgenstein's other writings on the subject. I will start with the few remarks Wittgenstein made about 'unprovable propositions during his series of lectures held in Cambridge in 1939. Furthermore, there are several remarks written in the period 1942-1944, which are published in (RFM VII). The section will conclude with a discussion of several remarks taken from the *Nachlass*.

4.1 RFM: *Teil* I, *Anhang* III

Sections 1–4: Introduction

The opening sections serve as an introduction to the problems raised by FIT. In the first section, Wittgenstein asks what we would think of a language which does not contain any questions and commands. We would not say of a question that it is true or false, but we might say such a thing about assertions of the form "I would like to know whether...":

> *Niemand würde doch von einer Frage (etwa, ob es draußen regnet) sagen, sie sei wahr oder falsch. Es ist freilich deutsch, dies von einem Satz "ich wünsche zu wissen, ob...", zu sagen.*[1] (RFM I A.III, 1)

The idea here is to show that it is possible to forge an analogy between two systems of language (or language-games), one in which we can say of sentences that those are true and false, whereas in the second we cannot do so for sentences which have the same meaning or grammar. Kienzler and Grève see this paragraph as warning for such an analogy,[2] but is not clear from the text that this is the case.

The second section deals with assertions. Wittgenstein observes that the plurality of the sentences we use are assertions. These are sentences with which we can play the game of truth functions. Now the act of 'assertion' does not add anything to the sentence which is asserted:

> *Denn die Behauptung ist nicht etwas, was zu dem Satz hinzutritt, sondern ein wesentlicher Zug des Spiels, das wir mit ihm spielen.*[3] (RFM I A.III, 2)

1. No one would say of a question (e.g. whether it is raining outside) that it is true or false. Of course it is English to say such of such a sentence as "I want to know whether...".
2. Kienzler and Grève 2016, 90–91.
3. For assertion is not something that gets added to the proposition, but an essential feature of the game we

Wittgenstein likes this to the game of chess in which a player wins when taking the others king. 'Winning' here is analogous to 'asserting a true proposition' and 'losing' to 'asserting a false proposition.' Now we may think of a game which looks like chess, but does not include this condition for winning, or even any condition for winning. In the first case, we are basically back to the same analogy made in the preceding section. We may have the same position on the board in both games, with the one meaning that the game has ended and there is a winner, whereas in the other game this may not apply. In the second case the question is raised whether we might still call this 'chess'. This question is not answered by Wittgenstein at this point, although from our discussions of the language-game conception we may infer that such a game would at least be in the family of chess games.

The third section consists of only one sentence, in which Wittgenstein denies that commands can be dived into a proposal and the commanding itself. This is analogous to the fact that we cannot separate propositions from them being asserted, as was discussed in the previous section.

In the fourth section Wittgenstein turns to the relation between propositions of arithmetic and the sentences from our ordinary language. He stresses that there is a connection between them, but that it could as well not exist:

> *Könnte man nicht Arithmetik treiben, ohne auf den Gedanken zu kommen, arithmetische* Sätze *auszusprechen, und ohne daß uns die Ähnlichkeit einer Multiplikation mit einem Satz je auffiele.*[4] (RFM I A.III, 4)

play with it.

4. Might we not do arithmetic without having the idea of uttering aritmetical *propositions*, and without ever having been struck by the similarity between a multiplication and a proposition?

The point is mainly that when uttering a certain proposition of arithmetic (such as $2+2=4$), it sounds like we are uttering an ordinary sentence:

> *Wir sind gewohnt, zu sagen "2 mal 2 ist 4" und das Verbum "ist" macht dies zum Satz und stellt scheinbar eine nahe Verwandschaft her mit allem, was wir 'Satz' nennen. Während es sich nur um eine sehr oberflächliche Beziehung handelt.*[5] (RFM I A.III, 4)

Sections 5–6: 'True but unprovable propositions'

After the introductory remarks in the preceding sections, the second group of sections finally begins discussing the main topic of the appendix. In the fifth section the imaginary interlocutor asks Wittgenstein a crucial question:

> *Gibt es wahre Sätze in Russells System, die nicht in seinem System zu beweisen sind? – Was nennt man denn einen wahren Satz in Russells System?*[6] (RFM I A.III, 5)

The first sentence of this section is important in several ways. In the first place, it mentions for the first time the topic of this appendix: the concept of 'true but unprovable propositions'. But in the second place it is significant for what it does *not* say. The name of Gödel is not mentioned, nor are his theorem and his proof. In the rest of the appendix Gödel also is nowhere referred to. This fact is often overlooked, with considerable consequences. Many (including Bernays, Kreisel and Rodych) have been led into believing that the remarks in this appendix are

5. We are used to saying "2 times 2 is 4", and the verb "is" makes this into a proposition, and apparently establishes a close kinship with everything that we call a 'proposition'. Whereas it is a matter only of a very superficial relationship.

6. Are there true propositions in Russell's system, which cannot be proved in this system?–What is called a true proposition in Russell's system, then?

about FIT, whereas it is actually only about the concept of 'true but unprovable propositions', which is mentioned by Gödel in the introduction to his proof of the Theorem. The mentioned commentators have rightfully noted that the proof does not involve this notion, but actually Wittgenstein is not interested in this.

The second sentence of section 5 is the question Wittgenstein will try to answer in section 6. The question is what it means to say that a proposition is true Russell's system. According to Wittgenstein, truth is not a property which is independent from the assertion of the proposition:

> *Was heißt denn, ein Satz* 'ist wahr'*?* '*p*' ist wahr $= p$. *(Dies ist die Antwort.)*[7] (RFM I A.III, 6)

In other words declaring a proposition true is equal to asserting the proposition. Wittgenstein proceeds by asking under what circumstances do we assert a proposition, and specifically how we do so in Russell's system:

> *Fragt man also in diesem Sinne: "Unter welchen Umständen behaupt man in Russells Spiel einen Satz?", so ist die Antwort: Am Ende eines seiner Beweise, oder als 'Grundgesetz' (Pp.). Anders werden in diesem System Behauptunssätze in den Russellschen Symbolen nicht verwendet.*[8] (RFM I A.III, 6)

There are different possible interpretations for this section. We might interpret Wittgenstein as saying that propositions within the system of *Principia Mathematica* (or other systems) *must* only be asserted at the end of a proof or as an axiom. This interpretation sees Wittgenstein as

7. For what does a proposition's '*being true*' mean? *'p' ist true* $= p$. (That is the answer.)

8. If, then, we ask in this sense: "Under what circumstances is a proposition asserted in Russell's game?" the answer is: at the end of one of his proofs, or as a 'fundamental law' (Pp.). There is no other way in this system of employing asserted propositions in Russell's symbolism.

proposing a normative account of mathematics. In such a case, the Gödelian sentence P, which says of itself that it is unprovable, cannot be part of Russell's system. This line of thought is taken by Rodych, who thinks that for Wittgenstein "*on his own terms*, a "true but unprovable" mathematical propositions is a *contradiction in terms*".[9]

On the other hand, we may also interpret Wittgenstein as saying that within mathematical practice, propositions *are* only asserted either at the end of their proofs or as fundamental law. According to this interpretation Wittgenstein is merely adopting an anthropological point of view, instead of prescribing how mathematics should be done. Such an account, which is, as I have argued in section 3.4, the most defensible, means that Wittgenstein is confronted by Gödel with a situation different from the one he encountered *before* the publication of Gödel's result. I think the latter interpretation to be more appropriate, which I will show in the remainder of this chapter.

Sections 7–8: The 'notorious paragraphs'

The third group of sections contains the 'notorious paragraph', as Floyd has called section 8.[10] But as Rodych has noted, we can also use this term for several other sections, including section 7.[11] I have followed this suggestion by Rodych in describing this third group of sections.

In the beginning of section 7, Wittgenstein returns to the questions raised in section 5: can we have propositions written in the symbolic notation provided by Russell which are true, but which are not provable? In this case, we must accept the following precondition for true propositions:

9. Rodych 1999, 180.
10. Floyd 2001, 284.
11. Rodych 2003, 312.

> *'Wahre Sätze', das sind also Sätze die in einem* andern *System wahr sind, d.h. in einem anderen Spiel mit Recht behauptet werden können.*[12] (RFM I A.III, 7)

Wittgenstein continues with explaining there is nothing wrong with such a statement at first sight. There is for instance nothing wrong with writing propositions of physics in Russell's symbolic notation. But on further evaluation, a slight complication appears. It is a fact that there exist propositions which are true in Euclidean geometry, but which are false in some other – non-Euclidean – geometric system. But looking at an example, Wittgenstein observes a startling consequence:

> *Können nicht Dreiecke – in einem andern System – ähnlich (* sehr *ähnlich) sein, die nicht gleiche Winkel haben? – "Aber das ist doch ein Witz! Sie sind ja dann nicht im selben Sinne einander 'ähnlich'!" – Freilich nicht; und ein Satz, der nicht in Russells System zu beweisen ist, ist in anderm Sinne 'wahr' oder 'falsch' als ein Satz der "Principia Mathematica".*[13] (RFM I A.III, 7)

What Wittgenstein is saying here is that a proposition can indeed be true in one system, but that does not mean that it is true in the same sense as in another system. For the Gödelian sentence P means that although it is unprovable in *Principia Mathematica* (and therefore true under the 'prose' interpretation) this does not mean it can really be 'true' of 'false' in this same system.

This is made more explicit in section 8. Wittgenstein begins with imagining someone would

12. 'True propositions', hence propositions which are true in *another* system, can rightly be asserted in another game.
13. May not triangles be–in another system–similar (*very* similar) which do not have equal angles?–"But that's just a joke! For in that case they are not 'similar' to one another in the same sense!"–Of course not; and a proposition which cannot be proved in Russell's system is "true" or "false" in a different sense from a proposition of *Principia Mathematica*.

claim that he has devised a proposition P, which can be made to mean: 'P is not provable in Russell's system.' We can infer from this definition that P is therefore true. But now we have a problem:

> *So wie wir fragen: "in welchem System 'beweisbar'?", so müssen wir auch fragen: "in welchem System 'wahr'?". In Russells System 'wahr' heißt, wie gesagt: in Russells System bewiesen; und 'in Russells System falsch' heißt: das Gegenteil sei in Russells System bewiesen.*[14] (RFM I A.III, 8)

Now Wittgenstein asks what it means to suppose that P is false. The answer is that in the Russellian sense, this means that we suppose that the opposite of P has been proved. Then a very important observation follows:

> ... ist das deine Annahme, *so wirst du jetzt die Deutung, er sei unbeweisbar, wohl aufgeben. Und unser dieser Deutung verstehe ich die Übersetzung in diesem deutschen Satz.*[15] (RFM I A.III, 8)

So if we assume that P is false (or: $\neg P$), it follows that we have to give up the natural-language interpretation of P. Wittgenstein goes on to claim that we have to give up this interpretation also when we suppose that P is provable, and also when we assume that P is true. He concludes with saying that the when P is false in another sense than in the Russellian, then it does not contradict the fact that it is proved in Russell's system. In the closing statement Wittgenstein invokes the comparison with chess from section 2:

14. Just as we ask: " 'provable' in what system?", so we must also ask: " 'true' in what system?" 'True in Russell's system' means, as was said: proved in Russell's system; and 'false in Russell's system means: the opposite has been proved in Russell's system.

15. *if that is your assumption,* you will now presumably give up the interpretation that it is unprovable. And by 'this interpretation' I understand the translation into this English sentence.

> *(Was im Schach "verlieren" heißt, kann doch in einem andern Spiel das Gewinnen ausmachen.)*[16] (RFM I A.III, 8)

Floyd and Putnam have argued that in this section Wittgenstein makes "a philosophical claim of great interest."[17] Their argument is that a proof of $\neg P$ means that the system PM is ω-inconsistent as a result of Gödel's proof. They deduce further that the "translation of P as 'P' is not provable is untenable" as a result of the ω-inconsistency of PM.[18] They credit Wittgenstein with the discovery of this fact. However, the concept of ω-inconsistency is not mentioned in this paragraph, nor is it mentioned in any other of his writings. Floyd and Putnam's claim that (RFM I A.III) was "written as notes for Wittgenstein himself" is incorrect, and therefore the claim that there was "no reason for their author to state explicitly everything that he knew in connection with them"[19] seems unjustified. If there was made "a philosophical claim of great interest", it is done by Floyd and Putnam themselves, not by Wittgenstein.

On the other hand, Rodych claims about this section that Wittgenstein makes a mistake, as he seems to think that the natural-language interpretation is somehow essential to Gödel's proof. We know this is not the case, and that the only essential interpretation of P is the number-theoretic one.[20] However, as I stressed before, Wittgenstein is really not interested in the technical aspects of the proof, and it is therefore inconceivable that he would treat the natural-language interpretation as essential to the proof.

16. (What is called "losing" in chess may constitute winning in another game.)
17. Floyd and Putnam 2000, 624.
18. 625.
19. 627.
20. Rodych 1999, 182.

Sections 9–10: Proving unprovability

In the subsequent sections, Wittgenstein continues to ask what it means that P asserts its own unprovability. This begins with the short section 9:

> *Was heißt es denn, P und "P ist unbeweisbar" seien der gleiche Satz? Es heißt, daß dies* zwei *deutschen Sätze in der und der Notation* einen *Ausdruck haben.*[21] (RFM I A.III, 9)

In the following section, Wittgenstein explores what would happen when P is proven:

> *Aber wenn dies [P] nun bewiesen wäre, oder wenn ich glaubte – vielleicht durch Irrtum – ich hätte es bewiesen, warum sollte ich den Beweis nicht gelten lassen und sagen, ich müsse meine Deutung* "unbeweisbar" *wieder zurückziehen.*[22] (RFM I A.III, 10)

Wittgenstein observes that we should give up the natural-language interpretation of P not only when P is false, but also when P is true. If Wittgenstein observations are correct, then such an interpretation of P would be untenable.

Sections 11–13: Contradiction

The next section is rather exemplary of what Wittgenstein does throughout the whole appendix. First, he argues that if the unprovability P was proved, then P would have been proved as well. And if this was done within Russell's system, then P simultaneously does and does not belong to this system. But rather than speaking for himself, he places the observation in the mouth

21. For what does it mean to say that P and "P is unprovable" are the same proposition? It means that these *two* English sentences have a *single* expression in such-and-such a notation.

22. But if this were now proved, or if I believed–perhaps through an error–that I had proved it, why should I not let the proof stand and say I must withdraw my interpretation *"unprovable"*?

of his imaginary interlocutor. After this observation, a short discussion ensues between a placid Wittgenstein and his startled interlocutor, starting with the former:

> *Das kommt davon, wenn man solche Sätze bildet. – Aber hier ist ja ein Widerspruch! – Nun so ist hier ein Widerspruch. Schadet er hier etwas?*[23] (RFM I A.III, 11)

The first sentence is taken by Rodych to refer to the *number-theoretic* construction of the proposition. From this assumption – which is equally absurd with respect to the rest Rodych's account, as it is untenable with respect to Wittgenstein's remarks – he infers that Wittgenstein criticizes the quantification over an infinite domain which is involved in the proof by Gödel.[24] I have already argued that Wittgenstein's opposition to this kind of quantification cannot be gathered from other writings of this period, and therefore Rodych's interpretation is unjustified.

On the other hand, with the last two sentences of section 11, Wittgenstein does bring one of his most important topics to the front: the problem of inconsistency. The placidness of Wittgenstein in these lines is on purpose. In much of his writings on mathematics he has argued against the mathematician's adamant insistence that no contradictions may occur. Wittgenstein held the view that there is nothing really to fear for. His views are perhaps best illustrated by the discussion he had with Alan Turing during his lectures in 1939:

> The question is: Why are people afraid of contradictions? It is easy to understand why they should be afraid of contradictions in orders, descriptions, etc., *outside* mathematics. The question is: Why should they be afraid of contradictions inside mathematics? Turing says, "Because something may go wrong with the application."

23. That is what comes of making up such sentences. –But there is a contradiction here!–Well, then there is a contradiction here. Does it do any harm here?
24. Rodych 1999, 185-186.

> But nothing need go wrong. And if something does go wrong–if the bridge breaks down–then your mistake was of the kind of using the wrong natural law. (LFM 217)

An echo of this is found in the two subsequent sections of the appendix.

In section 12, Wittgenstein starts with asking about the consequences of the contradiction which arises when one says "I lie. Therefore I do not lie. Therefore I lie." Does this do any harm? Is our use of the language compromised because such an inference is possible? Wittgenstein's answer is: no, it is not.

> *Der Satz* selbst *ist unbrauchbar, und ebenso dieses Schlüsseziehen; aber warom soll man es nicht tun? ... Es ist ein Sprachspiel, das Ähnlichkeit mit dem Spiel des Daumenfanges hat.*[25] (RFM I A.III, 12)

That Wittgenstein tries to deflate the importance of consistency is even more apparent from section 13:

> *Interesse erhält so ein Widerspruch nur dadurch, daßer Menschen gequält hat und dadurch zeigt, wie aus der Sprache quälende Probleme waschen können; und was für Dinge uns quälen können.*[26] (RFM I A.III, 13)

Some commentators have argued that Wittgenstein wants to deflate FIT on the premise of it being a logical paradox like the Liar's paradox. Notably, also Gödel himself, after having been shown Wittgenstein's remarks, has commented that Wittgenstein apparently did not understand his result because of this matter:

25. The proposition *itself* is unusable, and these inferences equally; but why should they not be made? ... It is a language game with some similarity to the game of thumb-catching.

26. Such a proposition is of interest only because it has tormented people, and because this shews both how tormenting problems can grow out of language, and what kind of things can torment us.

> As far as my theorem about undecidable propositions is concerned it is indeed clear from the passages you cite that Wittgenstein did not understand it (or pretended not to understand it). He interprets it as a kind of logical paradox, while in fact it is just the opposite, namely a mathematical theorem within an absolutely uncontroversial part of mathematics (finitary number theory or combinatorics). Incidentally, the whole passage you cite seems nonsense to me. See, e.g. the 'superstitious fear of mathematicians of contradictions'.[27]

This last sentence refers to another remark made later on in the appendix:

> *(Die abergläubische Angst und Verehrung der Mathematiker vor dem Widerspruch.)*[28]
>
> (RFM I A.III, 17)

Such a claim is, however, not warranted by these passages. My view on these is that Wittgenstein tries to show that if we hold on to the natural-language interpretation of P, a contradiction is obtained. But then the remark in section 13 draws our attention to the fact that when we have such a contradiction, there is no reason to be worried. We only know what it is that gets us worried, particularly the relation between 'proved' and 'true'. It is therefore that Wittgenstein turns back to the meaning of 'unprovable (proposition)'.

Sections 14-16: Proofs of unprovability

In section 14, Wittgenstein takes another point of view towards the unprovability of a proposition, and tries to forge a connection between FIT, and Euclidean geometry:

27. Gödel in a letter to Karl Menger. Quoted from (Floyd 1995, 409). Floyd, actually, accuses Gödel of confusing Wittgenstein's own voice and that of his imaginary interlocutor. However, she is this herself.

28. (The superstitious dread and veneration by mathematicians in face of contradiction.)

> *Ein Beweis der Unbeweisbarkeit ist quasi ein geometrischer Beweis ... Ganz analog einem Beweise etwa, daß die und die Konstruktion nicht mit Zirkel und Lineal ausführbar ist.*[29] (RFM I A.III, 14)

The consequence of such a proof, Wittgenstein asserts, would be that no one would bother to find such a construction, as is the case with the trisection of an angle. In this sense, such a proof carries with it an element of prediction:

> *Er muß – könnte man sagen – für uns ein* triftiger Grund *sein, die Suche nach einem Beweis (also einer Konstruktion der und der Art) aufzugeben.*
>
> *Ein Widerspruch ist als eine solche Vorhersage unbrauchbar.*[30] (RFM I A.III, 14)

Floyd has argued that this section implies that Wittgenstein saw the unprovability of P as analogous to the impossibility of construction the trisection of an angle. On her reading, Gödel's Theorem is for Wittgenstein "simply an impossibility proof".[31] I believe this is not the case. As Floyd herself notes, there "are important disanalogies" between the two proofs. The most important of these is that "it is not as if one constructed a triangle in Euclid which said of itself, "I am not constructible"."[32] This disanalogy looks insurmountable, and I believe Wittgenstein thought so as well. Furthermore, there is another objection to Floyd's account, which can be inferred from the last sentence.

29. A proof of unprovability is as it were a geometrical proof ... Quite analogous e.g. to a proof that such-and-such a construction is impossible with ruler and compass.
30. It must–we might say–be a *forcible reason* for giving up the search for a proof (i.e. for a construction of such-and-such a kind.)
A contradiction is unusable as such a prediction.
31. Floyd 1995, 400.
32. 409-410.

The section starts with the assumption that there is a proof of unprovability, and therefore it is a continuation of the reasoning started in section 11. As we have seen there, the assumption that the unprovability of P is proven, yields a contradiction. But, as Wittgenstein contends in the last sentence of section 14, a contradiction is incapable of making a prediction, for we do not no which side of the contradiction we should follow. But obviously this sets is apart from the proof of impossibility of a certain geometrical construction where we *do* know which path to follow. Therefore, I read this section as Wittgenstein trying to make sense by forging this analogy, however seeing that it fails.

In section 15 Wittgenstein enters familiar territory. The question is: when are we justified in believing that a certain proposition says "X is unprovable"? The answer is to be found in the proof of the proposition:

> *Nur der Beweis zeigt, was als das Kriterium der Unbeweisbarkeit gilt. Der Beweis ist ein Teil des Systems von Operationen, des Spiels, worin der Satz gebraucht wird, und zeigt uns seinem 'Sinn'.*[33] (RFM I A.III, 15)

We see here a replication of an argument Wittgenstein has made more often: if you want to know what is proven, you have to look at the proof. But we have also seen that the fact that a proposition is proven has a consequence, that is that it is being used as a rule in the language game. This would mean that the proof of unprovability of a proposition forces us to stop looking for a proof. However, Wittgenstein disputes this:

> *Es ist also die Frage ob der 'Beweis der Unbeweisbarkeit von P' hier ein triftiger*

33. The proof alone shews what counts as the criterion of unprovability. The proof is part of the system of operations, of the game, in which the proposition is used, and shews us its 'sense'.

> *Grund ist zur Annahme daß ein Beweis von P nicht gefunden werd.*[34] (RFM I A.III, 15)

This thought is elaborated upon in the following section:

> *Der Satz "P ist unbeweisbar" hat einen andern Sinn, nachdem – als ehe er bewiesen ist.*[35] (RFM I A.III, 16)

We know that Wittgenstein thought that to know the meaning of a proposition is to know when it is true. For mathematical propositions things are slightly different: they acquire their meaning through their proof. In the case of P we now see something unusual happening:

> *Ist er bewiesen, so ist er die Schluß des Unbeweisbarkeitsbeweises. – Ist er unbewiesen, so ist ja noch nicht* klar, was *als Kriterium seiner Wahrheit zu gelten hat, und sein Sinn is – kann man sagen – noch verschleichert.*[36] (RFM I A.III, 16)

What Wittgenstein is trying to say here is that, as P is presumed to be a proposition of mathematics, it has to obtain its meaning through its proof. But because P is constructed to be the same proposition as 'P is unprovable', it is impossible that P obtains his meaning that way. We seem to be in an inextricable position. Either we deny P its status as a mathematical proposition, or we have to give up some of our (or at least: Wittgenstein's) believes about mathematics. In the next section, Wittgenstein will write out the dilemma in detail.

34. Thus the question is whether the 'proof of the unprovability of P' is here a forcible reason for the assumption that a proof of P will not be found.

35. The proposition "P is unprovable" has a different sense afterwards–from before it was proved.

36. If it is proved, then it is the terminal pattern in the proof of unprovability.–If it is unproved, then *what* is to count as a criterion of its truth is not yet *clear,* and–we can say–its sense is still veiled.

Sections 17-20: Summarizing the critique

Section 17 is a particularly long section, which mainly serves as a summary of all the conclusions Wittgenstein has reached so far. The question is: what happens when P is proved or refuted? We can distinguish two ways in which P can be proved. The first way is by a proof of the unprovability of P. Then we have to look at the proof:

> *Nun, um zu sehen,* was *bewiesen ist, schau an den Beweis! Vielleicht ist hier bewiesen, daß die und die Form des Beweises nicht zu P führt.*[37] (RFM I A.III, 17)

The other way of proving P is *directly*, which means a proof of P is constructed. In that case we have a problem:

> *... dann folgt also der Satz "P ist unbeweisbar", und es muß sich nun zeigen, wie diese Deutung der Symbole von P mit der Tatsache des Beweises kollidiert und warum sie hier aufzugeben sei.*[38] (RFM I A.III, 17)

Although Wittgenstein professes some doubt about the natural-language interpretation of P in this passage, he does at this stage not argue for the unconditional rejection of this interpretation, as he did in the "notorious paragraphs".

Similarly, the refutation of P (or: the proof of $\neg P$) may be deduced either from a proof of P or by a direct proof of $\neg P$. In the first case, a proof of P means that P is not provable, and therefore that $\neg P$ is true. We are again in an inextricable situation.

> *Wenn mich jemand fragt: "Was ist der Fall – P, oder nicht-P?", so antworte ich: P*

37. Now, in order to see *what* has been proved, look at the proof. Perhaps it has here been proved that such-and-such forms of proof do not lead to P.

38. ... in that case there follows the proposition "P is unprovable", and it must now come out how this interpretation of the symbols of P collides with the fact of the proof, and why it has to be given up here.

> *steht am Ende eines Russellschen Beweises, du schreibt also im Russellschen System: P; anderseits ist es aber eben beweisbar und dies drückt man durch nicht-P, dieser Satz aber steht nicht am Ende eines Russellschen Beweises, gehört also nicht zum Russellschen System.*[39] (RFM I A.III, 17)

Wittgenstein notes that when the meaning 'P is unprovable' was given to P, the proof of P was not known, but we cannot say now that P means that this proof does not exist. He goes on:

> *Ist der Beweis hergestellt, so ist damit eine* neue Lage *geschaffen: Und wir haben nun zu entscheiden, ob wir* dies *einen Beweis (*noch *einen Beweis), oder ob wir* dies *noch die Aussage der Unbeweisbarkeit nennen wollen.*[40] (RFM I A.III, 17)

We reach the same conclusion here as before: we have to doubt what it means that P asserts its own unprovability. He concludes this section by examining the possibility that $\neg P$ is proven directly. In that case we know that it is possible to give a direct proof of P.

> *Das ist also wieder eine Frage der Deutung – es sei denn, daß wir nun auch einen direkten Beweis von P haben. Wäre es nun so, nun, so wäre es so.*[41] (RFM I A.III, 17)

The section is concluded with the remark cited at page 51. We are back at the question of contradiction, and we see again that Wittgenstein takes a laid-back response towards this problem.

39. If someone asks me "Which is the case, P, or not-P?" then I reply: P stands at the end of a Russellian proof, so you write P in the Russellian system; on the other hand, however, it is then provable and this is expressed by not-P, but this proposition does not stand at the end of a Russellian proof, and so does not belong to the Russellian system.

40. Once the proof has been constructed, this has created a *new situation:* and now we have to decide whether we will call *this* the statement of unprovability.

41. So this is once more a question of interpretation–unless we now also have a direct proof of P. If it were like that, well, that is how it would be.

In section 18, Wittgenstein, for the first time, assesses the possibility that P is false. On what grounds do we call P false?

> *Weil du einen Beweis siehst? – Oder aus andern Gründen? Dann machtes ja nichts. Man kann ja den Satz des Wiederspruchs sehr wohl falsch nennen, mit der Begrundung z. B., daß wir sehr oft mit gutem Sinn auf eine Frage antworten: "Ja und nein".*[42] (RFM I A.III, 18)

In other words, there is no reason we should not call P false, on whatever grounds we have for this. The earlier Wittgenstein would perhaps have argued that logic would prevent us from doing so, but the autonomy of logic has already been rejected in this stage of Wittgenstein's philosophical development. Logical laws are now seen by Wittgenstein as the results of linguistic conventions. We might just as well call P false as we might call the Law of Contradiction false; the latter obtains its validity only from the way we use it in our language. To give his argument more force, Wittgenstein closes this section with another example of a 'law' in logic which can be modified because of its use in our language:

> *Und desgleichen der Satz '$\neg\neg p = p$'; weil wir die Verdoppelung der Verneinung als eine* Verstärkung *der Verneinung verwenden und nicht bloß als ihre Aufhebung.*[43] (RFM I A.III, 18)

With this second example Wittgenstein links the discussion of FIT with *Anhang* II. In this appendix he already examined the possibility of the same modification of this law and its conse-

42. Because you can see a proof?–Or for other reasons? For in that case it doesn't matter. For one can quite well call the Law of Contradiction false, on the grounds that we very make often good sens by answering a question "Yes and no".

43. And the same for the proposition '$\neg\neg p = p$' because we employ double negation as a *strengthening* of the negation and not merely as its cancellation.

quences.

In section 19, Wittgenstein challenges the whole enterprise of asserting a proposition like P:

> *Du sagst: "..., also ist P wahr und unbeweisbar." Das heißt wohl: "Also P". Von mir aus – aber zu welchen Zweck schreibt du diese 'Behauptung' hin?*[44] (RFM I A.III, 19)

To illustrate the absurdity of such an assertion, Wittgenstein comes up with the following metaphor:

> *Das ist, als hätte jemand aus gewissen Prinzipien über Naturformen und Baustil abgeleitet, auf den Mount Everest, wo niemand wohnen kann, gehöre ein Schlößchen im Barockstile.*[45] (RFM I A.III, 19)

Wittgenstein ends the section with a rather sharp conclusion:

> *... wie könntest du mir die Wahrheit der Behauptung plausibel machen, da du sie ja zu nichts weiter brauchen kannst als zu jenen Kunststückchen?*[46] (RFM I A.III, 19)

With this we are back at Wittgenstein's demand that mathematical propositions must have a use to have meaning. But after all his inquiries, we are still nowhere near such a use, accept for doing some 'bits of legerdemain'. Kienzler and Grève have argued that Wittgenstein has failed to make any sense of Gödel's result. I believe, however, that the conclusion reached here is even stronger: Wittgenstein is trying to convince us, as readers, that there *is* no sense to be given

44. You say: "P is true and unprovable". That presumably means: "Therefore P". That is all right with me–but for what purpose do you write down this 'assertion'. [Note: Reesh uses 'presumably' as a translation of 'wohl'. In this case, I believe it would better be translated as 'surely' or 'no doubt'.]

45. It is as if someone had extracted from certain principles about natural forms and architectural style the idea that on Mount Everest, where no one can live, there belonged a châlet in the baroque style.

46. ... how could you make the truth of the assertion plausible to me, since you can make no use of it except to do these bits of legerdemain?

to a proposition which asserts its own unprovability, at least not within mathematical practice as we knew it. So either we have to reject the natural-language interpretation of P, or we are presented with a new situation, i.e. a situation in which we have to revise our commonly held opinions about the nature and the practice of mathematics.

With section 19 the discussion of 'true but unprovable propositions' is finished. The last section therefore serves only as an afterthought, in which Wittgenstein wants to remind us that

> *... die Sätze der Logik so konstruiert sind, daß sie als* Information keine *Anwendung in der Praxis haben. Man könnte also sehr wohl sagen, sie seien gar nicht* Sätze*; und daß man sie überhaupt hinschreibt, bedarf einer Rechtfertigung.*[47] (RFM I A.III, 20)

Wittgenstein is here exhibiting a view which was already present in his Tractarian period: that propositions of logic have themselves nothing to say. This view has in the later stages of his thinking lead to a complete overhaul of the role of logic in language, of which the rest of the section gives us another impression:

> *Fügt man diesen 'Sätzen' nun ein weiteres satzartiges Gebilde andrer Art hinzu, so sind wir hier schon erst recht im Dunkeln darüber, was dieses System von Zeichenkombinationen nun für eine Anwendung, für einen Sinn haben soll, denn der bloße* Satzklang *dieser Zeichenverbindungen gibt ihnen ja eine Bedeutung noch nicht.*[48] (RFM I A.III, 20)

47. ... the propositions of logic are so constructed as to have *no* application as *information* in practice. So it could very well be said that they were no *propositions* at all; and one's writing them down at all stands in need of justification.

48. Now if we append to these 'propositions' a further sentence-like structure of another kind, then we are all the more in the dark about what kind of application this system of sign-combinations is supposed to have; for the mere *ring of a sentence* is not enough to give these connexions of signs any meaning.

We have seen this thought also occurring in section 3.4 when we discussed the application of logical propositions as the Law of Excluded Middle in mathematics. In this case, the sentence '777 occurs in the expansion of π or 777 does not occur in the expansion of π' sounds like a proposition, but it is nevertheless unclear what meaning such a proposition has, as we need a proof for one of the conjuncts. And in the same sense, P may *sound* like a proposition, but that does not mean it is one.

4.2 Other writings and lectures

So far, I have examined Wittgenstein's most important inquiry on FIT. I have argued that Wittgenstein actually deals with the concept of 'true but unprovable propositions', rather than Gödel's result. On my reading, Wittgenstein argues that if we want to retain this concept, we find ourselves entangled in a new situation, in which we have to give up some of our commonly held believes about the nature and the practice of mathematics. In the following sections, I will survey other parts in which Wittgenstein has discussed either the Theorem or 'true but unprovable propositions' to see if his remarks in these writings or lectures are consistent with this interpretation.

4.2.1 Lectures on the Foundations of Mathematics

Wittgenstein's first time he returned to the topic was during his lectures on the foundations of mathematics in Cambridge. His first remark related to unprovable propositions occurs in lecture VI:

> If we prove that a certain mathematical proposition is not provable, then we may be said to be asserting a proposition of geometry; it is like asserting that the heptagon cannot be constructed. If we really prove that the heptagon cannot be constructed, it should be a proof which makes us give up trying–which is an empirical affair. And similarly with proving that a certain proposition is not provable. (LFM 56)

This is reminiscent of (RFM I A.III, 14) in the sense that unprovable propositions are likened to geometrical figures which are not constructible. However, this goes for unprovable propositions in general, but not for P, which is special as it asserts its own unprovability. The consequences of this peculiarity were already explored by Wittgenstein in (RFM I A.III, 14).

The only time Wittgenstein talks about FIT proper occurs in lecture XIX:

> One often hears statements about "true" and "false"–for example, that there are true mathematical statements which can't be proved in *Principia Mathematica*, etc. In such cases the thing is to avoid the words "true" and "false" altogether, and to get clear that to say that p is true is simply to assert p; and to say that p is false is simply to deny p or to assert $\neg p$. It is not a question of whether p is "true in a different sense". It is a question of whether we assert p. (LFM 188)

Floyd, in connection to this remark, asserts rightfully that this sums up "a crucial point" of Wittgenstein's philosophy:

> It is our use of words and of empirically given constructions that brings a symbolism to life within our ongoing practices of justification. ... Gödel's theorem shifted our use of [notions like truth, meaning and proof], and that is part of what makes it both

> a genuine contribution to mathematics, and a result of great interest to philosophy.[49]

This point is forcefully pressed in what follows during the lectures. For asserting is what we do with tautologies – at least in logic. Now Wittgenstein tries to devise new logics, in which we only assert contradictions. But he goes further:

> By the way, this is the way in which a proposition can assert of itself that it is not provable. Besides putting the assertion sign before contradictions I could put it before propositions like '$p \implies q$'. In the one case, '$\vdash p \wedge \neg p$' would mean '$p \wedge \neg p$ is refutable'; and in the other '$\vdash p \implies q$' would mean $p \implies q$ is not provable'. (LFM 189)

Unfortunately, what follows is an clearly inaccurate description of the lecture but I believe the point is clear: what we see as our criterion for assertion is merely a convention, which is challenged by Gödel.

4.2.2 RFM: *Teil* VII

Apart from (RFM I A.III), Wittgenstein discusses Gödel in the *Bemerkungen* only one time, in (RFM VII). This part is a selection from more comprehensive manuscripts, and is therefore not complete. The first remark about Gödel occurs in section 19:

> *Meine Aufgabe ist es nicht, über den Gödelschen Beweis, z. B., zu reden; sondern an ihm vorbei zu reden.*[50] (RFM VII, 19)

49. Floyd 2001, 300.
50. My task is, not to talk about (e.g.) Gödel's proof, but to by-pass it.

This is actually the first time he mentions *the proof* of FIT at all in his writings, as in the earlier ones, he was more concerned with possible proofs of P. What Wittgenstein means with '*vorbeireden an*' or 'to by-pass' the proof of FIT is not agreed upon by all commentators. Some, such as Rodych,[51] have argued that this means that Wittgenstein wants to ignore or reject the proof altogether. However, it is important to observe that Wittgenstein treats Gödel's proof here as an example. He could as well have taken Cantor's diagonal argument instead, which he has given a similar treatment in (RFM II). These proofs have in common that they have metaphysical consequences, rather than only mathematical ones. These alleged consequences are what Wittgenstein is trying to attack, and this by he wants to go beyond their proofs. This view is also consistent with the two remarks directly preceding the one cited above:

> *Meine Aufgabe ist es nicht, Russells Logik von* innen *anzugreifen, sondern von außen.*[52]
>
> *D.h.: nicht, sie mathematisch anzugreifen – sonst triebe ich Mathematik –, sondern ihre Stellung, ihr Amt.*[53] (RFM VII, 19)

Here again, Wittgenstein emphasizes that he is not interested in doing mathematics, but only in the position which are ascribed to some methods or theorems. What Wittgenstein gives here is essentially a summary of the 'quasi-revisionist' approach I attributed to him in section 3.4.

A more thorough discussion of our topic is contained in the sections 21 and 22. Section 21 seems at first sight to be a collection of loose observations about P. The first of these is about

51. source?
52. It is my task, not to attack Russell's logic from within, but from without.
53. That is to say, not to attack it mathematically–otherwise I should be doing mathematics–but its position, its office.

P as a mathematical statement:

> ... *der Satz, der von sich selbst aussagt, er sei unbeweisbar, ist als* mathematische *Aussage aufzufassen – denn das ist nicht* selbstverstandlich.[54] (RFM VII, 21)

There is a reason that this is not matter of course:

> *Hier nämlich ensteht leicht Verwirrung durch den bunten Gebrauch des Ausdrucks "dieser Satz sagt etwas von ... aus".*[55] (RFM VII, 21)

Wittgenstein is here arguing that propositions can be self-referential in several ways. One such way can be gathered from ordinary propositions of arithmetic:

> *In diesem Sinne sagt der Satz* '$625 = 25 \times 25$' *auch etwas über sich selbst aus: daß nämlich die linke Ziffer erhalten wird, wenn man die rechts stehenden multipliziert.*[56] (RFM VII, 21)

In what follows Wittgenstein questions the 'translation' of mathematical propositions into natural language:

> *Aber bist du sicher, daß du ihn recht ins Deutsche übersetzt hast? Ja gewiß, es scheint so. – Aber kann man da nicht fehlgehen?*[57] (RFM VII, 21)

The following remark concerns the meaning of Gödel's proposal of mapping propositions onto the system in which they are used:

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54. ... the proposition asserting of itself that it is unprovable is to be conceived as a *mathematical* assertion–for that is not a *matter of course*.

55. For here it is easy for confusion to occur through the veriegated use of the expression "this proposition asserts something of...".

56. In this sense the proposition '$625 = 25 \times 25$' also asserts something about itself: namely that the left-hand number is got by the multiplication of the numbers on the right.

57. But are you also certain that you have translated it correctly into English? Certainly it looks as if you had.– But isn't is possible to go wrong here?

> *Könnte man sagen: Gödel sagt, daß man einem mathematischen Beweis auch muß trauen können, wenn man ihn, praktisch, als den Beweis der Konstruierbarkeit der Satzfigur nach den Beweisregeln aufffassen will?*
> *Oder: Ein mathematischer Satz muß als Satz einer auf sich selbst wirklich anwendbaren Geometrie aufgefaß werden können. Und tut man das, so zeigt es sich, daß man sich auf einen Beweis in gewissen Fällen nicht verlassen kann.*[58] (RFM VII, 21)

In the closing remark of section 21, Wittgenstein brings the discussion back to the relation between truth and mathematics:

> *Die Grenzen der Empirie sind nicht unverbürgte Annnahmen oder intuitiv als richtig erkannte; sondern Arten und Weisen des Vergleichens und des Handelns.*[59] (RFM VII, 21)

According to the editors, Wittgenstein alludes to Russell's lecture 'The Limits of Empiricism'.[60] For Wittgenstein, what we consider as true (empirical) propositions, is merely a matter of grammar (a position Russell explicitly ascribes to Wittgenstein[61]). It seems that Wittgenstein wants to apply this to to propositions of mathematics as well: we assert true propositions at the end of a proof or as an axiom.

58. Could it be said: Gödel says that one must also be able to trust a mathematical proof when one wants to conceive it practically, as the proof that the propositional pattern can be constructed according to the rules of proof?
Or: a mathematical proposition must be capable of being conceived as a proposition of a geometry which is actually applicable to itself. And if one does this it comes out that in certain cases it is not possible to rely on a proof.

59. The limits of empiricism are not assumptions unguaranteed, or intuitively known to be correct: they are ways in which we make comparisons and in which we act.

60. Russell 1935.

61. 140.

Section 22 starts with a particularly long discussion of the translation of a proposition into numbers. After finishing our proof of FIT, we obtain a situation in which we have both a 'propositional sign' for P, and a number which corresponds with it. He continues to ask what we would be inclined to do with this result:

> *Was wäre nun zu tun? Ich denke mir, wir schenken unserer* Konstruktion *des* Satzzeichens *Glauben, also dem* geometrischen *Beweis. Wir sagen also, diese 'Satzfigur' ist aus jenen so und so gewinnbar. Und überstragen, nun, in eine andre Notation heißt das: diese Ziffer ist mittels dieser Operationen aus jenen zu gewinnen.*[62] (RFM VII, 22)

According to Wittgenstein, this does not mean that any special logic would have anything to do with either the proposition or the number. It is simply the case that the constructed proposition is just another notation for the constructed number:

> *... sie hatte die* Form *eines Satzes, aber wir vergleichen sie nicht mit andern Sätzen als Zeichen, welches dies oder jenes* sagt *einen* Sinn *hat.*[63] (RFM VII, 22)

The three following paragraphs are mainly concerned with the constructed 'propositional sign' or the number by which the proposition is denoted. These remarks are a bit puzzling, and do not really shed any light on Wittgenstein's overall opinion on 'true but unprovable' propositions, which why I skip them here.

62. What would have to be done here? I am supposing that we trust our *construction* or *propositional sign*; i.e. we trust the *geometrical* proof. So we say that this 'propositional pattern' can be obtained from those in such and such ways. And, merely translated into another notation, this means: this number can be got from those by means of these operations.

63. ... it had the *form* of a proposition but we don't compare it with other propositions as a sign *saying* this or that, making *sense.*

The three closing statements, however, brings us back on familiar soil. First of all, Wittgenstein asks us what counts as 'being convinced' by a proof.

> *Hier kommen wir wieder auf den Ausdruck "der Beweis überzeugt uns" zurück. Und was uns hier an der Überzeugung interessiert, ist weder ihr Ausdruck durch Stimme oder Gebärde, noch das gefühl der Befriedigung oder ähnliches; sondern ihre Bestätigung in der Verwendung des Bewiesenen.*[64] (RFM VII, 22)

We have already established, that there is not much use for P, accept for doing some "bits of legerdemain". Furthermore, we might ask ourselves what the significance of Gödel's result is:

> *Man kann mit Recht fragen, welche Wichtigkeit Gödel's Beweis für unsre Arbeit habe. Denn ein Stück Mathematik kann Probleme von der Art, die* uns *beunruhigen, nicht lösen. – Die Antwort ist: daß die Situation uns interessiert, in die ein solcher Beweis uns bringt. "Was sollen wir nun sagen?- das ist unser Thema.*[65] (RFM VII, 22)

That is not to say, of course, the result is not important. But the result just has not much significance for Wittgenstein's aims, which is giving clarity to what we are doing when we are doing mathematics. The final remark stresses this point even further:

> *So seltsam es klingt, so scheint meine Aufgabe, das Gödelsche Theorem betreffend, bloß darin zu bestehen, klar zu stellen, was in der Mathematik so ein Satz bedeutet, wie: "angenommen, man köntte dies beweisen".*[66] (RFM VII, 22)

64. Here once more we come back to the expression "the proof convinces us". And what interests us about conviction here is neither its expression by voice or gesture, not yet the feeling of satisfaction or anything of that kind; but its ratification in the use of what is proved.

65. It might justly be asked what importance Gödel's proof has for our work. For a piece of mathematics cannot solve problems of the sort that trouble *us*.–The answer is that the *situation*, into which such a proof brings us, is of interest to us. 'What are we to say now?'–That is our theme.

66. However queer it sounds, my task as far as concerns Gödel's proof seems merely to consist in making clear what such a proposition as: "Suppose this could be proved" means in mathematics.

4.2.3 The *Nachlass*

The last remarks I want to talk about are to be found among the unpublished manuscripts in the *Nachlass*. Wittgenstein mentions Gödel and his theorem on several occasions in his unpublished writings. In what follows, I will use those which have been found an discussed by Rodych.[67] Due to space limitations, I cannot evaluate all of them, so I have made a selection. These remarks come from (MS 117; 1938 & 1940), (MS 121; 1938-1939), (MS 163; 1941) and (MS 124; 1944).

The first remark I want to discuss comes from 1938, in which Wittgenstein offers us an logical deduction or proof of sorts:

> We accept the proposition [theorem[68]] that (for all [propositions] p)
>
> $\vdash \neg\Pi p \wedge p$
>
> If we now find a specific proposition P_1 for which
>
> $P_1 = \neg\Pi P_1$
>
> it follows that
>
> $\vdash \neg\Pi P_1 \wedge \neg\Pi P_1$. But $\vdash \neg\Pi P_1 \wedge \neg\Pi P_1 = \vdash \neg\Pi P_1 = \vdash P_1$
>
> Is this Gödel's train of thought? (MS 117, p. 147; August 1, 1938, Cambridge)

What Wittgenstein does here is (1) to assert that all propositions are true or not provable ($\vdash \neg\Pi p \wedge p$), (2) to construct a proposition P_1 which asserts its own unprovability and (3) deduce by logical inference that P_1 must therefore be true. Rodych considers this as Wittgenstein's "proof sketch" for the proof of FIT, and considers this as evidence that, at least in 1938, Wittgenstein had no correct understanding of Gödel's proof. I have, however, already argued that in that

67. Rodych 2002, 2003.
68. According to Rodych both 'proposition' and 'Theorem' are possible translations for '*Satz*'. See (Rodych 2003, fn. 25).

period Wittgenstein had no interest whatsoever for the proof offered by Gödel. This "proof sketch" should, I believe, be interpreted as a proof of the remark Gödel makes in the introduction of his paper:

> *Aus der Bemerkung, daß* $[R(q); q]$ *seine eigene Unbeweisbarkeit behauptet, folgt sofort, daß* $[R(q); q]$ *richtig ist, denn* $[R(q); q]$ *ist ja unbeweisbar (weil unentscheidbar). Der* im *System PM unentscheidbare Satz wurde also durch metamathematische Überlegungen doch entschieden.*[69] (Gödel 1931, 176)

So on my interpretation the question "Is this Gödel's train of thought?" does not pertain to the proof of FIT, but to the "metamathematical considerations" which establish the truth of P (or: P_1, $[R(q); q]$).

Apart from this, most remarks in the *Nachlass* are about the consequences the Gödel's result could have. These consequences could be mathematical or philosophical. I will consider the consequences for mathematics (or better: mathematical practice) first:

> Gödel's discovery is a mathematical discovery. Now if such a discovery can be regarded as an extension of grammar, what is the grammatical meaning of the construction? Could this also be expressed as follows: Which extra-mathematical application can we give to Gödel's theorem?
>
> What application do we have for a proposition that mathematically asserts its own unprovability? (MS 163, 41v; July 11, 1941)

Another remark by Wittgenstein, which was cut out from (RFM VII, 31), states:

69. From the remark that $[R(q); q]$ asserts its own unprovability, it follows at once that $[R(q); q]$ is correct, since $[R(q); q]$ is certainly unprovable (because undecidable). So the proposition which is undecidable *in the system* PM yet turns out to be decided by metamathematical considerations. (Gödel 1992, 41)

> What's unphilosophical in Gödel's essay is that he doesn't understand the relationship between mathematics and its application. In this, he maintains the murky notions of most mathematicians. (MS 124, 115r; March 5, 1944)

What is interesting here is that Wittgenstein explicitly calls for the *extra-mathematical* application of Gödel's theorem. Rodych sees this as evidence that Wittgenstein believes that "extra-system application is essential to mathematical propositionhood."[70] I have already argued that we should not interpret Wittgenstein to strictly when it comes to his demand for extra-mathematical application. In this case, we may recall what Wittgenstein said about the impossibility proofs in geometry. Those kind of proofs have an 'empirical' application in the sense that they predict that no construction of a certain figure will be found. Furthermore, Wittgenstein argued that in the case of P no such prediction ensues, so there is no application for this kind of propositions.

I want to close this section with some remarks about the philosophical significance Wittgenstein conferred on Gödel's result:

> Gödel shows us an unclarity in the concept of 'mathematics', which is indicated by the fact that mathematics was taken to be a system. (MS 121, 76r; Dec. 28, 1938)

> Gödel's theorem develops a problem that must appear in a much more elementary way. (And herein, it seems to me, lies both Gödel's great service to the philosophy of mathematics and, at the same time, the reason why it is not his particular theorem that interests us.) (MS 163, 39v-40r; July 8, 1941)

70. Rodych 2002, 387.

The first one is considered by Rodych to be very straightforward: before Gödel's proof, mathematics was taken to be a complete system, after Gödel's proof, this is no longer the case. Rodych refers here to the Hilbert program, which was shattered by the publication of the Incompleteness Theorems.[71] However, Wittgenstein's remark is not as straightforward as it seems. We have seen that in (RFM I A.III, 8), Wittgenstein holds that the truth of a proposition is dependent on the system in which it is used. The "unclarity" is a consequence of the fact that for P it is not clear in which system it is true: in PM or outside PM?

The second remark, then, makes it even more clear that the theorem itself was not what interested Wittgenstein. What he is interested in is an 'elementary' question: what do we mean when we say that a proposition is true? The solution, as we have seen, is to abandon the use of the words 'true' and 'false' as properties of a proposition altogether.

71. See (Zach 2003), particularly section 2.2, for a discussion of this topic.

Chapter 5

Conclusion

In the foregoing chapters I have done three things. First, I have explained what different views Wittgenstein held about mathematics in the period 1929-1944. I have described how he changed from a calculus conception to a language-game conception but how he retained a consistent anti-Platonist stance throughout his career. Second, I have defended my view on Wittgenstein's philosophy of mathematics. I have argued that application is an important aspect, and he demands that a mathematical proposition has an application. However, I have deviated from the traditionally held view that only *extra-mathematical* application could give meaning to mathematical propositions. Indeed, I have argued there is room for pure mathematics in Wittgenstein's philosophy. Third, I have advanced my view that Wittgenstein's philosophy of mathematics is revisionary, but only so in the sense that Wittgenstein wants to rid mathematics from the metaphysical implications attributed to some mathematical results. In other cases, Wittgenstein also tries to eradicate methods of mathematics which he sees as murky, mostly those in which the infinite is involved. For this kind of revisionism I have borrowed the term 'quasi-revisionism' from Frascolla. I am convinced that Wittgenstein has never attempted to do more with mathematics than any of this two things, and I believe there is ample textual evidence that he – being a philosopher – did not want to interfere with the work of mathematicians.

However, some commentators have argued that he has not been consistent in this position, and that on some topics he takes a strictly revisionist stance. I am however confident that the passages in which these topics are discussed are viable for interpretations consistent with my 'quasi-revisionist' conception of Wittgenstein. Indeed, the third thing I did in this thesis, is to compare my views of Wittgenstein with his remarks about FIT. To this purpose I adapted he method employed by Kienzler and Grève – scrutinizing (RFM I A.III) section-by-section – as I deemed this the most suitable way of making sense of Wittgenstein's writings. In addition to their work, I have added an evaluation of (several parts of) Wittgenstein's other writings about Gödel's result.

Now, the most important question which is to be answered is: was Wittgenstein right in seizing on FIT the way he did? The first answer we can give is: no. Wittgenstein talks mainly about 'true but unprovable propositions' whereas Gödel's result is actually a result in finitary number theory. This is the answer given by Gödel himself, but it is also favored by Victor Rodych, who has been very prolific in arguing this viewpoint. Nonetheless, I answer the question positively. The problem, I think, is that many commentators (among which, besides Gödel and Rodych, also Dummett, Kreisel and Bernays are included) have underappreciated the fact that Wittgenstein is not talking about FIT, nor its proof, *at all*. The whole discussion solely turns around the construction of a proposition which, according to Gödel's own 1931 paper, asserts its own unprovability and is therefore true.

According to Wittgenstein, the construction of this proposition P introduces "a new situation" into mathematics. Before Gödel, a proposition was true when it was proven, but this does not apply to P, which is true because it is unprovable. In (RFM I A.III) he tries to make sense of the notion of "true but unprovable" propositions. His first solution is to abandon the use of the adjectives 'true' and 'false' altogether. This is asserted in (RFM I A.III, 6), but also in LFM 188 and in the *Nachlass*. So this problem is solved. But there is another one which interests Wittgenstein: what is the use of P? We see Wittgenstein making several attempts of giving a meaning to it, but concludes that we can do nothing more than some 'bits of legerdemain'. Furthermore, it is even questionable if we can retain the translation of P into English; all inferences lead somehow to the conclusion that we would have to give up the natural-language interpretation of P.

The last thing I want to say concerns the lack of understanding of Gödel's work, of which Wittgenstein is often accused. Unfortunately, I have not been able to evaluate all the historical and circumstantial evidence which has been presented an discussed by several other commentators[1] – of which a lot can be said – so I confine myself to the writings I have cited in the foregoing. First of all, Wittgenstein never speaks of the proof of FIT in (RFM I A.III), but rather to the proofs of P. Neither does the presumed "proof sketch" pertain to the "train of thought" of Gödel in the proof itself, rather than to its introduction. So concerning Wittgenstein's understanding of Gödel's work in the period 1937-1938 the evidence remains inconclusive. In later writings, such as those in (RFM VII, 21-22), Wittgenstein talks about the translation of P into numbers,

1. See for instance (Floyd and Putnam 2000; Floyd 2001; Rodych 2003).

recognizing to some extent the number-theoretic character of the Theorem. So this could point to an enhanced understanding, or even an understanding which never has been lacking at all.

In the end, all comes back to the spirit in which Wittgenstein did his philosophy, and which he described in the foreword to *Philosophische Bemerkungen.* Contrasting this spirit with the one persistent in European and American civilization, he describes this one as expressing itself in "striving after clarity and perspicuity in no matter what structure." Gödel's result faced Wittgenstein with an unclarity in the structure of mathematics, one which he set out to resolve. Whether he succeeded is another story.

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