Symmetry Breaking in a U(1) extended non-minimal supersymmetric standard model

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Author :	Timo Blom
Supervisor :	Prof. Dr. van Holten
Second corrector :	Prof. Dr. Achúcarro

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Timo Blom

Huygens-Kamerlingh Onnes Laboratory, Leiden University
P.O. Box 9500, 2300 RA Leiden, The Netherlands

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Abstract

Symmetry breaking in realistic supersymmetric theories has proven to be difficult without the introduction of explicit supersymmetry breaking terms. In this thesis we investigate symmetry breaking through the Fayet-Iliopoulos mechanism in a $U(1)$ extended non-minimal supersymmetric standard model incorporating massive right-handed neutrinos and a new scalar field. We derive the potential of the theory and show the Fayet-Iliopoulos mechanism alone does not suffice to obtain realistic symmetry breaking. We conclude that explicit supersymmetry breaking terms are required to obtain realistic symmetry breaking in this model.

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Chapter 1

Introduction

By the middle of the 1970's the construction of the Standard Model was understood. With the addition of the electroweak interaction and the Brout-Englert-Higgs mechanism [1, 2] a quantum field theory was developed based on the gauge symmetries:

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

Today, over 40 years later, the Standard Model is still widely used explaining a vast range of experimental data to very high precision.

While highly succesful, the Standard Model suffers from a few problems. The first is an experimental problem which started with the discovery of neutrino oscillations, which was awarded this year's Nobel Prize in physics. In neutrino detection experiments electron neutrinos emitted from the sun are sometimes found to arrive on earth as mu-neutrinos or tau-neutrinos. By measuring the flux of the different incoming neutrino families, the probabilities of detecting different neutrino species are found to oscillate with distance.

An explanation for the observation of neutrino observations was given by Pontecorvo in [3] who proposed that the mass eigenstates of the neutrinos are linear combinations of the flavour eigenstates. Using Pontecorvo's approach, the neutrino oscillations can be linked to mass differences between the mass eigenstates, indicating that neutrinos are massive. This is not predicted by the minimal Standard Model, where all neutrinos are left-handed and massless. Next to massive neutrinos, this discovery also leads to *right-handed neutrinos* as massive chiral fermions cannot exist.

Another problem, a purely theoretical one, has to do with the stability of the

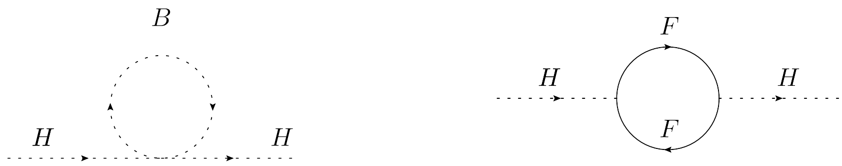

Figure 1.1: *Two processes contributing to the quantum corrections to the Higgs (H) mass. The left process involves a correction due to a boson (B), the right process involves a correction due to fermions (F).*

mass of the Higgs boson under quantum corrections. This problem is known as the *hierarchy problem* and it can be schematically illustrated [4] with the two Feynman diagrams shown in figure [1.1].

In the Feynman diagrams the left process involves a quantum correction to the Higgs mass due to a boson running in a loop, the right diagram involves a correction due to two fermions running in a loop. If one introduces a cut-off scale Λ for the loop process, the left process leads roughly to a correction to the squared Higgs mass m_H^2:

$$\Delta m_H^2 \propto \Lambda^2 + \alpha \ \ln(\frac{\Lambda}{m_b}) \tag{1.1}$$

where m_b is the mass of the boson in the loop and α is some proportionality factor. The right process contributes a correction equal in magnitude but with the characteristic *minus sign* for fermions:

$$\Delta m_H^2 \propto -\Lambda^2 + \beta \ \ln(\frac{\Lambda}{m_f}) \tag{1.2}$$

Usually the cut-off scale Λ is taken to be much larger than the Higgs mass m_H, sometimes as high as the Planck scale. We are therefore led to the following problem: why is m_H so small compared to Λ despite all quantum corrections to the mass?

A solution to the hierarchy problem starts by noting the relative minus sign between the contributions due to fermions and bosons in the loop. If one would associate a boson to each fermion appearing in the quantum corrections, the contributions proportional to Λ^2 might cancel each other, leaving a milder contribution proportional to $\ln(\Lambda)$. To this end a symmetry can be constructed which is called *supersymmetry*. To implement supersymmetry in a theory particles are given *superpartners*: for each boson a fermion is added, and vice versa.

In this thesis we will look at a supersymmetric model which could solve these problems. The model is based on the one described in [5], which introduces massive and right-handed neutrinos, a new gauged $U(1)$ symmetry and a new scalar field used to create Majorana mass terms for the neutrinos.

While supersymmetry offers a solution to the hierarchy problem, it has some of its own challenges. The main difficulty with supersymmetry is breaking it. Supersymmetry demands that superpartners have equal masses, however we know from experimental results that this is not the case.

Multiple mechanisms to break supersymmetry exist, examples are the O' Raifearteigh mechanism [6], dynamical supersymmetry breaking [7], the Fayet-Iliopoulos mechanism [8] and explicit supersymmetry breaking with soft breaking terms. The minimal supersymmetric Standard Model (MSSM), which is the model obtained after making the Standard Model supersymmetric, uses the last of these mechanisms to break supersymmetry by the introduction of a soft breaking Lagrangian.

The soft breaking Lagrangian contains explicit mass terms for superpartners of Standard Model particles along with additional interactions. These additional terms are compatible with the internal symmetries, they do however not respect supersymmetry. With the addition of the soft breaking Lagrangian a lot of new unknown paramters are introduced.

The Fayet-Iliopoulos mechanism could also be implemented in the Standard Model to try to break supersymmetry. This mechanism breaks supersymmetry in a model by introducing a so called *Fayet-Iliopoulos* term for each $U(1)$ symmetry. One Fayet-Iliopoulos term can therefore be introduced to break supersymmetry in the MSSM. This turns out to be insufficient to get realistic supersymmetry breaking in the MSSM.

In this thesis we are going to investigate supersymmetry breaking with the Fayet-Iliopoulos mechanism in a supersymmetric model based on the model described in article [5]. As this model introduces an extra $U(1)$ symmetry, one extra Fayet-Iliopoulos term can be introduced which could help break supersymmetry.

This thesis is built up as follows: after a short introduction to the Standard Model in chapter 2, we will look at the extension of the Standard Model in chapter 3. In chapter 4 we introduce supersymmetry and explain how supersymmetric gauge theories are constructed. After these preparatory chapters, we construct the supersymmetric model in chapter 5 based on the model presented in chapter 3. After this construction, we investigate symmetry breaking in this model and derive results from which we will draw conclusions in chapter 6. Following the conclusion, an appendix [A] can be found containing an overview of the conventions and definitions used in this thesis along with useful Majorana spinor identities, extra information on deriving supersymmetric actions, and the full supersymmetric action of the model presented in chapter 5.

Chapter 2

The Standard Model

The Standard Model of Particle Physics is a relativistic quantum field theory which is constructed along the lines of Quantum Electrodynamics (QED). Like its predecessor, the Standard Model is also a gauge theory, but unlike the gauge theory of QED, the Standard Model has multiple gauge symmetries corresponding to properties similar to electric charge. Not all of these symmetries are however directly visible in experiments, some of the symmetries are spontaneously broken. This symmetry breaking is connected to another big difference between the Standard Model and QED, the generation of masses for fields. Whereas mass terms in QED are explicitly added to the Lagrangian, this turns out not to be possible in the Standard Model, masses are generated through interactions with a scalar field, the Higgs field.

To get an understanding of the Standard Model we will begin with a short introduction to the tools needed to deal with the extra symmetries of the Standard Model. After this introduction we will take a look at the statement that the Standard Model is based on the symmetries

$$SU(3)_c \times SU(2)_L \times U(1)_Y \tag{2.1}$$

and see how this leads to structures on the fields. Following this, we will see how these symmetries are broken and masses for the fields are generated using the Brout-Englert-Higgs mechanism. We end this chapter by seeing how the Standard Model solves problems involving anomalies.

2.1 Generators and Representations

The Standard Model Lagrangian is invariant under transformations of the fields induced by elements from the continuous groups mentioned in expression [2.1]. The elements M of the Standard Model symmetry groups can be obtained by exponentiating a linear combination of only a finite number of Hermitian operators T^a:

$$\begin{aligned} M &= \exp\{i\alpha^a T^a\},\ \alpha \in \mathbb{R} \\ &= 1 + i\alpha^a T^a + \mathcal{O}(\alpha^2) \end{aligned} \tag{2.2}$$

These operators T^a are called the *generators* of the group. When determining the commutator of any two generators of a group, one obtains a linear combination of these same generators:

$$[T^a, T^b] = i f^{abc} T^c \tag{2.3}$$

The coefficients f^{abc} appearing in these commutation relations are called the *structure constants*. There are multiple sets of operators satisfying these commutation relations for given structure constants, these different sets of operators lead to different *representations* of the group.

Different representations lead to different generators and therefore to different symmetry transformations of the fields. To determine how a field transforms, one therefore has to know according to which representation it transforms and what the generators of this representation are.

2.1.1 Representations

The first representation we will need is the *fundamental representation*. For $SU(N)$ the fundamental representation consists of the set of $N \times N$ matrices which can operate on a N-dimensional vector space, we will denote these matrices by t^a. The generators are not yet uniquely defined, one has impose a normalisation condition, which is done by looking at the traces of products of generators. We choose them to be:

$$\mathrm{Tr}\left[t^a t^b\right] = \frac{1}{2}\delta^{ab} \tag{2.4}$$

Using these conventions the generators t^i of $SU(2)$ for example become one half times the Pauli matrices:

$$t^i = \frac{\sigma^1}{2} = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ t^2 = \frac{\sigma^2}{2} = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ t^3 = \frac{\sigma^3}{2} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{2.5}$$

For $SU(3)$ the matrices appearing in the generators analogous to the Pauli matrices are known as the Gell-Mann matrices. The generators for the $U(1)$

symmetry on a field can be taken, as in QED, to be proportional to a charge assigned to that field. We will use as the $U(1)$ symmetry generator t for a field with $U(1)$ charge q the following:

$$t = \frac{q}{2} \tag{2.6}$$

Closely linked to the fundamental representation is the *conjugate representation*, which has the generators:

$$T^a = -(t^a)^* \tag{2.7}$$

where t^a are the generators of the fundamental representation.

Next we need the representation known as the *adjoint representation* of a symmetry group. Its generators are given by the group's structure constants:

$$(T^a)_{bc} = if^{bac} \tag{2.8}$$

Using the Jacobi identity one can show that determining the commutator of two structure constants again leads to linear combination of structure constants as in equation [2.3] .

2.1.2 Transformations, Gauge fields and Covariant derivatives

Just as different representations lead to different generators, the field transformations depend on which generators or representation you choose for the symmetry transformations. We say that fields can be in different representations.

Fields which in a representation with generators $(T^a)_{ij}$ transform infinitesimally as

$$\phi_i \to (1 + i\alpha^a T^a)_{ij}\phi_j \tag{2.9}$$

where the α^i are transformation parameters. The range of the indices i, j depends on the dimension of the generators involved. The ϕ_i form a vector whose components are mixed under symmetry transformation, the vector of fields is called a *multiplet.*

To create Lagrangians which are invariant under local gauge transformation, the concepts of *covariant derivatives* and the *gauge field* of QED return in the Standard Model, but in a more general form. In the Standard Model there are multiple gauge fields, there is one gauge field for every generator of the different symmetry groups. Under gauge transformations the gauge fields associated to generators of each separate symmetry group mix with each other. The gauge fields A^a_μ transform in the adjoint representation but with an extra derivative term:

$$A^a_\mu \to A^a_\mu + \frac{1}{g}\partial_\mu \alpha^a + f^{abc} A^b_\mu \alpha^c \tag{2.10}$$

where g is the coupling constant for the symmetry group.

The covariant derivative for a field transforming in a representation with generators T^a, each which has an associated gauge field A^a_μ, is given by:

$$\nabla_\mu = \partial_\mu - igA^a_\mu T^a \tag{2.11}$$

In terms of these covariant derivatives the *field strength tensor* $F^a_{\mu\nu}$ is defined as:

$$[\nabla_\mu, \nabla_\nu] \equiv -igF^a_{\mu\nu} T^a \tag{2.12}$$

which leads to:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc} A^b_\mu A^c_\nu \tag{2.13}$$

2.2 Symmetries and Particles in the Standard Model

Now that we have learned about generators and representations, we are ready to discuss the symmetries and particle content of the Standard Model.

2.2.1 Symmetries of the Standard Model

As briefly stated before, the Standard Model Lagrangian is invariant under symmetry transformations of three symmetry groups. The first and simplest symmetry is the $U(1)_Y$ symmetry, which has an associated $U(1)_Y$ charge called *hypercharge*, denoted by Y. This hypercharge is not equal to the electric charge of the electromagnetic $U(1)_{EM}$ symmetry in QED. One might therefore be inclined to think the $U(1)_{EM}$ symmetry is lost in the Standard Model, however this is not the case. The $U(1)_{EM}$ symmetry turns out not to be a fundamental symmetry, but a symmetry which is obtained only after symmetry breaking, as we will discuss in the next section. The hypercharge is linked to the electric charge by the Gell-Mann-Nishijima relation:

$$Q = \frac{1}{2}Y + I_3 \tag{2.14}$$

where I_3 is a number linked to another symmetry of the Standard Model, the $SU(2)_L$ symmetry.

The third component of *isospin*, I_3, is a conserved number in weak interactions. Whether or not a fermion carries isospin is mainly linked to its *chirality* or handedness. We call a fermion left-handed if it is described by a spinor ψ which has eigenvalue -1 under application of γ_5:

$$\gamma_5\psi = -\psi \tag{2.15}$$

similarly right-handed particles are those described by spinors χ with eigenvalue $+1$:

$$\gamma_5\chi = \chi \tag{2.16}$$

One can create left-handed and right-handed particles by applying the projection operators on spinors:

$$\psi_L \equiv P_L\psi \equiv \frac{1}{2}(1-\gamma_5)\psi, \text{ and } \psi_R \equiv P_R\psi \equiv \frac{1}{2}(1+\gamma_5)\psi \tag{2.17}$$

In the Standard Model fermions are either left-handed or right-handed, only the left-handed fermions carry non-zero isospin, with values $I_3 = +\frac{1}{2}$ or $I_3 = -\frac{1}{2}$. Aside from the fermions, the Higgs field components are assigned isospin $I_3 = \pm\frac{1}{2}$ and the W-bosons are assigned isospin ±1 and 0. The $SU(2)_L$ symmetry acts on left-handed fermions, the W-bosons and the Higgs boson, right-handed fermions are left untransformed. As the fundamental representation of $SU(2)_L$ consists of 2-dimensional matrices, the $SU(2)_L$ symmetry acts on two-vectors of particles, these two-vectors are called *doublets*. Following the same terminology, we call the right-handed particles *singlets* under $SU(2)_L$, as they do not transform.

The $SU(2)$ doublets have two components, a “up” and a “down” component. The left-handed up components, $\psi_{u,L}$, of the $SU(2)_L$ components have values $I_3 = +\frac{1}{2}$, the left-handed down components, $\psi_{d,L}$, carry values $I_3 = -\frac{1}{2}$. We then write the left-handed doublets out like:

$$\Psi_L = \begin{pmatrix} \psi_{u,L} \\ \psi_{d,L} \end{pmatrix} \tag{2.18}$$

Gauge boson	Name	Symmetry	Number of Gauge bosons
B_μ	B-boson	$U(1)_Y$	1
W^i_μ	W-boson	$SU(2)_L$	3
G^i_μ	Gluon	$SU(3)_c$	8

Table 2.1: *A table with containing the gauge bosons of the Standard Model before electroweak symmetry breaking. They are spin-*1 *particles which transform in the adjoint representation of their corrresponding symmetry group.*

Because of the different transformation properties of left-handed and right-handed fermions, we call the Standard Model a chiral theory.

Similarly to the $SU(2)_L$ symmetry, the $SU(3)_c$ symmetry works on particles ordered in *triplets*. Only particles with a property called *colour* transform under $SU(3)$. Particles can be either red, blue or green, the particles are ordered in triplets Ψ_c as follows:

$$\Psi_c = \begin{pmatrix} \psi_r \\ \psi_b \\ \psi_g \end{pmatrix} \tag{2.19}$$

2.2.2 Particle content of the Standard Model

Having seen the symmetries of the Standard Model, it is time to show which particles exist in the Standard Model and how they are affected by the symmetry transformations. To do this, we begin by making a distinction between the gauge bosons and the other particles, which we shall call the matter particles. The main reason we do this, is because the two live in different representations of the symmetries and because the vector particles are bosons, while the far majority of matter particles are fermions.

As mentioned, the gauge bosons are linked to the symmetry groups, each corresponding to a specific generator. The gauge bosons are all spin-1 particles and as shown in equation [2.10] they transform according to the adjoint representation of the symmetry they belong to as listed in table [2.1] with an extra derivative term. There are $N^2 - 1$ generators and associated for $SU(N)$ gauge symmetries. Next, the matter particles of the Standard Model consist of fermions and one pair of bosons. The fermions are found to appear in three *generations* or *families* of particles, the first family consisting

Names	Particle	Families	Spin	$SU(3)$	$SU(2)$	I_3	Y	Q
Quarks	u_L	3	$\frac{1}{2}$	3	2	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$
	d_L	3	$\frac{1}{2}$	3	2	$-\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}$
	u_R	3	$\frac{1}{2}$	3	1	0	$\frac{4}{3}$	$\frac{2}{3}$
	d_R	3	$\frac{1}{2}$	3	1	0	$-\frac{2}{3}$	$-\frac{1}{3}$
Neutrinos	ν_L	3	$\frac{1}{2}$	1	2	$\frac{1}{2}$	-1	0
Charged leptons	e_L	3	$\frac{1}{2}$	1	2	$-\frac{1}{2}$	-1	-1
	e_R	3	$\frac{1}{2}$	1	1	0	-2	-1
Higgs boson	H^+	1	0	1	2	$\frac{1}{2}$	1	1
	H^0	1	0	1	2	$-\frac{1}{2}$	1	0

Table 2.2: *The matter particles in the Standard Model. The numbers $1, 2, 3$ in the $SU(2)$ and $SU(3)$ column indicate whether the particles transform respectively if the particle does not transform under the symmetry, transforms under the symmetry as part of a doublet or as a triplet.*

for example of the up quark, the down quark, the electron and the electron neutrino. Under symmetry transformations, the particles in each of the three families transform in the same way.

All matter particles carry hypercharge, and transform under the $U(1)_Y$ phase transformation depending on their Y charges as collected in table [2.2] along with their other transformation properties. Those particles which transform under $SU(2)$ as singlets are shown with a 1 in the corresponding column, the particles transforming as doublets under $SU(2)$ are shown with a 2, their positions in the doublets depending on their values of I_3.

The only fermion particles carrying colour, transforming as triplets under $SU(3)_c$, are the quarks. These particles transforming as triplets are listed with a 3 in table [2.2], where the triplets have components as shown in equation [2.19].

2.3 Spontaneous Symmetry Breaking

The $U(1)_Q$ symmetry of QED which depends on the electric charges Q of the fields, is no longer considered a fundamental symmetry in the Standard

Model, it is a symmetry which arises after spontaneous symmetry breaking of the $SU(2)_L \times U(1)_Y$ symmetry.

Spontaneous symmetry breaking in the standard model is connected to the problem of massive fields. Gauge invariance and the chiral nature of the theory forbids mass terms like

$$m^2 A_\mu A^\mu \text{ and } m\bar{\psi}\psi = m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L) \tag{2.20}$$

Masses in the Standard Model are therefore generated by the Brout-Engler-Higgs (BEH) mechanism. The scalar field responsible for the spontaneous symmetry breaking is the Higgs doublet, denoted by H, whose behaviour is governed by the potential for this field in the Lagrangian, which is given by:

$$V_\phi = \int d^4x \, [\lambda|H|^4 - \mu^2|H|^2] \tag{2.21}$$

The minimum of this potential is found for non-zero field values. To find it, we first perform a $SU(2)_L$ gauge transformation to eliminate one of the doublet fields:

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ H^0 \end{pmatrix} \tag{2.22}$$

The minimum value of the potential is reached for $|H^0|^2 = \frac{\mu^2}{2\lambda}$, we therefore say the field H^0 has acquired a vacuum expectation value. By using a $U(1)_Y$ transformation, we can choose H^0 to be real. Whereas the original potential was invariant under $SU(2)_L$ and $U(1)_Y$ gauge transformations, the resulting field configuration which minimises the potential, the vacuum, is not. We say the $SU(2) \times U(1)$ symmetry is broken, the $U(1)_Q$ is however a symmetry of the vacuum. After this symmetry breaking, known as *electroweak symmetry breaking*, the electric charges for the $U(1)_Q$ symmetry are determined by the isospin and the hypercharge according to the Gell-Mann-Nishijima relation [2.14].

To conclude this short explanation on the BEH mechanism, we can expand H^0 around the minimum solution we then have:

$$H^0 = \sqrt{\frac{\mu^2}{2\lambda}} + h \tag{2.23}$$

where we have performed a $U(1)$ gauge transformation to make H^0 real. The real field h, the perturbation around the vacuum, lives on in the theory as the Higgs particle.

2.4 Mass Generation

Now that we have seen that the Higgs doublet gets a vacuum expectation value, we can look at how masses are generated in the Standard Model. For a single scalar field, or scalar singlet, ϕ, the way to generate mass terms for a particle ψ would be to include a term

$$\phi\bar{\psi}\psi \tag{2.24}$$

in the Lagrangian, this leads to a mass proportional to $\langle\phi\rangle$. In the Standard Model we create these couplings, known as *Yukawa couplings*, keeping in mind the doublet structure of the Higgs fields and the left-handed fermions. To understand the mass terms in the Standard Model we first introduce the vector notation:

$$Q_L^i = \begin{pmatrix} u_L \\ d_L \end{pmatrix},\ L_L^i = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix},\ H^i = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \tag{2.25}$$

The mass terms we add have to be invariant under both $SU(2)_L$ and $U(1)_Y$ symmetry transformations and $U(1)_Q$ symmetries. The $U(1)_Q$ and $U(1)_Y$ charges are such that these terms are invariant under the two $U(1)$ transformations.

The mass terms in the Standard Model are generated by Yukawa couplings between the Higgs fields and the fermions. To give masses to the components in the lower part of the $SU(2)$ doublets, we introduce the following mass terms:

$$\mathcal{L}_{Yukawa,down} = \frac{i}{2}Y^d\bar{H}\bar{d}_R Q_L + \frac{i}{2}Y^e\bar{H}\bar{e}_R L_L + h.c. \tag{2.26}$$

where $\bar{H}$ denotes the Hermitian conjugate of H. In the mass terms above the objects Y are matrices which mix the different generations, meaning that implicitly one part of the mass term is for example:

$$\frac{i}{2}Y^d\bar{H}\bar{d}_R Q_L \equiv \frac{i}{2}Y^d_{ab}\bar{H}\bar{d}_{R,a}Q_{L,b} \tag{2.27}$$

where the indices a, b indicate to which generation the fields belong. This expression is invariant under the Standard Model symmetries, in particular it is $SU(2)$ invariant by contraction of the left-handed spinor with the conjugated Higgs field, the last of which transforms in the conjugate representation, the first of which transforms in the fundamental representation. To give masses to the upper part of the doublets with the same Higgs field, we have to create a $SU(2)$ invariant expression in a different way, we do this by contraction with the Levi-Civita symbol. This leads to the mass terms:

$$\mathcal{L}_{Yukawa,up} = \frac{i}{2}\epsilon_{ij}Y^u H^i \bar{u}_R Q_L^j + h.c. \tag{2.28}$$

To check that this expression is invariant under $SU(2)$ transformations one uses that the elements of this group have a unit determinant. One important thing to note is that *only one Higgs doublet* is used to generate both the mass terms for the up-type and the down-type particles, using both H and $\bar{H}$. We will later see that in supersymmetric extensions of the Standard Model one Higgs doublet does no longer suffice to generate all masses.

Now that we have seen how the fermions acquire their masses, we move on to show how gauge bosons acquire masses. Using the covariant derivatives, we can see that the mass terms for the gauge bosons come from the covariant derivative acting on the Higgs doublet:

$$\nabla_\mu H = (\partial_\mu - ig_B Y_H \mathbb{I}_{2\times 2} B_\mu - ig_W(\frac{\sigma^i}{2})W_\mu^i)H \tag{2.29}$$

where Y_H is the Y charge of the upper and lower component of the Higgs doublet. When the Higgs doublets now get a vacuum expectation value, these covariant derivatives turn into mass terms for the gauge fields.

2.5 Standard Model Lagrangian

Now that we have seen the technical details contained in the Standard Model, we can write down the Standard Model Lagrangian:

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}^2(G) - \frac{1}{4}F_{\mu\nu}^2(W) - \frac{1}{4}F_{\mu\nu}^2(B)\\ &- |\nabla_\mu H|^2 - \lambda|H|^4 + \mu^2|H|^2 + \sum_{\text{fermions}} \frac{i}{2}\bar{\psi}\not{\nabla}\psi\\ &+ (\frac{i}{2}\epsilon_{ij}Y^u H^i \bar{u}_R Q_L^j + \frac{i}{2}Y^d \bar{H}\bar{d}_R Q_L + \frac{i}{2}Y^e \bar{H}\bar{e}_R L_L + h.c.)\end{aligned} \tag{2.30}$$

This Lagrangian can be used to derive the Feynman rules for the Standard Model.

2.6 Anomalies

Anomalies are problems linked to divergences in chiral theories which destroy some important properties that quantum field theories should satisfy.

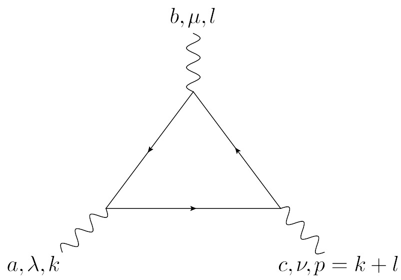

Figure 2.1: *A triangle diagram representing a contribution to the three-boson vertex. This triangle diagram leads to a divergence which cannot be regularised properly.*

We will not go into the details of anomalies here, we will just shortly state how anomalies can be avoided. More information can be found in [9] and [10]. The problem of anomalies can be found in the contributions to three-boson vertices through triangle diagrams as shown in figure [2.1].

This diagram contains a divergence which has to be regularised, we will denote the contribution of the diagram by $T^{\lambda\mu\nu}$. If the regularisation is done properly, one expects the Ward identities to hold:

$$k_\lambda T^{\lambda\mu\nu} = l_\mu T^{\lambda\mu\nu} = p_\nu T^{\lambda\mu\nu} = (k+l)_\nu T^{\lambda\mu\nu} = 0 \tag{2.31}$$

One can however not find a regularisation procedure which maintains the three Ward identities simultaneously when the diagram contains a term with an odd number number of γ_5 matrices. In a chiral gauge theory, the boson to fermion vertices in the Feynman diagrams contribute factors proportional to $t^a\gamma^\mu(1 \pm \gamma_5)$ where the t^a depends on the fermion in the loop. The projection operators from the vertices lead to a $\pm\gamma_5$ term, depending on the handedness of the fermion. The contribution of the term containing this $\pm\gamma_5$ term can be calculated and is proportional to:

$$\mathrm{Tr}[t^a\{t^b, t^c\}] \tag{2.32}$$

When one takes into account all fermions in the loop, the contributions to the amplitude of all terms containing γ_5 is proportional to:

$$\sum_{\text{L-fermions}} \mathrm{Tr}[t^a\{t^b, t^c\}] - \sum_{\text{R-fermions}} \mathrm{Tr}[t^a\{t^b, t^c\}] \tag{2.33}$$

This leads to a solution to the problem of the troublesome contribution: its contribution can be zero if the sums of the generators cancel each other. This is what happens in the Standard Model to solve the anomalous contribution, for this reason the Standard Model is said to be *anomaly free*.

To see how this cancellation works we will look at some examples. First consider the case of three $SU(2)$ gauge bosons interacting with each other through a triangle diagram. The $SU(2)$ bosons couple only to left-handed fermions, the $SU(2)$ generators are the Pauli matrices σ^i up to a multiplicative factor, which satisfy the anti-commutation relation:

$$\{t^i, t^j\} = \frac{1}{2}\delta^{ij} \tag{2.34}$$

The amplitude of three $SU(2)$ gauge bosons interacting through a triangle diagram is therefore proportional to:

$$\sum_{\text{L-fermions}} \text{Tr}[t^a\{t^b, t^c\}] = \sum_{\text{L-fermions}} \frac{1}{2}\text{Tr}[t^a]\delta^{bc} = 0 \tag{2.35}$$

The contribution to the triangle diagram vanishes as the Pauli matrices are traceless. Furthermore, as the $SU(2)$ bosons couple only to left-handed fermions there is no contribution due to right-handed fermions.

Another way the anomaly might show up is when the $U(1)_Y$ gauge boson interacts with two $SU(2)$ gauge bosons through the triangle diagram. Only the left-handed fermions couple to the $SU(2)$ gauge bosons with the $U(1)_Y$ generators given by $\frac{Y}{2}$. The contribution of the left-handed fermions becomes:

$$\sum_{\text{L-fermions}} \text{Tr}[t^a\{t^b, t^c\}] = \sum_{\text{L-fermions}} \frac{1}{2}\text{Tr}[\frac{Y}{2}]\delta^{bc} \tag{2.36}$$

where we have taken t^b and t^c to be the generators for the $SU(2)$ symmetry. If the two $SU(2)$ bosons are not identical, this contribution will vanish, otherwise it will be proportional to:

$$\sum_{\text{L-fermions}} Y = 3 \cdot 3 \cdot (\frac{1}{3} + \frac{1}{3}) + 3 \cdot (-1 - 1) = 0 \tag{2.37}$$

where both the number of families and the colour multiplicity have been taken into account.

Chapter 3

Extending the Standard Model

Right-handed neutrinos and neutrino masses are lacking in the minimal Standard Model. Multiple solutions have been proposed solve this, we are now going to look at one proposed model [5].

3.1 A new symmetry

Construction of the model starts by introducing right-handed neutrinos. As there are no observed interactions between right-handed neutrinos and Standard Model particles, they are assigned zero $U(1)_Y$ and $U(1)_{EM}$ charges and they are taken to be singlets under $SU(2)$ and $SU(3)$.

With the addition of right-handed neutrinos we can add an extra $U(1)$ symmetry to the Standard Model acting on right-handed particles, which we name the *R-symmetry*. We begin by assigning R-charges to the right-handed particles as shown in table [3.1].

Particles	R-charge
u_R	1
d_R	-1
ν_R	1
e_R	-1

Table 3.1: *The R-charges assigned to the right-handed Standard Model particles. These charges can be used to create new anomaly free $U(1)$ symmetries.*

This new $U(1)_R$ symmetry is anomaly free. However, it turns out we can construct a more general anomaly free $U(1)$ symmetry, which we call the $U(1)_X$ symmetry, with the following charges for fermions:

$$X = \alpha Y + \beta R \tag{3.1}$$

where α and β are yet unspecified constants. As the anomalous contribution of the triangle diagrams involving $U(1)_X$ bosons vanishes independent of the choice for α and β, the symmetry can be gauged.

Gauging the symmetry

When the $U(1)_X$ symmetry is gauged, we have to add an extra gauge boson to the Standard Model. We will denote this gauge boson by C_μ. This new gauge boson cannot be massless, as that would lead to an extra infinite range force which has not been observed. The solution is to make the gauge boson very massive, leading to a short ranged force.

To give mass to the new gauge boson in this model, a new complex scalar field ϕ is introduced. The scalar field is taken to be a singlet under $SU(2)$ and $SU(3)$, it can carry hypercharge as well as X-charge. The scalar field is assigned hypercharge η and a unit X-charge. The original Higgs doublet is assigned X-charge ξ. The scalar field ϕ is given a vacuum expectation value along with the original Higgs doublet H by the modified potential:

$$V = \frac{\lambda_1}{4}(|H|^2 - v_1^2)^2 + \frac{\lambda_2}{4}(|\phi|^2 - v_2^2)^2 + \frac{\lambda_m}{4}(|\phi|^2 - v_2^2)(|H|^2 - v_1^2) \tag{3.2}$$

With the addition of the right-handed neutrinos and the scalar field ϕ the matter content of the Standard Model and its set of $U(1)$ charges is changed, as shown in table [3.2]. If the $U(1)_X$ transformations are to be a symmetry of the Standard Model, we will have to check if the original terms in the Lagrangian are gauge invariant. To check gauge invariance we look at the total X-charges of the Yukawa couplings as listed in table [3.3]. From this we find that to implement the new $U(1)_X$ symmetry the X-charge of the Higgs doublet has to be $\xi = \alpha + \beta$.

Particle	Y-charge	X-charge	Q-charge
u_L	$\frac{1}{3}$	$\frac{\alpha}{3}$	$\frac{2}{3}$
d_L	$\frac{1}{3}$	$\frac{\alpha}{3}$	$-\frac{1}{3}$
u_R	$\frac{4}{3}$	$\frac{4\alpha}{3}+\beta$	$\frac{2}{3}$
d_R	$-\frac{2}{3}$	$-\frac{2\alpha}{3}-\beta$	$-\frac{1}{3}$
ν_L	-1	$-\alpha$	0
ν_R	0	β	0
e_L	-1	$-\alpha$	-1
e_R	-2	$-2\alpha-\beta$	-1
H^+	1	ξ	1
H^0	1	ξ	0
ϕ	η	1	$\frac{\eta}{2}$

Table 3.2: *The Standard Model matter particles with their associated $U(1)$ charges. The X-charges connected to the new $U(1)_X$ symmetry are given by equation [3.1].*

Yukawa coupling term	Total X-charge
$\epsilon_{ij}Y^u H^i \bar{u}_R Q_L^j$	$\xi-\alpha-\beta$
$Y^d \bar{H}\bar{d}_R Q_L$	$-\xi+\alpha+\beta$
$Y^e \bar{H}\bar{e}_R L_L$	$-\xi+\alpha+\beta$

Table 3.3: *The Yukawa coupling terms of the Standard Model with their overal X-charge. The overal X-charge has to be zero in order for the Yukawa coupling to be gauge-invariant.*

3.2 Giving mass to neutrinos

With the new scalar field ϕ and right-handed neutrinos we can add new terms to the Lagrangian. First of all Yukawa couplings to the right-handed neutrinos are introduced:

$$\frac{i}{2}\epsilon_{ij}Y^\nu H^i \bar{\nu}_R L_L^j + h.c. \tag{3.3}$$

Particle	Y-charge	X-charge	Q-charge
u_L	$\frac{1}{3}$	$\frac{\alpha}{3}$	$\frac{2}{3}$
d_L	$\frac{1}{3}$	$\frac{\alpha}{3}$	$-\frac{1}{3}$
u_R	$\frac{4}{3}$	$\frac{4\alpha}{3}+\frac{1}{2}$	$\frac{2}{3}$
d_R	$-\frac{2}{3}$	$-\frac{2\alpha}{3}-\frac{1}{2}$	$-\frac{1}{3}$
ν_L	-1	$-\alpha$	0
ν_R	0	$\frac{1}{2}$	0
e_L	-1	$-\alpha$	-1
e_R	-2	$-2\alpha-\frac{1}{2}$	-1
H^+	1	$\alpha+\frac{1}{2}$	1
H^0	1	$\alpha+\frac{1}{2}$	0
ϕ	0	1	0

Table 3.4: *Matter content of the Standard Model with their $U(1)$ charges, if the Majorana mass terms of equation [3.4] are to be included.*

The only other possible gauge-invariant term is the Majorana mass term:

$$\frac{i}{2}(\kappa^{\nu}_{ij}\phi\bar{\nu}_{R,i}C\bar{\nu}^T_{R,j} - \bar{\kappa}^{\nu}_{ij}\bar{\phi}\nu^T_{R,i}C\nu_{R,j}) \tag{3.4}$$

where C is the charge conjugation matrix on which more information can be found in appendix [A.1] along with other information on Majorana spinors. The object κ^{ν} is a matrix mixing the right-handed neutrinos of different families. After spontaneous symmetry breaking of the $U(1)_X$ symmetry, this term will lead to a *Majorana mass* for neutrinos.

The total X-charge of this Majorana mass term is $2\beta - 1$, to allow this combination in the Lagrangian we therefore have to pick $\beta = \frac{1}{2}$. The total Y-charge of this term is η, we are therefore lead to $\eta = 0$. With β and η fixed, there is still some freedom in determining the X-charges by choosing α, as listed in table [3.4].

3.3 Gauge boson masses

If we want to introduce the Majorana mass terms for right-handed neutrinos, we are led to the choice of $\eta = 0$. This choice slightly alters the calculation of the gauge boson masses as presented in [5]. The potential [3.2] leads to the vacuum expectation values for H and ϕ. Using the $SU(2)$ gauge transformation and the two $U(1)$ symmetries we can choose the vacuum expectation values to be:

$$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}, \ \langle \phi \rangle = v_\phi \tag{3.5}$$

where v and v_ϕ can simultaneously be chosen real as ϕ carries zero Y-charge. The gauge boson masses are found by writing out the terms:

$$\begin{aligned} &|\nabla H|^2 + |\nabla \phi|^2 \\ &= \frac{1}{4} g_W^2 v^2 |W_{1,\mu} - iW_{2,\mu}|^2 + \frac{1}{4} v^2 (g_B B_\mu - g_W W_{3,\mu} + g_C \delta C_\mu)^2 + \frac{1}{4} v_\phi^2 g_C^2 (C\mu)^2 \\ &= \frac{1}{2} g_W^2 v^2 W^+ \cdot W^- + \frac{1}{2} m_Z^2 Z_\mu^2 + \frac{1}{2} m_{Z'}^2 (Z'_\mu)^2 \end{aligned} \tag{3.6}$$

where we have defined:

$$\delta \equiv \alpha + \frac{1}{2} \tag{3.7}$$

This leads to the charged bosons:

$$W_\mu^\pm = \frac{W_{1,\mu} \mp iW_{2,\mu}}{\sqrt{2}} \tag{3.8}$$

with masses

$$m_{W^\pm}^2 = \frac{1}{2} g_W^2 v^2 \tag{3.9}$$

and the neutral bosons:

$$Z_\mu = \frac{g_B A_1 B_\mu - g_W A_1 W_{3,\mu} + g_C \delta A_2 C_\mu}{\sqrt{(g_B^2 + g_W^2) A_1^2 + g_C^2 \delta^2 A_2^2}} \tag{3.10}$$

and

$$Z'_\mu = \frac{g_B A_3 B_\mu - g_W A_3 W_{3,\mu} + g_C \delta A_4 C_\mu}{\sqrt{(g_B^2 + g_W^2) A_3^2 + g_C^2 \delta^2 A_4^2}} \tag{3.11}$$

Where we have defined the coefficients

$$\begin{cases} A_1 = (\gamma v^2 - g_C^2 v_\phi^2 - \sqrt{(\gamma v^2 + v_\phi^2 g_C^2)^2 - 4 g_C^2 v^2 v_\phi^2 (g_B^2 + g_W^2)} \\ A_2 = (\gamma v^2 + g_C^2 v_\phi^2 - \sqrt{(\gamma v^2 + v_\phi^2 g_C^2)^2 - 4 g_C^2 v^2 v_\phi^2 (g_B^2 + g_W^2)} \\ A_3 = (\gamma v^2 - g_C^2 v_\phi^2 + \sqrt{(\gamma v^2 + v_\phi^2 g_C^2)^2 - 4 g_C^2 v^2 v_\phi^2 (g_B^2 + g_W^2)} \\ A_4 = (\gamma v^2 + g_C^2 v_\phi^2 + \sqrt{(\gamma v^2 + v_\phi^2 g_C^2)^2 - 4 g_C^2 v^2 v_\phi^2 (g_B^2 + g_W^2)} \end{cases} \tag{3.12}$$

with

$$\gamma \equiv g_W^2 + g_B^2 + g_C^2\delta^2 \tag{3.13}$$

The masses for the neutral bosons are given by:

$$\begin{cases} m_Z^2 = \frac{1}{4}(\gamma v^2 + g_C^2 v_\phi^2 - \sqrt{(\gamma v^2 + v_\phi^2 g_C^2)^2 - 4 g_C^2 v^2 v_\phi^2 (g_B^2 + g_W^2)} \\ m_{Z'}^2 = \frac{1}{4}(\gamma v^2 + g_C^2 v_\phi^2 + \sqrt{(\gamma v^2 + v_\phi^2 g_C^2)^2 - 4 g_C^2 v^2 v_\phi^2 (g_B^2 + g_W^2)} \end{cases} \tag{3.14}$$

The massless photon is given by the combination:

$$A_\mu = \frac{g_W B_\mu + g_B W_{3,\mu}}{\sqrt{g_B^2 + g_W^2}} \tag{3.15}$$

3.4 Seesaw mechanism

Let us now look at the mass eigenstates of the neutrinos when the Majorana mass terms are included. The neutrino mass eigenstates are determined by the following part of the Lagrangian:

$$\frac{i}{2}\epsilon_{ij} Y^\nu H^i \bar{\nu}_R L_L^j + \frac{i}{2}(\kappa^\nu \phi \bar{\nu}_R C \bar{\nu}_R^T - \bar{\kappa}^\nu \bar{\phi} \nu_R^T C \nu_R) + (\frac{i}{2}\epsilon_{ij} Y^\nu H^i \bar{\nu}_R L_L^j)^\dagger \tag{3.16}$$

We will make the calculation of the mass eigenstates easier by assuming the matrices Y^ν and κ^ν appearing are real and diagonal. Suppose that for one neutrino family the diagonal components of these matrices are y^ν and k^ν. After spontaneous symmetry breaking the fields H and ϕ obtain vacuum expectation values, the terms contributing to the neutrino masses are then:

$$-\frac{i}{2} y^\nu v \bar{\nu}_R \nu_L + \frac{i}{2} k^\nu v_\phi (\bar{\nu}_R C \bar{\nu}_R^T - \nu_R^T C \nu_R) - \frac{i}{2} y^\nu v \bar{\nu}_L \nu_R \tag{3.17}$$

Next we define

$$m \equiv -y^\nu v,\ M = k^\nu v_\phi \tag{3.18}$$

and we introduce the Majorana spinors:

$$\alpha \equiv \nu_L + (\nu_L)^c,\ \beta \equiv (\nu_R)^c + \nu_R \tag{3.19}$$

Using these definitions we can rewrite equation [3.17] to:

$$\frac{im}{2}(\bar{\alpha}\beta + \bar{\beta}\alpha) + \frac{iM}{2}\bar{\beta}\beta \tag{3.20}$$

If we take $M >> m$ the mass eigenstates are approximated by α and β with masses:

$$m_\alpha \approx \frac{m^2}{M} \tag{3.21}$$

$$m_\beta \approx M \tag{3.22}$$

One can see that the masses depend on M in the opposite ways, one increases with M whereas the other decreases with M. This mechanism is known as the *Seesaw mechanism*. If M is indeed taken to be very large, this mechanism explains why right-handed neutrinos are hardly seen due to their high masses, and simultaneously it explains why left-handed neutrinos appear to be almost massless.

Chapter 4

Supersymmetry

In this chapter we start by deriving some properties of supersymmetry using Hilbert space language after which we will see how supersymmetry is implemented in gauge field theories. We end this chapter with a short discussion of the Minimal Supersymmetric Standard Model. Even though we will look at several aspects of supersymmetry, many other aspects are left out. Two good starting points for learning more about supersymmetry are [4] and [11].

4.1 Supersymmetry in Hilbert space

Supersymmetry is generated in Hilbert space by the fermionic generators Q_α and $Q^\dagger_\alpha$. The operators respectively turn bosonic states, denoted by $|B\rangle$, into fermionic states, denoted by $|F\rangle$, and vice versa:

$$\begin{aligned} Q_\alpha|B\rangle &= |F\rangle \\ Q^\dagger_\alpha|F\rangle &= |B\rangle \end{aligned} \tag{4.1}$$

The set of bosons and fermions which transform into each other under supersymmetry transformations is called a *multiplet*. The anti-commutation and commutation relations the fermionic operators Q and $Q^\dagger$ satisfy are limited by the Haag–Lopuszanski–Sohnius theorem [12]. The operators satisfy the anti-commutation relations:

$$\{Q_\alpha, Q_\beta\} = \{Q^\dagger_\alpha, Q^\dagger_\beta\} = 0 \tag{4.2}$$

$$\{Q_\alpha, Q^\dagger_\beta\} = \frac{1}{2}(\gamma_\mu\gamma^0)_{\alpha\beta}P^\mu \tag{4.3}$$

The commutation relations with the generators of Lorentz transformation $M_{[\mu\nu]}$ and four-momentum P^μ are given by:

$$[M_{[\mu\nu]}, Q_\alpha] = -\frac{1}{2} i (\gamma_{\mu\nu})_{\alpha\beta} Q_\beta \tag{4.4}$$
$$[P^\mu, Q_\alpha] = 0 \tag{4.5}$$

where $\gamma_{\mu\nu} \equiv \frac{1}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$. These (anti-)commutation relations can be used to derive some important properties of supersymmetric models.

As the supersymmetry generators do not commute with the Lorentz transformation generators and the anti-commutator of Q and $Q^\dagger$ is the translation operator P^μ, supersymmetry is said to be a spacetime symmetry and therefore commutes with internal symmetries. Particles in the same multiplet therefore have the same transformation properties under internal symmetries. From equation [4.5] follows that particles in the same multiplets have equal masses. Realistic supersymmetric theories therefore need broken supersymmetry, as no bosons and fermions with equal masses are known.

Next, taking the trace of equation [4.3] over all spinor indices leads to [13]:

$$P^0 = \frac{1}{2} \mathrm{Tr}[QQ^\dagger + Q^\dagger Q] \tag{4.6}$$

If we define a supersymmetric ground state $|0\rangle$ as one that is annihilated by all Q_α and $Q^\dagger_\alpha$, the energy in the supersymmetric ground state is zero:

$$\langle 0|P^0|0\rangle = \frac{1}{2}\langle 0|\mathrm{Tr}[QQ^\dagger + Q^\dagger Q]|0\rangle = 0 \tag{4.7}$$

As P^0 is proportional to the trace of a squared operator we have for general ground states:

$$\frac{1}{2}\langle 0|\mathrm{Tr}[QQ^\dagger + Q^\dagger Q]|0\rangle = \frac{1}{2}\langle 0|\mathrm{Tr}[(Q + Q^\dagger)^2]|0\rangle \geq 0 \tag{4.8}$$

from which follows that when supersymmetry is broken the ground state energy is positive.

Another useful result we will not derive here (see for example reference [4]), is that the number of fermionic degrees of freedom in a multiplet has to be equal to the number of bosonic degrees of freedom.

4.2 A simple supersymmetric field theory

To implement supersymmetry on fields we will first look at the basic example of a supersymmetric action for the *chiral multiplet*. The chiral multiplet contains three fields: a left-handed projection of a Majorana spinor field ψ_L, a complex scalar field ϕ and an *auxiliary field* F. The action for the chiral multiplet is given by:

$$S = \int d^4x[\, -\partial\bar{\phi}\cdot\partial\phi + i\bar{\psi}_L\gamma\cdot\partial\psi_L + \bar{F}F] \tag{4.9}$$

This action is invariant up to boundary terms under the following supersymmetry transformations:

$$\begin{aligned} \delta\phi &= -i\sqrt{2}\bar{\epsilon}_R\psi_L \\ \delta\psi_L &= \sqrt{2}(\gamma\cdot\phi\epsilon_R + F\epsilon_L) \\ \delta F &= -i\sqrt{2}\bar{\epsilon}_L\gamma\cdot\partial\psi_L \end{aligned} \tag{4.10}$$

In these transformation ϵ is a constant Majorana spinor, which acts as a parameter of the transformation. The supersymmetry transformations satisfy the following commutator algebra:

$$[\delta(\epsilon_1), \delta(\epsilon_2)]X = 2i\bar{\epsilon}_1\gamma^\mu\epsilon_2\partial_\mu X \tag{4.11}$$

where X represents any of the fields in the chiral multiplet. We therefore see that the commutator of two supersymmetry transformations leads to a translation on the fields. This is linked to the statement that supersymmetry is a spacetime symmetry, as was concluded from equation [4.3]. The auxiliary field has been included to close the commutator algebra when the fields are off-shell. The degrees of freedom of the auxiliary field are furthermore needed to make sure the number of fermionic degrees of freedom equals the number of bosonic degrees of freedom when the fields are off-shell.

The action contains no kinetic terms for the auxiliary fields F and $\bar{F}$, they can therefore be *eliminated* from the action using their equations of motion. These are given by:

$$F = \bar{F} = 0 \tag{4.12}$$

leading to the action:

$$S = \int d^4x[\, -\partial\bar{\phi}\cdot\partial\phi + i\bar{\psi}_L\gamma\cdot\partial\psi_L] \tag{4.13}$$

To create supersymmetric field theories with interactions, we can add to the action given by equation [4.9] another action:

$$S = \int d^4x[F\frac{\partial W(\phi)}{\partial \phi} + \bar{F}\frac{\partial \bar{W}(\bar{\phi})}{\partial \bar{\phi}} + \frac{i}{2}\frac{\partial^2 W(\phi)}{\partial^2 \phi}\bar{\psi}_R\psi_L + \frac{i}{2}\frac{\partial^2 \bar{W}(\bar{\phi})}{\partial^2 \bar{\phi}}\bar{\psi}_L\psi_R] \tag{4.14}$$

where the term $W(\phi)$ appearing in the action is called the *superpotential*. For this action to be invariant under the supersymmetry transformations the superpotential must be a holomorphic function of the scalar field ϕ, that is to say, it can only depend on ϕ and not on $\bar{\phi}$. This action is separately invariant under supersymmetry transformations, a more generalised version of this action can be used to create interactions between different chiral multiplets by making the superpotential a holomorphic function of the scalar fields appearing in the different chiral multiplets.

If we add the two separately supersymmetry invariant actions [4.9] and [4.14], the equations of motions of the auxiliary fields become:

$$\begin{aligned} F &= -\frac{\partial \bar{W}(\bar{\phi})}{\partial \bar{\phi}} \\ \bar{F} &= -\frac{\partial W(\phi)}{\partial \phi} \end{aligned} \tag{4.15}$$

This leads to the action:

$$\begin{aligned} S = \int d^4x[&-\partial\bar{\phi}\cdot\partial\phi + i\bar{\psi}_L\gamma\cdot\partial\psi_L \\ &- |\frac{\partial W(\phi)}{\partial\phi}|^2 + \frac{i}{2}\frac{\partial^2 W(\phi)}{\partial^2\phi}\bar{\psi}_R\psi_L + \frac{i}{2}\frac{\partial^2 \bar{W}(\bar{\phi})}{\partial^2\bar{\phi}}\bar{\psi}_L\psi_R] \end{aligned} \tag{4.16}$$

From this we can see the potential is given by:

$$V_F = |\frac{\partial W(\phi)}{\partial \phi}|^2 \tag{4.17}$$

This potential is either zero or positive, as shown in the previous section. Moreover we can see from this potential that to create a renormalizable theory, the superpotential must contain terms consisting of products of up to three fields.

The actions we have seen above are not yet suitable for gauge theories. Therefore, to be able to discuss more realistic quantum field theories we will have to introduce some more machinery. In appendix [A.3.2] the construction of supersymmetry multiplets and supersymmetric actions is briefly explained.

4.3 Supersymmetric gauge theories

To be able to create supersymmetric gauge theories we introduce the *vector multiplet*. The vector multiplet contains a gauge field A^a_μ, a Majorana spinor field λ^a and an auxiliary field D^a. We denote the vector multiplet by $(A^a_\mu, \lambda^a, D^a)$. The indices a indicate the different gauge bosons involved for the specific gauge theory. The action for the supersymmetric vector multiplet is given by:

$$S = \int d^4x(-\frac{1}{4}F^{a,\mu\nu}F^a_{\mu\nu} + \frac{i}{2}\bar{\lambda}^a\gamma \cdot \nabla\lambda^a + \frac{1}{2}D^aD^a) \tag{4.18}$$

As we have seen before with the chiral multiplet, the auxiliary field D^a is non-propagating and can be eliminated using its equation of motion. This leads to an extra contribution to the potential.

The action is invariant under the supersymmetry transformations:

$$\begin{aligned} \delta A^a_\mu &= -i\bar{\epsilon}\gamma_\mu\lambda^a \\ \delta\lambda^a &= -F_{\mu\nu}\gamma^{\mu\nu}\epsilon + iD^a\gamma_5\epsilon \\ \delta D^a &= \bar{\epsilon}\gamma_5\gamma \cdot \nabla\lambda^a \end{aligned} \tag{4.19}$$

Under gauge transformations fields λ^a and D^a transform in the adjoint representation without the additional derivative term involved in gauge field transformations. The field λ^a for example transforms as:

$$\lambda^a \to \lambda^a + f^{abc}\lambda^b\alpha^c \tag{4.20}$$

where α^i is a transformation parameter.

Next we can look at the supersymmetric action for a set of chiral multiplets with fields $(\phi_i, \psi_{L,i}, F_i)$ coupled to gauge fields through covariant derivatives. The $\psi_{L,i}$ are left-handed projections of Majorana spinor fields ψ_i. If the fields transform under gauge transformations with generators t^a, the supersymmetric action is given by:

$$\begin{aligned} S = &\int d^4x[-\nabla\phi_i \cdot \nabla\bar{\phi}_i + i\bar{\psi}_{L,i}\gamma \cdot \nabla\psi_{L,i} + \bar{F}_iF_i] \\ &+ \int d^4x[\sqrt{2}g(\bar{\psi}_{i,L}(t^a)_{ij}\phi_j\lambda^a_R - \bar{\lambda}^a_R\bar{\phi}_i(t^a)_{ij}\psi_{j,L}) + gD^a\bar{\phi}_i(t^a)_{ij}\phi_j] \end{aligned} \tag{4.21}$$

An additional action for the superpotential can be added which is also supersymmetry invariant:

$$S_W = \int d^4x [\frac{\partial W}{\partial \phi_i} F_i + \frac{i}{2} \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \bar{\psi}_{R,i} \psi_{L,j} + \frac{\partial \bar{W}}{\partial \bar{\phi}_i} \bar{F}_i + \frac{i}{2} \frac{\partial^2 \bar{W}}{\partial \bar{\phi}_i \partial \bar{\phi}_j} \bar{\psi}_{L,i} \psi_{R,j}] \tag{4.22}$$

where the superpotential W is a holomorphic function of the scalar fields in the chiral multiplets. The action is invariant under the simultaneous supersymmetry transformations given by equations [4.19] and:

$$\begin{aligned} \delta \phi_i &= -i\sqrt{2} \bar{\epsilon}_R \psi_{L,i} \\ \delta \psi_{L,i} &= \sqrt{2}(\gamma \cdot \nabla \phi_i \epsilon_R + F_i \epsilon_L) \\ \delta F_i &= -i\sqrt{2} \bar{\epsilon}_L \gamma \cdot \nabla \psi_{L,i} - 2g \bar{\epsilon}_L \lambda^a_R t^a_{ij} \phi_j \end{aligned} \tag{4.23}$$

where the indices i, j indicate the components of different chiral multiplets.

To construct the supersymmetric gauge theories which contain matter particles interacting with gauge bosons, one has to use both actions [4.18] and [4.21]. There will then be two types of contributions to the potential after elimination of the auxiliary fields. The first part is obtained by eliminating all F-terms using their equations of motion. The contribution of these auxiliary fields is a generalisation of [4.17], it is:

$$V_F = \sum_i |\frac{\partial W}{\partial \phi_i}|^2 \tag{4.24}$$

Next, there is a contribution of the D-terms, their equations of motion are given by:

$$D^a = -g\bar{\phi} t^a \phi \tag{4.25}$$

They lead to a contribution to the potential:

$$V_D = \sum_a \frac{1}{2} (g \bar{\phi} t^a \phi)^2 \tag{4.26}$$

The two terms [4.24],[4.26] together form the *scalar potential*.

4.4 Symmetry breaking and Fayet-Iliopoulos terms

Internal symmetry breaking and supersymmetry breaking can be obtained in supersymmetric gauge theories if the complete scalar potential admits a vacuum which is not invariant under the internal symmetries which are to be broken, and if the potential in the ground-state is non-zero.

Looking at the contributions [4.24],[4.26] obtained after eliminating the auxiliary fields F_i and D^a shows that the contribution to the potential due to the D-fields can always be set to zero by choosing all scalar fields expectation values to be zero. The non-zero contribution to the potential needed to break supersymmetry could be obtained by adding a term linear in the fields ϕ_i in the superpotential. This would lead to a V_F of the form:

$$V_F = |c + f(\phi_i)|^2 \tag{4.27}$$

where $f(\phi_i)$ is a function of the scalar fields ϕ_i with no constant term. The scalar potential might not allow a zero energy ground state if these linear terms are included. However, in a gauge theory without fields which are singlets under all gauge symmetries these linear terms break gauge invariance due to for example the part of the action:

$$\int d^4x[\frac{\partial W}{\partial \phi_i} F_i] \tag{4.28}$$

where one has keep in mind that F_i transforms in the same way as ϕ_i.

This shows the inability to break supersymmetry without gauge singlets using the contribution through the superpotential and F-terms, known as the O'Raifeartaigh mechanism [6] or F-term breaking. We therefore turn to D-term symmetry breaking, also known as the *Fayet-Iliopoulos* mechanism [8]. We begin by looking more closely at the supersymmetry transformation of the auxiliary field D^a:

$$\delta D^a = \bar{\epsilon}\gamma_5\gamma \cdot \nabla\lambda^a \tag{4.29}$$

If D^a is part of a $U(1)$ vector multiplet, both the field D^a and λ^a are in the adjoint representation and do not transform under internal symmetry transformations, as can be seen from equation [4.20] using that the structure constants of Abelian groups disappear. In this case the covariant derivative appearing in the supersymmetry transformation [4.29] reduces to a normal derivative. Therefore a $U(1)$ auxiliary D-field transforms as a total derivative under supersymmetry transformations. We can therefore add the following supersymmetric and gauge invariant part to the action if D is part of a $U(1)$ vector multiplet:

$$S_{F.I.} = \int d^4x[g\xi D] \tag{4.30}$$

where g is the coupling constant for the $U(1)$ symmetry and ξ is a new constant parameter. This term is known as the *Fayet-Iliopoulos* term. We can add a Fayet-Iliopoulos term for each $U(1)$ symmetry of the theory. The result of including the Fayet-Iliopoulos term will be to modify the equation of motion for the $U(1)$ auxiliary field D:

$$D = -g\bar{\phi}_i Q_{ij}\phi_j - g\xi \tag{4.31}$$

where Q_{ij} is the matrix of $U(1)$ charges. The contribution of the D-term to the scalar potential is then modified to:

$$V_D = \frac{1}{2}(g\bar{\phi}_i Q_{ij}\phi_j + g\xi)^2 \tag{4.32}$$

The Fayet-Iliopoulos term could induce symmetry breaking if $g\xi \neq 0$ as this contribution does not equal zero when the fields obtain no vacuum expectation values. Supersymmetry is broken if the contributions to the potential given by equations [4.24], [4.26] for non-Abelian multiplets, and [4.32] for $U(1)$ vector multiplets if Fayet-Iliopoulos terms are added, cannot simultaneously equal zero.

4.5 Minimal Supersymmetric Standard Model

The Minimal Supersymmetric Standard Model (MSSM) is the model which is obtained by constructing a supersymmetric gauge theory containing all Standard Model particles and interactions. The construction of the MSSM starts by creating chiral multiplets to contain the Standard Model scalar fields and fermions. As mentioned before, the particles in multiplets have the same transformation properties. Looking at the transformation properties of the Standard Model [2.2], we see it is not possible to group Standard Model scalar particles with Standard Model fermions in chiral multiplets. We will therefore have to add new superpartners for all existing scalar particles and fermions.

4.5.1 A second Higgs doublet

Constructing the MSSM forces us to make a modification to the Standard Model. The reason for this modification, is due to the Standard Model Yukawa couplings:

$$\mathcal{L}_{Yukawa} = \frac{i}{2}\epsilon_{ij}Y^u H^i \bar{u}_R Q_L^j + \frac{i}{2}Y^d \bar{H}\bar{d}_R Q_L + \frac{i}{2}Y^e \bar{H}\bar{e}_R L_L + \text{h.c.} \tag{4.33}$$

We have noted in section [2.4] that both the Higgs doublet H and its Hermitian conjugate $\bar{H}$ are involved in generating the fermion mass terms. In creating the MSSM we have to construct a superpotential W such that it will reproduce these Standard Model Yukawa couplings. In the MSSM these are given by:

$$\frac{\partial^2 W}{\partial\phi_i \partial\phi_j}\bar{\psi}_{R,i}\psi_{L,j} + \text{h.c.} \tag{4.34}$$

Comparing this with equation [4.33], we would have to include both H and $\bar{H}$ in the superpotential. This is however not allowed in a supersymmetric model, as the superpotential has to be a holomorphic function of the fields.

To solve this problem, we therefore introduce a second Higgs doublet. We denote the Higgs doublet giving mass to particles in the upper parts of the $SU(2)$ doublets by H_1, and the one giving mass to lower parts we denote by H_2. In terms of these two Higgs doublets, we write the Standard Model Yukawa couplings:

$$\mathcal{L} = \frac{i}{2}\epsilon_{ij}H_1^i \bar{u}_R Y^u Q_L^j - \frac{i}{2}\epsilon_{ij}H_2^i(\bar{d}_R Y^d Q_L^j + \bar{e}_R Y^e L_L^j) + \text{h.c.} \tag{4.35}$$

where we have used the doublet notation:

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix},\ H_2 = \begin{pmatrix} H_2^0 \\ H_2^- \end{pmatrix} \tag{4.36}$$

To keep the terms gauge invariant, we have to assign hypercharge 1 to H_1 and hypercharge -1 to H_2. To the upper component of H_2 zero electric charge is assigned, to the lower component electric charge -1 is assigned.

With the introduction of the extra Higgs doublet, the chiral multiplets in the MSSM can be constructed. The scalar components are listed in the first column of table [4.1], the fermion part is listed in the second column. The auxiliary fields have been left out of the table, but are part of the chiral multiplets. We use the convention that only left-handed chiral multiplets are used in the construction of the supersymmetric theory. We therefore include right-handed fermions in chiral multiplets by using their charge conjugated spinors. The right-handed electron for example is contained in a left-handed chiral multiplet as $(e_R)^c$, which is a left-handed fermion. Superpartners are denoted by placing a tilde over their Standard Model partner symbol. For example, $\tilde{e}_R$ is the scalar partner of the right-handed electron. The subscript R is used to identify the particle with its superpartner, as the scalar particle

Scalar	Fermion	$SU(3)$	$SU(2)$	$U(1)_Y$	$U(1)_{EM}$
$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	3	2	$\frac{1}{3}$	$(\frac{2}{3}, -\frac{1}{3})$
$\tilde{u}_R$	$(u_R)^c$	$\bar{3}$	1	$-\frac{4}{3}$	$\frac{2}{3}$
$\tilde{d}_R$	$(d_R)^c$	$\bar{3}$	1	$\frac{2}{3}$	$-\frac{1}{3}$
$(\tilde{\nu}_L, \tilde{e}_L)$	(ν_L, e_L)	1	2	-1	$(0, -1)$
$\tilde{e}_R$	$(e_R)^c$	1	1	2	-1
(H_1^+, H_1^0)	$(\tilde{H}_1^+, \tilde{H}_1^0)$	1	2	1	$(1, 0)$
(H_2^0, H_2^-)	$(\tilde{H}_2^0, \tilde{H}_2^-)$	1	2	-1	$(0, -1)$

Table 4.1: *The scalar particles and left-handed fermions contained in the chiral multiplets of the MSSM listed with their transformation properties. The auxiliary fields F^i have been left out.*

Gauge boson	Gaugino	Symmetry
B_μ	λ_B	$U(1)_Y$
W_μ^a	λ_W^a	$SU(2)_L$
G_μ^a	λ_G^a	$SU(3)_c$

Table 4.2: *The particle content of the vector multiplets of the MSSM. The auxiliary fields D^a have been left out. The fields transform in the adjoint representation of their corresponding group.*

has zero spin it could not be left-handed or right-handed.

Looking at the assignment of hypercharges reveals another reason to include two Higgs doublets in the MSSM. As extra left-handed fermions are introduced, there will be extra terms contributing to the anomaly shown in figure [2.1]. By introducing two Higgs doublets with opposite Y charges these contributions cancel.

The vector multiplets contained in the MSSM are listed in table [4.2]. In the first column the gauge bosons are listed, in the second column their fermion superpartners are listed. All fields in the vector multiplet transform in the adjoint representation of their associated symmetry group according to equation [2.10] for the gauge bosons and according to [4.20] for the other fields.

As for the nomenclature, the scalar partners of the Standard Model fermions are often given the names of their corresponding fermions with an *s-* prefixed. Examples are selectrons and squarks. The names of the fermion superpartners of Standard Model scalar particles end with *-ino*. We have the Higgsino as fermion superpartner to the Higgs boson, and gauginos as the superpartners to the gauge bosons.

4.5.2 Deriving the potential

Now that we have determined the particle content of the MSSM, we can determine the action which consists of the terms introduced in section [4.3]. To begin, we construct the superpotential of the MSSM, using the the Standard Model Yukawa couplings given by equation [4.35]. It is found to be:

$$W = \epsilon_{ij} Y^u H_1^i \tilde{u}_R \tilde{Q}^j - \epsilon_{ij} H_2^i (Y^d \tilde{d}_R \tilde{Q}^j + Y^e \tilde{e}_R \tilde{L}^j) + \mu \epsilon_{ij} H_1^i H_2^j \tag{4.37}$$

where we have used the notation $\tilde{Q}^i$ and $\tilde{L}^i$ for the doublets:

$$\tilde{Q}^i = \begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}, \ \tilde{L}^i = \begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix} \tag{4.38}$$

The superpotential can contain terms of mass dimension up to three. Other gauge-invariant terms could have been added to the superpotential, but the MSSM does not include them. This is due to the concept of *R-parity* which is used in the MSSM. To each particle R-parity is assigned with value:

$$P_R = (-1)^{3(B-L)} (-1)^{2s} \tag{4.39}$$

In the MSSM only terms with a multiplicative R-parity of +1 are included. The introduction of R-parity excludes terms violating baryon number and lepton number from the superpotential, ensuring for example that proton decay does not occur.

Conservation of R-parity has some other important consequences. All Standard Model particles have R-parity +1, supersymmetric partners have R-parity -1. Therefore Standard Model particles can only produce supersymmetric particles in pairs, and processes involving supersymmetric particles can only result in an odd number of supersymmetric particles. Moreover, the lightest supersymmetric particle is stable, and is therefore a candidate for dark matter.

With the superpotential, the scalar potential of the MSSM can be determined. It is given by:

$$\begin{aligned} V = &\frac{1}{2}\sum_a g_a^2(\bar{\phi}t^a\phi)^2 + \sum_{\phi_i \neq H_1, H_2} |\frac{\partial W}{\partial \phi_i}|^2 \\ &+ |\mu|^2(|H_1^+|^2 + |H_1^0|^2 + |H_2^0|^2 + |H_2^-|^2) \end{aligned} \tag{4.40}$$

As this potential is a sum of squares containing only terms proportional to fields and the superpotential does not have any terms linear in fields, minimising this potential leads to a zero vacuum expectation value for the Higgs fields: there is no internal symmetry breaking or supersymmetry breaking.

4.5.3 Soft supersymmetry breaking

To break the internal symmetries and supersymmetry, an extra piece is added to the Lagrangian of the MSSM. It is called the soft supersymmetry breaking term, denoted $\mathcal{L}_{Soft}$, and it breaks the symmetries explicitly. It is given by:

$$\begin{aligned} \mathcal{L}_{Soft} = &-M_{H_1}^2|H_1|^2 - M_{H_2}^2|H_2|^2 - (b\epsilon_{ij}H_1^iH_2^j + h.c.) \\ &+ \frac{i}{2}(M_B\bar{\tilde{B}}B + M_W\bar{\tilde{W}}^aW^a + M_G\bar{\tilde{G}}^aG^a) \\ &- \bar{\tilde{Q}}M_{\tilde{Q}}^2\tilde{Q} - \bar{\tilde{L}}M_{\tilde{L}}^2\tilde{L} - \bar{\tilde{u}}_RM_{\tilde{u}}^2\tilde{u}_R - \bar{\tilde{d}}_RM_{\tilde{d}}^2\tilde{d}_R - \bar{\tilde{e}}_RM_{\tilde{e}}^2\tilde{e}_R \\ &- (\bar{\tilde{u}}_Ra_u\epsilon_{ij}H_2^i\tilde{Q}^j + \bar{\tilde{d}}_Ra_d\epsilon_{ij}H_1^i\tilde{Q}^j - \bar{\tilde{e}}_Ra_e\epsilon_{ij}H_1^i\tilde{L}^j + h.c.) \end{aligned} \tag{4.41}$$

where the objects $M_{H_1}^2, M_{H_2}^2, b, M_B, M_W$ and M_G are scalars and the other $M_{\tilde{X}}^2$ and a_X terms are matrices acting on the different families. Supersymmetry is explicitly broken by adding mass terms and interaction terms for the scalar fields, along with mass terms for the gauginos. The soft-breaking Lagrangian leads to a potential which has a ground state in which the two Higgs fields get vacuum expectation values.

The name soft-breaking is given as the term is constructed such that the quadratic dependence on the cut-off of scalar mass corrections which supersymmetry was meant to fix, does not reappear. The addition of the soft-breaking terms leads to the introduction of more than 100 free parameters.

Chapter 5

The supersymmetric model

We now construct the supersymmetric version of the extended Standard Model as introduced in chapter [3]. In doing this we use concepts which have been used in constructing the MSSM. We begin by determining the particle content of the model, after which we construct its supersymmetric action. As we have seen in the previous section, the MSSM is not capable of breaking either internal symmetry realistically or supersymmetry without the introduction of explicit soft supersymmetry breaking terms. In this model we introduce the Fayet-Iliopoulos terms to the action and derive the scalar potential to see if realistic symmetry breaking can be obtained without soft supersymmetry breaking terms. The minima of the potential are hard to find, and it is hard to determine which ground state occurs for which choice of parameters in the Lagrangian. We will therefore focus on the ground states which could be interesting phenomenologically and calculate the Higgs masses around these ground states.

5.1 The Particle content

To construct the model, we begin by introducing a second Higgs doublet. Just as we have seen in the construction of the MSSM in subsection [4.5.1], reproducing the Standard Model Yukawa couplings with one Higgs doublet in the supersymmetric theory requires the introduction of the complex conjugate of the Higgs doublet to the superpotential. This is however not possible as the superpotential has to be a holomorphic function of the fields.

The matter particles of the model are all ordered in chiral multiplets, requiring the addition of superpartners for all Standard Model particles. With the introduction of the new scalar field ϕ in the extended model a new chiral multiplet has to be added to the supersymmetric theory leading to a new left-handed fermion. As the scalar field ϕ carries $U(1)$ charges, the new fermion will also carry these $U(1)$ charges. This fermion will therefore contribute to the anomaly discussed in section [2.6], rending the supersymmetric theory anomalous.

To make the theory anomaly free, we introduce one new chiral multiplet as the *simplest solution*. We will rename the original scalar field of the model presented in chapter [3] from ϕ to ϕ_1. The new chiral multiplet then contains a scalar field which we will denote by ϕ_2 and a left-handed fermion denoted by $\tilde{\phi}_2$. To these new fields we assign $U(1)$ charges opposite to those of the ϕ_1 field.

In this supersymmetric extension we also introduce chiral multiplets containing the right-handed neutrinos and a vector multiplet containing the new gauge boson. We do not have to worry about the contributions to the anomaly of the right-handed neutrinos as the X-charge it carries was constructed such that the theory was anomaly free. This continues to hold in the supersymmetric version. The new gaugino which is introduced does not bring any new contributions to the anomaly either, as it has no chiral interactions. Remembering that we only use left-handed scalar multiplets, the chiral multiplet content of the supersymmetric version of the extended model is shown in table [5.1]. Pairing the gauge bosons with gauginos leads to the vector multiplet content shown in table [5.2].

5.2 The supersymmetric action

With the particle content of the supersymmetric extension given in tables [5.1] and [5.2], we can start constructing the action for the theory. For this we use the formulas [4.18],[4.21], [4.22] and [4.30] as given in chapter [4] for the supersymmetric actions of vector multiplets and gauged chiral multiplets.

The expression for the full supersymmetric action is built up of only a few basic pieces, the full expression is found in the appendix [A.4]. It is constructed by first adding the separate actions for all gauged chiral multiplets and vector multiplets. The $SU(2)$ vector multiplet for example contributes:

$$S = \int d^4x[-\frac{1}{4}F(W)^{a,\mu\nu}F(W)^a_{\mu\nu} + \frac{i}{2}\bar{\lambda}^a_W\gamma\cdot\nabla\lambda^a_W + \frac{1}{2}D^a_W D^a_W] \tag{5.1}$$

Scalar	Fermion $\frac{1}{2}$	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_X$	$U(1)_{EM}$
$\tilde{Q} = (\tilde{u}_L, \tilde{d}_L)$	$Q = (u_L, d_L)$	3	2	$\frac{1}{3}$	$\frac{\alpha}{3}$	$(\frac{2}{3}, -\frac{1}{3})$
$\tilde{u}_R$	$(u_R)^c$	$\overline{3}$	1	$-\frac{4}{3}$	$-\frac{4}{3}\alpha - \frac{1}{2}$	$-\frac{2}{3}$
$\tilde{d}_R$	$(d_R)^c$	$\overline{3}$	1	$\frac{2}{3}$	$\frac{2}{3}\alpha + \frac{1}{2}$	$\frac{1}{3}$
$\tilde{L} = (\tilde{\nu}_L, \tilde{e}_L)$	$L = (\nu_L, e_L)$	1	2	-1	$-\alpha$	$(0, -1)$
$\tilde{e}_R$	$(e_R)^c$	1	1	2	$2\alpha + \frac{1}{2}$	1
$\tilde{\nu}_R$	$(\nu_R)^c$	1	1	0	$-\frac{1}{2}$	0
$H_1 = (H_1^+, H_1^0)$	$\tilde{H}_1 = (\widetilde{H}_1^+, \widetilde{H}_1^0)$	1	2	1	$\alpha + \frac{1}{2}$	$(1, 0)$
$H_2 = (H_2^0, H_2^-)$	$\tilde{H}_2 = (\widetilde{H}_2^0, \widetilde{H}_2^-)$	1	2	-1	$-\alpha - \frac{1}{2}$	$(0, -1)$
ϕ_1	$\widetilde{\phi}_1$	1	1	0	1	0
ϕ_2	$\widetilde{\phi}_2$	1	1	0	-1	0

Table 5.1: *The scalar and fermion components of the chiral multiplets in this supersymmetric version of the model as presented in chapter [3]. A new chiral multiplet containing the scalar field ϕ_2 and fermion field $\tilde{\phi}_2$ is introduced to make sure the theory is anomaly free. The auxiliary F fields have been left out but are part of the chiral multiplets.*

As an example of the contribution of a gauged chiral multiplet, the action for the multiplet containing Q is given by:

$$
\begin{aligned}
S = \int d^4x[&-\nabla\tilde{Q}\cdot\nabla\tilde{Q} + i\bar{Q}\gamma\cdot\nabla Q + \bar{F}_Q F_Q \\
&+ \sqrt{2}g_S(\bar{Q}(\frac{\lambda^a}{2})\tilde{Q}\lambda^a_{G,R} - \bar{\lambda}^a_{G,R}\bar{\tilde{Q}}(\frac{\lambda^a}{2})Q) + g_S D^a_G \bar{\tilde{Q}}(\frac{\lambda^a}{2})\tilde{Q} \\
&+ \sqrt{2}g_W(\bar{Q}(\frac{\sigma^a}{2})\tilde{Q}\lambda^a_{W,R} - \bar{\lambda}^a_{W,R}\bar{\tilde{Q}}(\frac{\sigma^a}{2})Q) + g_W D^a_W \bar{\tilde{Q}}(\frac{\sigma^a}{2})\tilde{Q} \\
&+ \sqrt{2}g_B(\bar{Q}(\frac{1}{6})\tilde{Q}\lambda_{B,R} - \bar{\lambda}_{B,R}\bar{\tilde{Q}}(\frac{1}{6})Q) + g_B D_B \bar{\tilde{Q}}(\frac{1}{6})\tilde{Q} \\
&+ \sqrt{2}g_C(\bar{Q}(\frac{\alpha}{6})\tilde{Q}\lambda_{C,R} - \bar{\lambda}_{C,R}\bar{\tilde{Q}}(\frac{\alpha}{6})Q) + g_C D_C \bar{\tilde{Q}}(\frac{\alpha}{6})\tilde{Q}]
\end{aligned}
\tag{5.2}
$$

Gauge boson	Gaugino	Gauge Symmetry
G^a_μ	λ^a_G	$SU(3)_c$
W^a_μ	λ^a_W	$SU(2)_L$
B_μ	λ_B	$U(1)_Y$
C_μ	λ_C	$U(1)_X$

Table 5.2: *The gauge bosons and corresponding gauginos contained in the vector multiplets. Both transform in the adjoint representation of the symmetry group they are associated to. Each multiplet also contains an auxiliary field D which has been left out.*

Aside from these terms, we add the following part containing the contribution of the superpotential:

$$S_W = \int d^4x [\frac{\partial W}{\partial \phi^i} F^i + \frac{i}{2} \frac{\partial^2 W}{\partial \phi^i \partial \phi^j} \bar{\psi}^i_R \psi^j_L + \frac{\partial \bar{W}}{\partial \bar{\phi}^i} \bar{F}^i + \frac{i}{2} \frac{\partial^2 \bar{W}}{\partial \bar{\phi}^i \partial \bar{\phi}^j} \bar{\psi}^i_L \psi^j_R]$$

where the ϕ^i indicate all scalar fields contained in the chiral multiplets and the ψ^i_L indicate the fermions. Finally we add the two Fayet-Iliopoulos terms:

$$S_{F.I.} = \int d^4x [g_B \xi D_B + g_C \zeta D_C]$$

where ξ and ζ are unspecified constants.

We now need to determine the superpotential to have the full supersymmetric action for the extended Standard Model. To do so, we add to the superpotential of the MSSM all possible terms involving gauge invariant combinations of the new scalar fields ϕ_1 and ϕ_2 respecting R-parity leading to a renormalizable quantum field theory. As there is still some freedom left in choosing the $U(1)$ charges, we have collected all $SU(3)$ and $SU(2)$ invariant combinations respecting R-parity in table [5.3] together with the constraints on the parameters in the $U(1)$ charges.

If we want to reproduce the Majorana mass terms for the right-handed neutrinos we have to include the $\phi_1 \tilde{\nu}_{R,i} \tilde{\nu}_{R,j}$ term in the superpotential. To see why, let us look at a technical detail in creating supersymmetric models which we have skimped so far. The chiral fields $\psi_{L,i} = P_L \psi_i$ appearing in the chiral multiplets [4.3] are left-handed projections of Majorana fields ψ_i. Therefore in creating supersymmetric actions using expressions [4.21] and [4.22] we have to keep in mind that the ψ_i are Majorana spinors. Note that any chiral field χ_L can be written as the left-handed projection of a Majorana spinor as follows:

$$\chi_L = P_L \chi_L = P_L(\chi_L + (\chi_L)^c) \tag{5.3}$$

where $(\chi_L + (\chi_L)^c)$ is a Majorana spinor. We have included right-handed Standard Model particles ψ_R in left-handed chiral multiplets by using their charge conjugate $(\psi_R)^c$. We can also write these as left-handed projections of Majorana spinors:

$$(\psi_R)^c = P_L(\psi_R)^c = P_L(\psi_R + (\psi_R)^c) \tag{5.4}$$

We can now understand how the Majorana mass terms result from the $\phi_1 \tilde{\nu}_{R,i} \tilde{\nu}_{R,j}$ term in the superpotential. The chiral multiplet containing $\tilde{\nu}_{R,i}$

Term	Overal Y-charge	Overal X-charge	Conditions
$\phi_1\phi_2$	0	0	None
$\phi_1\tilde{e}_{R,i}\tilde{e}_{R,j}$	$\eta+4$	$4\alpha+2\beta+1$	$\eta=-4,\ 4\alpha+2\beta=-1$
$\phi_1\tilde{e}_{R,i}\tilde{\nu}_{R,j}$	$\eta+2$	$1+2\alpha$	$\eta=-2,\ \alpha=-\frac{1}{2}$
$\phi_1\tilde{\nu}_{R,i}\tilde{\nu}_{R,j}$	η	$1-2\beta$	$\eta=0,\ \beta=\frac{1}{2}$
$\phi_1\epsilon_{ij}H_1^i\tilde{L}^j$	η	$1+\beta$	$\eta=0,\ \beta=-1$
$\phi_1\epsilon_{ij}H_2^i\tilde{L}^j$	$\eta-2$	$1-2\alpha-\beta$	$\eta=2,\ 2\alpha+\beta=1$
$\phi_2\tilde{e}_{R,i}\tilde{e}_{R,j}$	$-\eta+4$	$4\alpha+2\beta-1$	$\eta=4,\ 4\alpha+2\beta=1$
$\phi_2\tilde{e}_{R,i}\tilde{\nu}_{R,j}$	$-\eta+2$	$2\alpha-1$	$\eta=2,\ \alpha=\frac{1}{2}$
$\phi_2\tilde{\nu}_{R,i}\tilde{\nu}_{R,j}$	$-\eta$	$-1-2\beta$	$\eta=0,\ \beta=-\frac{1}{2}$
$\epsilon_{ij}\phi_2H_1^i\tilde{L}^j$	$-\eta$	$\beta-1$	$\eta=0,\ \beta=1$
$\epsilon_{ij}\phi_2H_2^i\tilde{L}^j$	$-\eta-2$	$-1-2\alpha-\beta$	$\eta=-2,\ 2\alpha+\beta=-1$

Table 5.3: *Collection of all possible $SU(3)$ and $SU(2)$ invariant terms in the superpotential involving the scalar fields ϕ_1 and ϕ_2 leading to a renormalizable theory. The constraints on the parameters appearing in the $U(1)$ charges to obtain $U(1)$ invariance are also listed. Adding the term $\phi_1\tilde{\nu}_{R,i}\tilde{\nu}_{R,j}$ leads to the Majorana mass terms for right-handed neutrinos.*

contains also the left-handed spinor $(\nu_R)^c$, which can be written as the left-handed projection of the Majorana spinor $\chi_\nu = \nu_R + (\nu_R)^c$. The Majorana mass terms will then come from the superpotential action, more specifically from the part:

$$\begin{aligned}
&\frac{i}{2}\frac{\partial^2 W}{\partial\tilde{\nu}_{R,i}\partial\tilde{\nu}_{R,j}}\bar{\chi}_{\nu,i}P_L\chi_{\nu,j} + \frac{i}{2}\frac{\partial^2 \bar{W}}{\partial\bar{\tilde{\nu}}_{R,i}\partial\bar{\tilde{\nu}}_{R,j}}\bar{\chi}_{\nu,i}P_R\chi_{\nu,j}\\
&= \frac{i}{2}\kappa_{ij}\phi_1\bar{\chi}_{\nu,i}P_L\chi_{\nu,j} + \frac{i}{2}\bar{\kappa}_{ij}\bar{\phi}_1\bar{\chi}_{\nu,i}P_R\chi_{\nu,j}\\
&= \frac{i}{2}\kappa_{ij}\phi_1\bar{\nu}_{R,i}(\nu_{R,j})^c + \frac{i}{2}\bar{\kappa}_{ij}\bar{\phi}_1(\bar{\nu^c_{R,i}})\nu_{R,j}\\
&= \frac{i}{2}\kappa_{ij}\phi_1\bar{\nu}_{R,i}C(\bar{\nu}_{R,j})^T - \frac{i}{2}\bar{\kappa}_{ij}\bar{\phi}_1\nu^T_{R,i}C\nu_{R,j}
\end{aligned} \tag{5.5}$$

which is the Majorana mass term as we have seen in section [3.2].

From table [5.3] we can see that including the term $\kappa_{ij}\phi_1\tilde{\nu}_{R,i}\tilde{\nu}_{R,j}$ requires $\eta = 0$ and $\beta = \frac{1}{2}$. If this term is included, the only other possible term in the superpotential involving the new scalar fields is $\phi_1\phi_2$. So, the superpotential leading to the action with Majorana mass terms for right-handed neutrinos is given by:

$$\begin{aligned} W = &\epsilon_{ij} H_1^i (Y^u \tilde{u}_R \tilde{Q}^j + Y^\nu \tilde{\nu}_R \tilde{L}^j) - \epsilon_{ij} H_2^i (Y^d \tilde{d}_R \tilde{Q}^j + Y^e \tilde{e}_R \tilde{L}^j) \\ &+ \kappa_{ij} \phi_1 \tilde{\nu}_{R,i} \tilde{\nu}_{R,j} + \mu \epsilon_{ij} H_1^i H_2^j + \lambda \phi_1 \phi_2 \end{aligned} \tag{5.6}$$

where κ_{ij} is an unspecified matrix acting on the different families of right-handed sneutrinos and λ is a new parameter. The family indices on the Y-matrices have been left out.

5.3 Deriving the scalar potential

Knowing the superpotential, we can derive the scalar potential of the model. For this purpose the relevant terms of the supersymmetric action are the terms of the form:

$$\frac{1}{2} D^a D^a, \bar{F}^i F^i, g D^a \bar{\phi} t^a \phi, \frac{\partial W}{\partial \phi_i} F^i, \frac{\partial \bar{W}}{\partial \bar{\phi}_i} \bar{F}^i \text{ and } \xi D \tag{5.7}$$

These are the terms that will contribute to the scalar potential after elimination of the auxiliary fields. We start by determining the equations of motion for the D_W^a fields:

$$D_W^a = -g_W (\bar{\tilde{Q}} (\frac{\sigma^a}{2}) \tilde{Q} + \bar{\tilde{L}} (\frac{\sigma^a}{2}) \tilde{L} + \bar{H}_1 (\frac{\sigma^a}{2}) H_1 + \bar{H}_2 (\frac{\sigma^a}{2}) H_2) \tag{5.8}$$

Next the equation of motion for the D_B, where we have to take into account the contribution of its Fayet-Iliopoulos term:

$$\begin{aligned} D_B = -g_B (&\frac{1}{6} \bar{\tilde{Q}} \tilde{Q} - \frac{2}{3} \bar{\tilde{u}}_R \tilde{u}_R + \frac{1}{3} \bar{\tilde{d}}_R \tilde{d}_R - \frac{1}{2} \bar{\tilde{L}} \tilde{L} + \bar{\tilde{e}}_R \tilde{e}_R \\ &+ \bar{H}_1 H_1 - \bar{H}_2 H_2 + \xi) \end{aligned} \tag{5.9}$$

And the equation of motion for D_C which also involves a piece due to the other Fayet-Iliopoulos term:

$$\begin{aligned} D_C = -g_C (&\frac{\alpha}{6} \bar{\tilde{Q}} \tilde{Q} - (\frac{2\alpha}{3} + \frac{1}{4}) \bar{\tilde{u}}_R \tilde{u}_R + (\frac{\alpha}{3} + \frac{1}{4}) \bar{\tilde{d}}_R \tilde{d}_R - \frac{\alpha}{2} \bar{\tilde{L}} \tilde{L} \\ &+ (\alpha + \frac{1}{4}) \bar{\tilde{e}}_R \tilde{e}_R - \frac{1}{4} \bar{\tilde{\nu}}_R \tilde{\nu}_R + \frac{1}{2} (\alpha + \frac{1}{2}) \bar{H}_1 H_1 - \frac{1}{2} (\alpha + \frac{1}{2}) \bar{H}_2 H_2 \\ &+ \frac{1}{2} \bar{\phi}_1 \phi_1 - \frac{1}{2} \bar{\phi}_2 \phi_2 + \zeta) \end{aligned} \tag{5.10}$$

From the $SU(3)$ vector multiplet we have the D_S term with equation of motion:

$$D_S^a = -g_S (\bar{\tilde{Q}} (\frac{\lambda^a}{2}) \tilde{Q} + \bar{\tilde{u}} (\frac{\lambda^a}{2}) \tilde{u} + \bar{\tilde{d}} (\frac{\lambda^a}{2}) \tilde{d}) \tag{5.11}$$

where the λ^a are the Gell-Mann matrices.

We will not explicitly give all equations of motion for the auxiliary F-fields. We restrict ourselves as an example to the equation of motion for the auxiliary fields F which is part of the multiplet $(\tilde{u}_{L,i}, u_{L,i}, F^i_{u_L})$ where the i is the family index. Its equation of motion is:

$$\begin{aligned} F^i_{u_L} &= -\frac{\partial \bar{W}}{\partial \bar{\tilde{u}}_{L,i}} \\ &= -(H^0_1 \tilde{u}_{R,j} Y^u_{ji} + H^-_2 \tilde{d}_{R,j} Y^d_{ji})^* \end{aligned} \tag{5.12}$$

The other equations of motion are determined similarly. With all equations of motions determined, the scalar potential is obtained by adding all contributions of the D and F auxiliary fields and is found to be:

$$\begin{aligned} V &= \sum_D \frac{1}{2} D^2 + \sum_F |F|^2 \\ &= \frac{1}{2} g_S^2 \sum_a (\bar{\tilde{Q}}(\frac{\lambda^a}{2})\tilde{Q} + \bar{\tilde{u}}(\frac{\lambda^a}{2})\tilde{u} + \bar{\tilde{d}}(\frac{\lambda^a}{2})\tilde{d})^2 \\ &+ \frac{1}{2} g_W^2 \sum_a (\bar{\tilde{Q}}(\frac{\sigma^a}{2})\tilde{Q} + \bar{\tilde{L}}(\frac{\sigma^a}{2})\tilde{L} + \bar{H}_1(\frac{\sigma^a}{2})H_1 + \bar{H}_2(\frac{\sigma^a}{2})H_2)^2 \\ &+ \frac{1}{2} g_B^2 (\frac{1}{6}\bar{\tilde{Q}}\tilde{Q} - \frac{2}{3}\bar{\tilde{u}}_R\tilde{u}_R + \frac{1}{3}\bar{\tilde{d}}_R\tilde{d}_R - \frac{1}{2}\bar{\tilde{L}}\tilde{L} + \bar{\tilde{e}}_R\tilde{e}_R + \frac{1}{2}\bar{H}_1H_1 - \frac{1}{2}\bar{H}_2H_2 + \xi)^2 \\ &+ \frac{1}{2} g_C^2 (\frac{\alpha}{6}\bar{\tilde{Q}}\tilde{Q} - (\frac{2\alpha}{3} + \frac{1}{4})\bar{\tilde{u}}_R\tilde{u}_R + (\frac{\alpha}{3} + \frac{1}{4})\bar{\tilde{d}}_R\tilde{d}_R - \frac{\alpha}{2}\bar{\tilde{L}}\tilde{L} + (\alpha + \frac{1}{4})\bar{\tilde{e}}_R\tilde{e}_R \\ &- \frac{1}{4}\bar{\tilde{\nu}}_{R,i}\tilde{\nu}_{R,i} + \frac{1}{2}(\alpha + \frac{1}{2})\bar{H}_1H_1 - \frac{1}{2}(\alpha + \frac{1}{2})\bar{H}_2H_2 + \frac{1}{2}\bar{\phi}_1\phi_1 - \frac{1}{2}\bar{\phi}_2\phi_2 + \zeta)^2 \\ &+ \sum_i |-H^0_1\tilde{u}_{R,j}Y^u_{ji} + H^-_2\tilde{d}_{R,j}Y^d_{ji}|^2 + |H^+_1\tilde{u}_{R,j}Y^u_{ji} - H^0_2\tilde{d}_{R,j}Y^d_{ji}|^2 \\ &+ \sum_i |-H^0_1\tilde{\nu}_{R,j}Y^\nu_{ji} + H^-_2\tilde{e}_{R,j}Y^e_{ji}|^2 + |H^+_1\tilde{\nu}_{R,j}Y^\nu_{ji} - H^0_2\tilde{e}_{R,j}Y^e_{ji}|^2 \\ &+ \sum_i |H^+_1Y^u_{ij}\tilde{d}_{L,j} - H^0_1Y^u_{ij}\tilde{u}_{L,j}|^2 + |H^+_1Y^\nu_{ij}\tilde{e}_{L,j} - H^0_1Y^\nu_{ij}\tilde{\nu}_{L,j}|^2 \\ &+ \sum_i |H^0_2Y^d_{ij}\tilde{d}_{L,j} - H^-_2Y^d_{ij}\tilde{u}_{R,j}|^2 + |H^0_2Y^e_{ij}\tilde{e}_{L,j} - H^-_2Y^e_{ij}\tilde{\nu}_{L,j}|^2 \\ &+ |\tilde{u}_{R,i}Y^u_{ij}\tilde{d}_{L,j} + \tilde{\nu}_{R,i}Y^\nu_{ij}\tilde{e}_{L,j} + \mu H^-_2|^2 + |\tilde{u}_{R,i}Y^u_{ij}\tilde{u}_{L,j} + \tilde{\nu}_{R,i}Y^\nu_{ij}\tilde{\nu}_{L,j} + \mu H^0_2|^2 \\ &+ |\tilde{d}_{R,i}Y^d_{ij}\tilde{d}_{L,j} + \tilde{e}_{R,i}Y^e_{ij}\tilde{e}_{L,j} + \mu H^0_1|^2 + |\tilde{d}_{R,i}Y^d_{ij}\tilde{u}_{L,i} + \tilde{e}_{R,i}Y^e_{ij}\tilde{\nu}_{L,j} + \mu H^+_1|^2 \\ &+ |\kappa_{ij}\tilde{\nu}_{R,i}\tilde{\nu}_{R,j} + \lambda\phi_2|^2 + |\lambda\phi_1|^2 \end{aligned} \tag{5.13}$$

5.4 Finding the ground state

We are now going to determine the ground state of the theory and see if it leads to the spontaneous breaking of internal symmetry and supersymmetry. Before we embark on this calculation, we make some remarks.

5.4.1 Preventing electromagnetic symmetry breaking

First of all we cannot allow the scalar fields carrying electric charge to get vacuum expectation values as this would lead to the breaking of the electromagnetic symmetry. To see how this has consequences for our model let us take a look at the full potential given in expression [5.13]. The minimum value of the potential is either equal to zero or positive, a zero potential would therefore be the minimum value. In case the Fayet-Iliopoulos parameters are both positive, $\xi > 0$ and $\zeta > 0$, the potential could for example be zero by giving vacuum expectation values only to the fields $\tilde{u}_R$ and $\tilde{\nu}_{R,i}$ in such a way that the following two squares containing the Fayet-Iliopoulos parameters are set to zero:

$$\begin{aligned}
&\frac{1}{2}g_B^2(\frac{1}{6}\bar{\tilde{Q}}\tilde{Q} - \frac{2}{3}\bar{\tilde{u}}_R\tilde{u}_R + \frac{1}{3}\bar{\tilde{d}}_R\tilde{d}_R - \frac{1}{2}\bar{\tilde{L}}\tilde{L} + \bar{\tilde{e}}_R\tilde{e}_R + \frac{1}{2}\bar{H}_1H_1 - \frac{1}{2}\bar{H}_2H_2 + \xi)^2 \\
&+ \frac{1}{2}g_C^2(\frac{\alpha}{6}\bar{\tilde{Q}}\tilde{Q} - (\frac{2\alpha}{3} + \frac{1}{4})\bar{\tilde{u}}_R\tilde{u}_R + (\frac{\alpha}{3} + \frac{1}{4})\bar{\tilde{d}}_R\tilde{d}_R - \frac{\alpha}{2}\bar{\tilde{L}}\tilde{L} + (\alpha + \frac{1}{4})\bar{\tilde{e}}_R\tilde{e}_R \\
&- \frac{1}{4}\bar{\tilde{\nu}}_{R,i}\tilde{\nu}_{R,i} + \frac{1}{2}(\alpha + \frac{1}{2})\bar{H}_1H_1 - \frac{1}{2}(\alpha + \frac{1}{2})\bar{H}_2H_2 + \frac{1}{2}\bar{\phi}_1\phi_1 - \frac{1}{2}\bar{\phi}_2\phi_2 + \zeta)^2
\end{aligned} \tag{5.14}$$

In this way the value of the potential would be zero in the ground state, and a charged scalar field would obtain a vacuum expectation value. Similar reasoning applies to other values of the parameters ξ and ζ.

As the potential [5.13] does not exclude vacua where charged scalar fields obtain vacuum expectation values, we can conclude that the *Fayet-Iliopoulos mechanism by itself is not sufficient to obtain realistic symmetry breaking in this model*. We will therefore explore the option of adding explicit mass terms for the electrically charged scalar fields to see if these are sufficient together with the Fayet-Iliopoulos mechanism to lead to realistic symmetry breaking. The explicit mass terms are given by:

$$\mathcal{L}_{Soft} = -\bar{\tilde{Q}}M_{\tilde{Q}}^2\tilde{Q} - \bar{\tilde{L}}M_{\tilde{L}}^2\tilde{L} - \bar{\tilde{u}}_RM_{\tilde{u}}^2\tilde{u}_R - \bar{\tilde{d}}_RM_{\tilde{d}}^2\tilde{d}_R - \bar{\tilde{e}}_RM_{\tilde{e}}^2\tilde{e}_R \tag{5.15}$$

By giving the charged scalar fields high masses they will be excluded from obtaining non-zero vacuum expectation values. These mass terms break supersymmetry explicitly. We will however not have to introduce mass terms for the charged Higgs fields H_1^+ and H_2^-, as we will see they will not obtain vacuum expectation values if the neutral Higgs fields H_1^0 and H_2^0 get vacuum expectation values.

As the charged scalar fields, except for H_1^+ and H_2^-, will not get vacuum expectation values due to their high masses, we set these massive charged scalar fields to zero in the potential leading to:

$$\begin{aligned}V = &\frac{1}{8}g_W^2(|H_1^+|^4 + |H_1^0|^4 + |H_2^-|^4 + |H_2^0|^4 + 2|H_1^+|^2|H_1^0|^2 + 2|H_1^+|^2|H_2^0|^2\\&- 2|H_1^0|^2|H_2^0|^2 - 2|H_1^+|^2|H_2^-|^2 + 2|H_1^0|^2|H_2^-|^2 + 2|H_2^0|^2|H_2^-|^2\\&+ 4\bar{H}_1^+\bar{H}_2^-H_1^0H_2^0 + 4H_1^+H_2^-\bar{H}_1^0\bar{H}_2^0) + \frac{1}{8}g_B^2(|H_1|^2 - |H_2|^2 + 2\xi)^2\\&+ \frac{1}{8}g_C^2(-\frac{1}{2}\bar{\tilde{\nu}}_{R,i}\tilde{\nu}_{R,i} + (\alpha + \frac{1}{2})(|H_1|^2 - |H_2|^2) + |\phi_1|^2 - |\phi_2|^2 + 2\zeta)^2\\&+ |\mu|^2(|H_2^-|^2 + |H_2^0|^2 + |H_1^0|^2 + |H_1^+|^2) + |\kappa_{ij}\tilde{\nu}_{R,i}\tilde{\nu}_{R,j} + \lambda\phi_2|^2 + |\lambda\phi_1|^2\\&+ (|H_1^0|^2 + |H_1^+|^2)\sum_i |\tilde{\nu}_{R,j}Y^\nu_{ji}|^2 \end{aligned} \tag{5.16}$$

5.4.2 Minimisation constraints

To find the ground state of this potential we must minimise the potential. To start, we can use the $SU(2)$ symmetry to remove one of the charged components of the Higgs doublets to zero in the minimum configuration.

$$\langle H_1\rangle = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} \overset{SU(2)_L}{\rightarrow} \begin{pmatrix} 0 \\ H_1^0 \end{pmatrix} \tag{5.17}$$

This leads to the two following minimisation conditions:

$$\begin{aligned}&\frac{\partial V}{\partial H_1^0}|_{H_1^+=0} = 0 \qquad (5.18)\\&= \bar{H}_1^0(g_W^2(|H_1^0|^2 - |H_2^0|^2 + |H_2^-|^2) + \frac{1}{4}g_B^2(|H_1^0|^2 - |H_2^0|^2 - |H_2^-|^2 + 2\xi)\\&+ \frac{1}{4}(\alpha - \frac{1}{2})g_C^2(-\frac{1}{2}\bar{\tilde{\nu}}_{R,i}\tilde{\nu}_{R,i} + (\alpha + \frac{1}{2})(|H_1|^2 - |H_2|^2) + |\phi_1|^2 - |\phi_2|^2 + 2\zeta)\\&+ |\mu|^2 + \sum_i |\tilde{\nu}_{R,j}Y^\nu_{ji}|^2)\end{aligned}$$

and

$$\frac{\partial V}{\partial H_2^0}|_{H_1^+=0} = 0 \tag{5.19}$$
$$= \bar{H}_2^0(-g_W^2(|H_1^0|^2 - |H_2^0|^2 - |H_2^-|^2) - \frac{1}{4}g_B^2(|H_1^0|^2 - |H_2^0|^2 - |H_2^-|^2 + 2\xi)$$
$$- \frac{1}{4}(\alpha - \frac{1}{2})g_C^2(-\frac{1}{2}\bar{\tilde{\nu}}_{R,i}\tilde{\nu}_{R,i} + (\alpha + \frac{1}{2})(|H_1|^2 - |H_2|^2) + |\phi_1|^2 - |\phi_2|^2 + 2\zeta) + |\mu|^2)$$

We are interested in extrema where both neutral Higgs components get vacuum expectation values, allowing us to generate masses. If we exclude the possibilities $H_1^0 = 0$ and $H_2^0 = 0$, we can combine the two equations above to get the minimisation constraint:

$$2|\mu|^2 + \sum_i |\tilde{\nu}_{R,j}Y_{ji}^\nu|^2 + 2g_W^2|H_2^-|^2 = 0 \tag{5.20}$$

As this is a sum of squares, a ground state where H_1^0 and H_2^0 get vacuum expectation values therefore requires $\mu = 0$, $H_2^- = 0$, and $\tilde{\nu}_{R,j}Y_{ji}^\nu = 0$. The field H_2^- gets no vacuum expectation value leaving the $U(1)_{EM}$ symmetry unbroken. We do however lose our explicit Higgs mass terms if we choose $\mu = 0$ as the term

$$\mu\epsilon_{ij}H_1^iH_2^j \tag{5.21}$$

leads roughly to the following mass terms in the scalar potential:

$$|\mu|^2(|H_1|^2 + |H_2|^2) \tag{5.22}$$

We can get more constraints on the vacuum expectation values by looking at two more minimisation conditions:

$$\frac{\partial V}{\partial \phi_1} = \bar{\phi}_1(|\lambda|^2 + \frac{1}{4}g_C^2(-\frac{1}{2}\bar{\tilde{\nu}}_{R,i}\tilde{\nu}_{R,i} + (\alpha + \frac{1}{2})(|H_1|^2 - |H_2|^2) + |\phi_1|^2 - |\phi_2|^2 + 2\zeta))$$
$$= 0 \tag{5.23}$$

and

$$\frac{\partial V}{\partial \phi_2} = \frac{1}{4}g_C^2\bar{\phi}_2(-\frac{1}{2}\bar{\tilde{\nu}}_{R,i}\tilde{\nu}_{R,i} + (\alpha + \frac{1}{2})(|H_1|^2 - |H_2|^2) + |\phi_1|^2 - |\phi_2|^2 + 2\zeta)$$
$$+ \lambda(\kappa_{ij}\tilde{\nu}_{R,i}\tilde{\nu}_{R,j} + \lambda\phi_2)^*$$
$$= 0 \tag{5.24}$$

In case ϕ_1 gets a vacuum expectation, we can combine the equations above to get:

$$\bar{\lambda}\kappa_{ij}\tilde{\nu}_{R,i}\tilde{\nu}_{R,j} = 0 \tag{5.25}$$

5.4.3 Minimising the potential

We are now interested in finding the minima of the potential. Instead of finding all minima of the potential and discussing which is the minimum under which choice of parameters, we adopt a different strategy. We focus on finding only the minima which could be phenomenologically interesting ground states and determine whether they lead to a realistic theory. We will do this in the next section by calculating the Higgs masses for these ground states. To be phenomenologically interesting the minima need to lead to non-zero vacuum expectation values for the fields H_1^0, H_2^0 and ϕ_1. With this constraint in mind we will find one set of potentially interesting ground states.

To get started, the extrema with non-zero values for the fields H_1^0 and H_2^0 require as we have seen in the previous subsection: $\mu = H_2^- = 0$ and $\tilde{\nu}_{R,j} Y^\nu_{ji} = 0$. We will first *characterise* the minima with heuristic reasoning instead of finding them by direct minimisation, which is tedious due to the large amount of unknown parameters. We begin by making a distinction between the case where the matrix Y^ν can satisfy the constraint $\tilde{\nu}_{R,j} Y^\nu_{ji} = 0$ for non-zero $\tilde{\nu}_{R,i}$ and the case where it cannot. We will assume $\lambda \neq 0$ in both cases.

Case 1. Y^ν does allow non-zero $\tilde{\nu}_{R,i}$ in the ground state

In this case the potential [5.16] evaluated in the ground state is given by the following expression, where the fields should all be evaluated in their ground state values:

$$\begin{aligned}
V = {} & \frac{1}{8} g_W^2 (|H_1^0|^4 + |H_2^0|^4 - 2|H_1^0|^2 |H_2^0|^2) + \frac{1}{8} g_B^2 (|H_1^0|^2 - |H_2^0|^2 + 2\xi)^2 \\
& + \frac{1}{8} g_C^2 (-\frac{1}{2} \bar{\tilde{\nu}}_{R,i} \tilde{\nu}_{R,i} + (\alpha + \frac{1}{2})(|H_1^0|^2 - |H_2^0|^2) + |\phi_1|^2 - |\phi_2|^2 + 2\zeta)^2 \\
& + |\kappa_{ij} \tilde{\nu}_{R,i} \tilde{\nu}_{R,j} + \lambda \phi_2|^2 + |\lambda \phi_1|^2 \\
= {} & \frac{1}{8} g_W^2 x^2 + \frac{1}{8} g_B^2 (x + 2\xi)^2 + \frac{1}{8} g_C^2 (-\frac{1}{2} \bar{\tilde{\nu}}_{R,i} \tilde{\nu}_{R,i} + (\alpha + \frac{1}{2}) x + |\phi_1|^2 - |\phi_2|^2 + 2\zeta)^2 \\
& + |\kappa_{ij} \tilde{\nu}_{R,i} \tilde{\nu}_{R,j} + \lambda \phi_2|^2 + |\lambda \phi_1|^2
\end{aligned} \tag{5.26}$$

where $x \equiv |H_1^0|^2 - |H_2^0|^2$. We start by noting that the first three squares in the potential contain factors of x, the last two squares do not. When looking for the minimum of the potential, the value of x therefore depends only on

ϕ_1, ϕ_2 and $\tilde{\nu}_{R,i}$ in the combination

$$z \equiv |\phi_1|^2 - |\phi_2|^2 - \frac{1}{2}\bar{\tilde{\nu}}_{R,i}\tilde{\nu}_{R,i} \tag{5.27}$$

through the cross term of x with z coming from the third square. In the analysis which follows we will keep the value of z and with it also x constant. This allows us to see how we can lower the potential by varying the fields ϕ_1, ϕ_2 or $\tilde{\nu}_{R,i}$. *Supposing* the value of x in the minimum has been found as a function of z, we can distinguish in the minima of the potential between the three cases $z > 0$, $z = 0$ and $z < 0$. Knowing that either of these is the case, we will show that apart from H_1^0 and H_2^0 either ϕ_1 or ϕ_2 and $\tilde{\nu}_{R,i}$ will get non-zero vacuum expectation values, or none of the fields ϕ_1, ϕ_2 or $\tilde{\nu}_{R,i}$.

Subcase 1. $z > 0$

Suppose ϕ_2 and $\tilde{\nu}_{R,i}$ are not equal to zero in expression [5.26]. By lowering $|\phi_2|$ and $|\tilde{\nu}_{R,i}|$ continuously to zero while keeping z fixed, the contribution:

$$|\kappa_{ij}\tilde{\nu}_{R,i}\tilde{\nu}_{R,j} + \lambda\phi_2|^2 \tag{5.28}$$

vanishes. To keep z fixed, $|\phi_1|$ also has to be continuously lowered. The term

$$|\lambda\phi_1|^2 \tag{5.29}$$

is then lowered as well. So in the case that $z > 0$ in the minimum, the potential is minimised when the fields $\tilde{\nu}_{R,i}$ and ϕ_2 vanish. Only ϕ_1 will get a non-zero vacuum expectation value in which case the constraint given by [5.25] is satisfied.

Subcase 2. $z = 0$

If $z = 0$ in the minimum, the contributions

$$|\kappa_{ij}\tilde{\nu}_{R,i}\tilde{\nu}_{R,j} + \lambda\phi_2|^2 + |\lambda\phi_1|^2 \tag{5.30}$$

to the potential can be minimised by taking $\phi_1 = \phi_2 = \bar{\nu}_{R,i} = 0$.

Subcase 3. $z < 0$

In this case ϕ_1 will not get a vacuum expectation value. To see why, suppose $\phi_1 \neq 0$, by continuously reducing $|\phi_1|$ to zero while keeping z fixed, the value of the contribution $|\lambda\phi_1|^2$ is lowered. To keep z fixed we can perform the transformations:

$$\begin{cases} \tilde{\nu}_{R,i} \to \alpha\tilde{\nu}_{R,i} \\ \phi_2 \to \alpha^2\phi_2 \end{cases} \tag{5.31}$$

with $|\alpha| < 1$. In doing so, we have:

$$|\kappa_{ij}\tilde{\nu}_{R,i}\tilde{\nu}_{R,j} + \lambda\phi_2|^2 + |\lambda\phi_1|^2 \to |\alpha|^4|\kappa_{ij}\tilde{\nu}_{R,i}\tilde{\nu}_{R,j} + \lambda\phi_2|^2 \tag{5.32}$$

which lowers the contribution to the potential. This tells us that in this case the value of expression [5.26] can be lowered by reducing ϕ_1 to zero, therefore the field ϕ_1 gets no non-zero vacuum expectation value.

Case 2. Y^ν does not allow non-zero $\tilde{\nu}_{R,i}$ in the ground state

In this case the right-handed sneutrino fields $\tilde{\nu}_{R,i}$ do not get vacuum expectation values if the fields H_1^0 and H_2^0 get vacuum expectation values through the constraint given by equation [5.20]. Setting the right-handed sneutrino fields to zero in the potential leads to:

$$\begin{aligned} V = &\frac{1}{8}g_W^2(|H_1^0|^4 + |H_2^0|^4 - 2|H_1^0|^2|H_2^0|^2) + \frac{1}{8}g_B^2(|H_1^0|^2 - |H_2^0|^2 + 2\xi)^2 \\ &+ \frac{1}{8}g_C^2((\alpha + \frac{1}{2})(|H_1^0|^2 - |H_2^0|^2) + |\phi_1|^2 - |\phi_2|^2 + 2\zeta)^2 \\ &+ |\lambda\phi_2|^2 + |\lambda\phi_1|^2 \\ = &\frac{1}{8}g_W^2x^2 + \frac{1}{8}g_B^2(x + 2\xi)^2 + \frac{1}{8}g_C^2((\alpha + \frac{1}{2})x + |\phi_1|^2 - |\phi_2|^2 + 2\zeta)^2 \\ &+ |\lambda\phi_2|^2 + |\lambda\phi_1|^2 \end{aligned} \tag{5.33}$$

The same reasoning as before can be applied. Keeping $z \equiv |\phi_1|^2 - |\phi_2|^2$ fixed, we can minimise $|\lambda\phi_2|^2 + |\lambda\phi_1|^2$ by setting either ϕ_1 equal to zero if $z < 0$ or ϕ_2 equal to zero if $z > 0$. In case $z = 0$ we can minimise the potential by making both fields vanish.

Having examined these two cases, we are led to the conclusion that either ϕ_1 or ϕ_2 and ν_R get vacuum expectation values. In what follows we will focus on the case where the field ϕ_1 gets a vacuum expectation, or equivalently assuming that $z > 0$ in the minimum as described above. The reason we do this is because the cases where $z < 0$ and $z = 0$ are not compatible with phenomenology, as the field ϕ_1 will not obtain a vacuum expectation value in these cases. The model requires ϕ_1 to get a non-zero vacuum expectation value in order to create the Majorana mass terms as shown in equation [3.4].

5.4.4 Vacuum expectation values

We now determine the vacuum expectation values in the case where the Higgs doublets and scalar field ϕ_1 get non-zero vacuum expectation values. To do so, we set the sneutrino fields $\tilde{\nu}_{R,i}$ and the extra scalar field ϕ_2 to zero. This leads to the potential:

$$\begin{aligned} V = & \frac{1}{8}g_W^2(|H_1^0|^2 - |H_2^0|^2)^2 + \frac{1}{8}g_B^2(|H_1^0|^2 - |H_2^0|^2 + 2\xi)^2 \\ & + \frac{1}{8}g_C^2((\alpha + \frac{1}{2})(|H_1^0|^2 - |H_2^0|^2) + |\phi_1|^2 + 2\zeta)^2 + |\lambda|^2|\phi_1|^2 \end{aligned} \tag{5.34}$$

Notice that the potential only depends on the variable $|H_1^0|^2 - |H_2^0|^2$. Using this as a new variable, finding the vacuum expectation values amounts to solving:

$$\frac{\partial V}{\partial(|H_1^0|^2 - |H_2^0|^2)} = \frac{\partial V}{\partial|\phi_1|} = 0 \tag{5.35}$$

which leads to the ground state following phenomenologically interesting vacuum expectation values

$$\phi_1^2 = -4\zeta - \frac{4|\lambda|^2}{g_C^2} + \frac{4\delta\xi g_B^2 - 8\delta^2|\lambda|^2}{g_W^2 + g_B^2} \text{ and } |H_1^0|^2 - |H_2^0|^2 = \frac{4\delta|\lambda|^2 - 2\xi g_B^2}{g_W^2 + g_B^2} \tag{5.36}$$

where $\delta \equiv \alpha + \frac{1}{2}$. These are only possible vacuum expectation values if the right hand side of the squared vacuum expectation value of ϕ_1 is positive. The value of the potential in the minimum is positive for these vacuum expectation values, which is another indication that supersymmetry is broken.

As we have discussed before, the potential has other minima as well. Numerical analysis shows that for certain parameter choices this is the minimum of the potential. We have reasoned that the minima other than this one lead to ground states which are not phenomenologically interesting as not all the fields H_1^0, H_2^0 and ϕ_1 get non-zero vacuum expectation values. For general parameter choices we can conclude that either the ground state is one of these other minima of the theory, making the model incompatible with phenomenology, or this minimum is the ground state. We will now analyse this minimum as the ground state of the model to see if it is compatible with phenomenology.

5.5 Tree level Higgs masses

We will now calculate the Higgs masses of the perturbations of H_1, H_2 and ϕ_1 around the ground state with vacuum expectation values given by equation [5.36]. We will use the following notation for the vacuum expectation values:

$$\langle H^0_{1,2}\rangle \equiv v_{1,2}, \ \langle \phi_1 \rangle \equiv v_\phi \tag{5.37}$$

We have not yet used the full gauge freedom in choosing the vacuum expectation values, so far only the $SU(2)_L$ gauge freedom has been used in equation [5.17]. We can use the gauge freedom of the two $U(1)$ symmetries to make the vacuum expectation values of H_1 and ϕ_1 real. This can be done as ϕ_1 does not transform under $U(1)_Y$ gauge transformations as can be seen from table [5.1], while H_1 and H_2 do transform. The three fields do however all transform under the $U(1)_X$ symmetry. With these choices the relative phase between H_1 and H_2 is still undetermined as only their absolute values matter in the ground state as seen in equation [5.36].

To begin the calculation of the masses we expand the fields around the ground states:

$$H_1 = \begin{pmatrix} H_1^+ \\ v_1 + h_1 \end{pmatrix}, \ H_2 = \begin{pmatrix} v_2 + h_2 + ig_2 \\ H_2^- \end{pmatrix}, \ \phi_1 = v_\phi + h_\phi \tag{5.38}$$

5.5.1 Charged Higgs masses

Collecting the quadratic terms containing the charged scalar fields, we find the following terms in the expansion of the scalar potential [5.13] around the ground state:

$$\begin{pmatrix} \overline{(H_1^+)} & H_2^- \end{pmatrix} \begin{pmatrix} \frac{1}{2} g_W^2 |v_2|^2 & \frac{1}{2} g_W^2 v_1 v_2 \\ \frac{1}{2} g_W^2 v_1 \bar{v}_2 & \frac{1}{2} g_W^2 v_1^2 \end{pmatrix} \begin{pmatrix} H_1^+ \\ \overline{(H_2^-)} \end{pmatrix} \tag{5.39}$$

From this we extract the massless eigenstates

$$S_1^+ = \frac{1}{|v|}(-v_1 H_1^+ + \bar{v}_2 \overline{H_2^-}) \tag{5.40}$$

where $v \equiv \sqrt{v_1^2 + v_2^2}$. The eigenstate S_1^+ and its conjugate $S_1^- \equiv \overline{S_1^+}$ are Goldstone modes corresponding the massive charged bosons $W^\pm$. To see they are Goldstone modes, note that these modes vanish if we switch to the unitarity gauge with $H_1^+ = 0$. Next, we also find the massive eigenstates:

$$S_2^+ = \frac{1}{|v|}(v_2 H_1^+ + v_1 \overline{H_2^-}) \tag{5.41}$$

and its complex conjugate S_2^- which is another massive eigenstate. Both carry masses:

$$m_{S_2}^2 = \frac{1}{2} g_W^2 v^2 \tag{5.42}$$

5.5.2 Neutral Higgs masses

The mass eigenstates of the uncharged perturbations are determined by collecting the terms from the expansion of the potential given by equation [5.13] around the ground state as shown in equation [5.38]:

$$(h_1\ h_2\ g_2\ h_\phi) M^2 (h_1\ h_2\ g_2\ h_\phi)^T \tag{5.43}$$

where M^2 is the matrix

$$M^2 = \begin{pmatrix} \frac{1}{2}\gamma(v_1)^2 & -\frac{1}{2}\gamma v_1\ Re(v_2) & -\frac{1}{2}\gamma v_1\ Im(v_2) & \frac{1}{2} g_C^2 \delta v_1 v_\phi \\ -\frac{1}{2}\gamma v_1\ Re(v_2) & \frac{1}{2}\gamma\ Re(v_2)^2 & \frac{1}{2}\gamma\ Re(v_2)\ Im(v_2) & -\frac{1}{2} g_C^2 \delta v_\phi\ Re(v_2) \\ -\frac{1}{2}\gamma v_1\ Im(v_2) & \frac{1}{2}\gamma\ Re(v_2)\ Im(v_2) & \frac{1}{2}\gamma (Im(v_2))^2 & -\frac{1}{2} g_C^2 \delta v_\phi\ Im(v_2) \\ \frac{1}{2} g_C^2 \delta v_1 v_\phi & -\frac{1}{2} g_C^2 \delta v_\phi\ Re(v_2) & -\frac{1}{2} g_C^2 \delta v_\phi\ Im(v_2) & \frac{1}{2} g_C^2 (v_\phi)^2 \end{pmatrix} \tag{5.44}$$

where we have defined

$$\delta \equiv \alpha + \frac{1}{2} \tag{5.45}$$

and

$$\gamma \equiv g_W^2 + g_B^2 + \delta^2 g_C^2 \tag{5.46}$$

The eigenvalues of the matrix M^2 are found to be:

$$\begin{cases} m_{S_3}^2 = 0 \\ m_{S_4}^2 = 0 \\ m_{S_5}^2 = \frac{1}{4}(\gamma v^2 + g_C^2 (\phi_1^0)^2 - \sqrt{(\gamma v^2 + g_C^2 v_\phi^2)^2 - 4 g_C^2 (g_W^2 + g_B^2) v_\phi^2 v^2}) \\ m_{S_6}^2 = \frac{1}{4}(\gamma v^2 + g_C^2 (\phi_1^0)^2 + \sqrt{(\gamma v^2 + g_C^2 v_\phi^2)^2 - 4 g_C^2 (g_W^2 + g_B^2) v_\phi^2 v^2}) \end{cases} \tag{5.47}$$

with eigenstates:

$$\begin{cases} S_3 = \frac{1}{\sqrt{v_1^2 + (Im(v_2))^2}} (Im(v_2) h_1 + v_1 g_2) \\ S_4 = \frac{1}{\sqrt{v_1^2 + (Re(v_2))^2}} (Re(v_2) h_1 + v_1 h_2) \\ S_5 = \frac{1}{\sqrt{(g_c^2 \delta v_\phi A_1)^2 + v^2 A_2^2}} (v_1 A_2 h_1 - Re(v_2) A_2 h_2 - Im(v_2) A_2 g_2 + g_C^2 \delta v_\phi A_1 h_\phi) \\ S_6 = \frac{1}{\sqrt{(g_c^2 \delta v_\phi A_3)^2 + v^2 A_4^2}} (v_1 A_4 h_1 - Re(v_2) A_4 h_2 - Im(v_2) A_4 g_2 + g_C^2 \delta v_\phi A_3 h_\phi) \end{cases} \tag{5.48}$$

Where the following abbreviations have been used:

$$\begin{cases} A_1 \equiv \gamma v^2 + g_C^2 v_\phi^2 - \sqrt{(\gamma v^2 + g_C^2 v_\phi^2)^2 - 4g_C^2(g_W^2 + g_B^2)v_\phi^2 v^2} \\ A_2 \equiv \gamma^2 v^2 - \gamma g_C^2 v_\phi^2 + 2g_C^4 \delta^2 v_\phi^2 - \gamma\sqrt{(\gamma v^2 + g_C^2 v_\phi^2)^2 - 4g_C^2(g_W^2 + g_B^2)v_\phi^2 v^2} \\ A_3 \equiv \gamma v^2 + g_C^2 v_\phi^2 + \sqrt{(\gamma v^2 + g_C^2 v_\phi^2)^2 - 4g_C^2(g_W^2 + g_B^2)v_\phi^2 v^2} \\ A_4 \equiv \gamma^2 v^2 - \gamma g_C^2 v_\phi^2 + 2g_C^4 \delta^2 v_\phi^2 - \gamma\sqrt{(\gamma v^2 + g_C^2 v_\phi^2)^2 + 4g_C^2(g_W^2 + g_B^2)v_\phi^2 v^2} \end{cases} \tag{5.49}$$

5.5.3 Analysis Higgs masses

Comparison with gauge boson masses

The Higgs masses can be compared with the gauge boson masses of the supersymmetric theory whose calculation is similar to the one found in section [3.3]. The only change is the replacement of the factor v^2 in equations [3.9] and [3.14] by $v_1^2 + v_2^2$. Using our definition $v^2 \equiv v_1^2 + v_2^2$ we have the following charged gauge boson masses:

$$m_{W^\pm}^2 = \frac{1}{2} g_W^2 v^2 \tag{5.50}$$

The masses for the neutral bosons are:

$$\begin{cases} m_Z^2 = \frac{1}{4}(\gamma v^2 + g_C^2 v_\phi^2 - \sqrt{(\gamma v^2 + v_\phi^2 g_C^2)^2 - 4g_C^2 v^2 v_\phi^2 (g_B^2 + g_W^2)} \\ m_{Z'}^2 = \frac{1}{4}(\gamma v^2 + g_C^2 v_\phi^2 + \sqrt{(\gamma v^2 + v_\phi^2 g_C^2)^2 - 4g_C^2 v^2 v_\phi^2 (g_B^2 + g_W^2)} \end{cases} \tag{5.51}$$

The masses of the charged bosons are equal to the masses of the states $S_{1,2}^+$, the masses of the states $S_{3,4}$ are equal to the masses of the neutral bosons.

The appearance of scalar particles with masses equal to those of a massive gauge bosons is a property of spontaneous internal symmetry breaking in supersymmetric models, sometimes known as the super Higgs mechanism. After spontaneous internal symmetry breaking the chiral multiplets whose fields get a vacuum expectation merge with the vector multiplets whose vector bosons become massive to form massive vector multiplets. The chiral multiplets contain Goldstone modes whose degrees of freedom are transferred to the vector bosons. After the merging the new multiplets contain a massive real scalar field and a massive vector boson with a total of 4 bosonic degrees of freedom along with a Dirac fermion which has 4 fermionic degrees of freedom. The Dirac fermion is obtained after spontaneous symmetry breaking by the mixing of the Majorana spinor fields from the chiral multiplets and the vector multiplets. Despite the breaking of supersymmetry in this model, this effect still persists. More information on massive vector multiplets can be found in [14] and [15].

The massless eigenstates

The massless eigenstates are Goldstone bosons which arise due to the breaking of a global symmetry of the potential [5.34] which we will now show. Let us denote by ϕ the set of scalar fields with $\langle\phi\rangle = 0$. After gauge transformations [5.17] the potential is a function of ϕ and the fields H_1^0, $Re(H_2^0)$, $Im(H_2^0)$ and ϕ_1:

$$V = V(\phi, H_1^0, Re(H_2^0), Im(H_2^0), \phi_1) \tag{5.52}$$

where we have split H_2^0 into its real and imaginary part for the argument to come. We can see from equation [5.34] that when the fields which obtain no vacuum expectation values are removed from the potential, the resulting potential is only dependent on the quantity:

$$(H_1^0)^2 - |H_2^0|^2 \tag{5.53}$$

This quantity is invariant under the two transformations:

$$\begin{pmatrix} H_1^0 \\ Re(H_2^0) \end{pmatrix} \to \begin{pmatrix} H_1^0 \\ Re(H_2^0) \end{pmatrix} + \begin{pmatrix} 0 & \theta \\ \theta & 0 \end{pmatrix} \begin{pmatrix} H_1^0 \\ Re(H_2^0) \end{pmatrix} \tag{5.54}$$

and

$$\begin{pmatrix} H_1^0 \\ Im(H_2^0) \end{pmatrix} \to \begin{pmatrix} H_1^0 \\ Im(H_2^0) \end{pmatrix} + \begin{pmatrix} 0 & \theta \\ \theta & 0 \end{pmatrix} \begin{pmatrix} H_1^0 \\ Im(H_2^0) \end{pmatrix} \tag{5.55}$$

where $\theta << 1$ is a transformation parameter. The general potential given by equation [5.13] does not only depend on the fields H_1^0 and H_2^0 in this combination shown in [5.53], and is therefore not invariant under these transformations. The potential given by equation [5.34] with the set of scalar fields ϕ evaluated in their ground state, $V(\langle\phi\rangle = 0, H_1^0, H_2^0, \phi_1)$, is however invariant under these transformations. This means that:

$$\begin{aligned} &V(\phi = 0, H_1^0, Re(H_2^0), Im(H_2^0), \phi_1) \\ &= V(\phi = 0, H_1^0 + \theta Re(H_2^0), Re(H_2^0) + \theta H_1^0, Im(H_2^0), \phi_1) \\ &= V(\phi = 0, H_1^0, Re(H_2^0), Im(H_2^0), \phi_1) + \theta Re(H_2^0)\frac{\partial V}{\partial H_1^0}|_{\phi=0} + \theta H_1^0 \frac{\partial V}{\partial Re(H_2^0)}|_{\phi=0} \end{aligned} \tag{5.56}$$

From which we find:

$$Re(H_2^0)\frac{\partial V}{\partial H_1^0}|_{\phi=0} + H_1^0\frac{\partial V}{\partial Re(H_2^0)}|_{\phi=0} = 0 \tag{5.57}$$

Next we can take the derivative of this term with respect to any of the fields H_1^0, $Re(H_2^0)$, $Im(H_2^0)$ and ϕ_1 which we denote by X and evaluate these derivatives in the ground state (G.S) values for all fields. Using that the first derivatives of the potential disappear in the ground state, we find:

$$Re(H_2^0)\frac{\partial^2 V}{\partial H_1^0 \partial X}|_{G.S.} + H_1^0\frac{\partial^2 V}{\partial Re(H_2^0)\partial X}|_{G.S.} = 0 \tag{5.58}$$

Denoting with Z the fields contained in the set ϕ, all second derivatives:

$$\frac{\partial^2 V}{\partial X \partial Z}|_{G.S.} \tag{5.59}$$

are also found to vanish when evaluated in the ground state. Denoting any of the scalar fields in the theory by S_i, the mass matrix of perturbations around the ground state is given by:

$$\frac{\partial^2 V}{\partial S_i \partial S_j}|_{G.S.} \tag{5.60}$$

with the basis $S_i - \langle S_i \rangle|_{G.S}$. We have therefore found the massless perturbation:

$$\langle Re(H_2^0)\rangle h_1 + \langle H_1^0 \rangle h_2 = Re(v_2)h_1 + v_1 h_2 \tag{5.61}$$

where $h_{1,2}$ are the perturbations around the ground state as defined by equation [5.38]. The massless eigenstate associated to transformation [5.55] are similarly found to be:

$$Im(v_2)h_1 + v_1 g_2 \tag{5.62}$$

Chapter 6

Conclusion

In this thesis we have researched whether we could break internal symmetry and supersymmetry using Fayet-Iliopoulos terms in a supersymmetric version of an extension of the Standard Model. We started by making the model described in article [5] compatible with supersymmetry. To do so we had to introduce a second Higgs doublet, and we had to make sure the theory was anomaly free. As the newly introduced scalar field ϕ_1 made a contribution to the triangle diagram anomaly, we chose to add a single chiral multiplet to the theory containing a new scalar field ϕ_2. We chose this as the simplest solution to the anomaly, though other interesting solutions might exist involving multiple new multiplets.

The approach we have taken in breaking symmetries is different from the one taken in the MSSM where symmetry breaking requires the introduction of many soft breaking terms which explicitly break supersymmetry. When starting with minimising the potential [5.13] we found that this model also requires the introduction of supersymmetry breaking mass terms for electrically charged scalar fields to avoid vacuum expectation values for charged scalar fields and thereby violation of the electromagnetic $U(1)$ symmetry.

With the introduction of these heavy mass terms, we derived the potential [5.16] along with minimisation conditions. These conditions lead us to consider the two potentials [5.26] and [5.33]. The minima of these potentials could be distinguished by the value of the combination:

$$z \equiv |\phi_1|^2 - |\phi_2|^2 - \frac{1}{2}\bar{\tilde{\nu}}_{R,i}\tilde{\nu}_{R,i} \tag{6.1}$$

in the minimum. The cases $z \leq 0$ lead to minima without a vacuum expectation value for the field ϕ_1, which makes them incompatible with phenomenology. We therefore focused on finding the minima in the only phenomenologically interesting case $z > 0$ and found the vacuum expectation values [5.36].

To see if the model with these vacuum expectation values is compatible with phenomenology we look at the results of the particle masses obtained in section [5.5]. The tree level masses for the charged perturbations, given by equation [5.42], are equal to those of the charged gauge bosons, given by equation [5.50]. Next to these masses, we also found two massless and two massive neutral eigenstates with masses given by equation [5.47]. The massive eigenstates have masses equal to those of the two neutral gauge bosons given by equation [5.51]. The appearance of massive vector multiplets with gauge bosons and scalar particles with equal masses due to the super Higgs mechanism shows supersymmetry has not been broken properly.

Massless scalar particles and scalar particles with masses equal to gauge bosons have not been found in experiments. The ground state we have focussed on is therefore incompatible with phenomenology, all other ground states have already been found to be incompatible with phenomenology. By these results we are therefore led to the conclusion that the Fayet-Iliopoulos mechanism and the inclusion of soft supersymmetry breaking mass terms for the charged scalar fields as given by equation [5.15] do not lead to a realistic model. Additional explicit mass terms have to be added to equation [5.15] for the electrically neutral scalar fields as well as the charged Higgs fields to make this model compatible with phenomenology.

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Appendix A

Appendix

A.1 Conventions and Definitions

In this thesis, the conventions of [16] are followed. We begin by choosing the Minkowski metric to be:

$$\eta_{\mu\nu} = \mathrm{diag}(-,+,+,+) \tag{A.1}$$

The Dirac matrices γ_μ satisfy the following algebra:

$$\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu} \tag{A.2}$$

under Hermitian conjugation we have:

$$\gamma_\mu^\dagger = \gamma_0\gamma_\mu\gamma_0 \tag{A.3}$$

Aside from these four Dirac matrices, the fifth gamma matrix is defined as:

$$\gamma_5 \equiv i\gamma_0\gamma_1\gamma_2\gamma_3 \tag{A.4}$$

We define the following matrix:

$$\gamma_{\mu\nu} \equiv \frac{1}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu) \tag{A.5}$$

One set of Dirac matrices satisfying equation [A.2] is:

$$\gamma_0 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \ \gamma_i = \begin{pmatrix} 0 & -i\sigma_i \\ i\sigma_i & 0 \end{pmatrix}, \ \gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{A.6}$$

where σ_i are the Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{A.7}$$

Using the Dirac matrices, we define the *Feynman Slash* notation:

$$\not{A} \equiv \gamma^\mu A_\mu \tag{A.8}$$

and the usual bar notation for the *Dirac adjoint*:

$$\bar{\psi} \equiv \psi^\dagger \gamma_0 \tag{A.9}$$

With these conventions, the Dirac equation for a spinor ψ, coupled to gauge fields through the covariant derivative ∇_μ, becomes:

$$(\not{\nabla} + m)\psi = 0 \tag{A.10}$$

The Dirac equation can be obtained from the Lagrangian:

$$\mathcal{L} = i\bar{\psi}(\not{\nabla} + m)\psi \tag{A.11}$$

To discus Majorana spinors, we introduce the *charge conjugation* matrix C with the properties:

$$C^T = -C \text{ and } C^{-1}\gamma_\mu C = -\gamma_\mu^T \tag{A.12}$$

With this matrix, we define the charge conjugate spinor:

$$\psi^c \equiv C\bar{\psi}^T \tag{A.13}$$

We call a spinor a *Majorana spinor* if it satisfies:

$$\psi^c = \psi \tag{A.14}$$

A.2 Majorana Spinor identities

When working with Majorana spinors, it is often necessary to rewrite expressions involving spinors into more useful forms. We will discuss a way of finding these identities here. In this thesis, we use the 4-component notation for Majorana spinors. In the literature the 2-component *van der Waerden* notation, also known as dotted/undotted notation, is also frequently used. More information on relating the two is found in [17].

A useful tool in rewriting spinorial expressions involves the *Fierz decomposition.* Using the inner product $(M_1, M_2) = \mathrm{Tr}(M_1 M_2)$ for two 4×4 matrices M_1 and M_2, one can show that the set of matrices $(\mathbb{1}, \gamma_\mu, \gamma_{\mu\nu}, \gamma_5\gamma_\mu, \gamma_5)$ forms a basis for the 4×4 matrices. This allows the following decomposition of a 4×4 matrix M:

$$
\begin{aligned}
M = &\frac{1}{4}\mathrm{Tr}(M)\mathbb{1}_{4\times 4} + \frac{1}{4}\mathrm{Tr}(M\gamma^\mu)\gamma_\mu - \frac{1}{2}\mathrm{Tr}(M\gamma^{\mu\nu})\gamma_{\mu\nu} && \text{(A.15)}\\
&- \frac{1}{4}\mathrm{Tr}(M\gamma_5\gamma^\mu)\gamma_5\gamma_\mu + \frac{1}{4}\mathrm{Tr}(M\gamma_5)\gamma_5 && \text{(A.16)}
\end{aligned}
$$

An important thing to remember is the anticommutation of spinor components. Using the two spinors ψ, χ and a matrix M this leads to:

$$
\begin{aligned}
\psi_a \chi_b &= -\chi_b \psi_a \\
(\bar{\psi} M \chi)^T &= -\chi^T M^T \bar{\psi}^T
\end{aligned}
\tag{A.17}
$$

With these things in mind one can derive the following collection of identities for Majorana spinors α and β:

$$
\begin{aligned}
\bar{\alpha}\beta &= \bar{\beta}\alpha \\
\bar{\alpha}\gamma_\mu\beta &= -\bar{\beta}\gamma_\mu\alpha \\
\bar{\alpha}\gamma_5\beta &= \bar{\beta}\gamma_5\alpha \\
\bar{\alpha}\gamma_\mu\gamma_5\beta &= \bar{\beta}\gamma_\mu\gamma_5\alpha \\
\alpha\bar{\beta} - \beta\bar{\alpha} &= \frac{1}{2}\bar{\alpha}\gamma^\mu\beta\gamma_\mu - \bar{\alpha}\gamma^{\mu\nu}\beta\gamma_{\mu\nu} \\
(\alpha_R\bar{\beta}_R - \beta_R\bar{\alpha}_R)\psi_L &= -\frac{1}{2}\bar{\beta}\gamma_\mu\alpha\gamma^\mu\psi_L \\
(\alpha_L\bar{\beta}_L - \beta_L\bar{\alpha}_L)\psi_R &= \frac{1}{2}\bar{\alpha}\gamma_\mu\beta\gamma^\mu\psi_R \\
\alpha_L\bar{\beta}_R - \beta_L\bar{\alpha}_R &= \bar{\alpha}_R\gamma^{\mu\nu}\beta_L\gamma_{\mu\nu}
\end{aligned}
\tag{A.18}
$$

A.3 Constructing multiplets and supersymmetric actions.

The multiplets appearing in supersymmetry can be constructed using what is known as *multiplet calculus*. We will follow an approach described in chapter 14 of [13]. Alternative methods based on *superspace formalism* exist, on which more information can be found in references [4] and [18].

A.3.1 Constructing supersymmetry multiplets

Construction of the multiplets begins with the choice of a first component with a specific supersymmetry transformation. The supersymmetry transformations contain as a transformation parameter a Majorana spinor ϵ. Supersymmetry transformations for specific field types must be of the same field type: the transformation of a scalar field for example has to be a scalar as well. One then chooses the most general transformation rule compatible with this requirement, thereby introducing unknown fields in the transformations. The unknown fields are constrained by demanding two supersymmetry transformation $\delta(\epsilon_1), \delta(\epsilon_2)$ satisfy the relation:

$$[\delta(\epsilon_1), \delta(\epsilon_2)]X = 2\bar{\epsilon}_1\gamma^\mu\epsilon_2\partial_\mu X \tag{A.19}$$

for all fields X in the multiplet. Fields in the supersymmetry transformations which remain unconstrained in this process are added to the multiplet.

To clarify the procedure we explain the construction of the chiral multiplet introduced in section [4.2]. We begin its construction by demanding the first component is a complex scalar field ϕ whose supersymmetry transformation contains the right-handed projection of the Majorana spinor transformation parameter $\bar{\epsilon}_R$. As the variation has to be a scalar, we introduce an unknown Majorana spinor field ψ along with a normalisation factor to get the supersymmetry transformation:

$$\begin{aligned}\delta\phi &= -i\sqrt{2}\bar{\epsilon}_R\psi \\ &= -i\sqrt{2}\bar{\epsilon}_R\psi_L\end{aligned} \tag{A.20}$$

We then add the new field ψ_L to the chiral multiplet. To get its supersymmetry transformation we start with the most general supersymmetry transformation:

$$\delta\psi_L = \sqrt{2}P_L(A_1 + \gamma_\mu A_2^\mu + \gamma_{\mu\nu}A_3^{\mu\nu})\epsilon \tag{A.21}$$

where A_1, A_2^μ and $A_3^{\mu\nu}$ are unspecified fields. Demanding that equation [A.19] holds, determines A_2^μ and $A_3^{\mu\nu}$. The scalar field A_1 is however left unconstrained. We add this unconstrained field to the chiral multiplet, and repeat the above procedure to find all fields are then constrained. Renaming $A_1 = F$, we have then obtained the chiral *scalar multiplet* (ϕ, ψ_L, F) with its supersymmetry transformations:

$$\begin{aligned}\delta\phi &= -i\sqrt{2}\bar{\epsilon}_R\psi_L \\ \delta\psi_L &= \sqrt{2}(\gamma\cdot\phi\epsilon_R + F\epsilon_L) \\ \delta F &= -i\sqrt{2}\bar{\epsilon}_L\gamma\cdot\partial\psi_L\end{aligned} \tag{A.22}$$

Other chiral multiplets can be constructed by starting with a different scalar field with this transformation rule. In general, by starting from different first components and different associated supersymmetry transformations other supersymmetry multiplets can be created.

A.3.2 Construction supersymmetric actions

To create supersymmetric actions by start by noting that the variation of the auxiliary field F of a chiral scalar multiplet is a total derivative. We could therefore create a supersymmetric action by taking:

$$S = \int d^4x\ F \tag{A.23}$$

We can obtain more interesting actions by *multiplying multiplets.* To obtain the action for the chiral scalar multiplet for example, we first create a second chiral multiplet which has as its first component the Hermitian conjugate of the auxiliary field of a chiral multiplet $\bar{F}$. Starting with this field, we can create the multiplet $(\bar{F}, \gamma \cdot \partial\psi_R, \Box\bar{\phi})$ whose third component can also be checked to transform as a total derivative.

We definine the multiplication of two chiral multiplets to be $(\phi_1, \psi_{L,1}, F_1)$ and $(\phi_2, \psi_{L,2}, F_2)$:

$$\begin{aligned}&(\phi_1, \psi_{L,1}, F_1) \times (\phi_2, \psi_{L,2}, F_2)\\ =&(\phi_1\phi_2,\ \phi_1\psi_{L,2} + \phi_2\psi_{L,1},\ \phi_1 F_2 + \phi_2 F_1 + i\bar{\psi}_1 P_L \psi_2)\end{aligned} \tag{A.24}$$

The third component of this multiplet can again be checked to transform as a total derivative. Performing this multiplication on the multiplets (ϕ, ψ_L, F) and $(\bar{F}, \gamma \cdot \partial\psi_R, \Box\bar{\phi})$ leads to the a multiplet whose third component is:

$$\phi\Box\bar{\phi} + \bar{F}F + i\bar{\psi}_R\gamma \cdot \partial\psi_R \tag{A.25}$$

which according to the multiplication of multiplets transforms as a total derivative under supersymmetry transformations. This third component therefore leads to the supersymmetric action for the chiral multiplet:

$$\begin{aligned}S_{SM} &= \int d^4x[\phi\Box\bar{\phi} + \bar{F}F + i\bar{\psi}_R\gamma \cdot \partial\psi_R]\\ &\cong \int d^4x[-\partial\phi \cdot \partial\bar{\phi} + \bar{F}F + i\bar{\psi}_L\gamma \cdot \partial\psi_L]\end{aligned} \tag{A.26}$$

where the two actions are equal up to boundary terms.

The same procedure of creating multiplets and actions can be performed for a chiral multiplet whose first component is the superpotential $W(\phi)$ with ϕ part of a chiral multiplet. This leads to the action for the superpotential as shown in equation [4.22]. The procedure can also be used to create *real multiplets* starting with a real scalar field C as a first component with transformation:

$$\delta C = \bar{\epsilon}\gamma_5\xi \tag{A.27}$$

Repeating the process leads to a multiplet with six components, three of which can be eliminated using a *supergauge transformation* leading to the vector multiplet which can be used to derive the vector multiplet action given by equation [4.18]. More information on the creation of vector multiplets with this method can be found in [13].

A.4 The full supersymmetric action

In this section the full action of the extended supersymmetric model is given. The action consists of the separate actions for all vector multiplets and chiral multiplets coupled to the vector multiplets, along with the contributions of the superpotential and Fayet-Iliopoulos terms. The terms are constructed using formulas [4.18], [4.21],[4.22] and [4.30] along with tables [5.1] and [5.2]. As the full action would be rather long, it is split up into different parts.

The first term is the action for the vector multiplets:

$$\begin{aligned}
S = \int d^4x[&-\frac{1}{4}F(G)^{a,\mu\nu}F(G)^a_{\mu\nu} + \frac{i}{2}\bar{\lambda}^a_G\gamma\cdot\nabla\lambda^a_G + \frac{1}{2}D^a_G D^a_G \\
&-\frac{1}{4}F(W)^{a,\mu\nu}F(W)^a_{\mu\nu} + \frac{i}{2}\bar{\lambda}^a_W\gamma\cdot\nabla\lambda^a_W + \frac{1}{2}D^a_W D^a_W \\
&-\frac{1}{4}F(B)^{\mu\nu}F(B)_{\mu\nu} + \frac{i}{2}\bar{\lambda}_B\gamma\cdot\nabla\lambda_B + \frac{1}{2}D_B D_B \\
&-\frac{1}{4}F(C)^{\mu\nu}F(C)_{\mu\nu} + \frac{i}{2}\bar{\lambda}_C\gamma\cdot\nabla\lambda_C + \frac{1}{2}D_C D_C]
\end{aligned} \tag{A.28}$$

The part containing the action for the quarks is given by:

$$\begin{aligned}
S_{Quarks} = \int d^4x[&-\nabla\tilde{Q}\cdot\nabla\tilde{Q} + i\bar{Q}_L\gamma\cdot\nabla Q_L + \bar{F}_Q F_Q \\
&+\sqrt{2}g_S(\bar{Q}_L(\frac{\lambda^a}{2})\tilde{Q}\lambda^a_{G,R} - \bar{\lambda}^a_{G,R}\bar{\tilde{Q}}(\frac{\lambda^a}{2})Q_L) + g_S D^a_G\bar{\tilde{Q}}(\frac{\lambda^a}{2})\tilde{Q} \\
&+\sqrt{2}g_W(\bar{Q}_L(\frac{\sigma^a}{2})\tilde{Q}\lambda^a_{W,R} - \bar{\lambda}^a_{W,R}\bar{\tilde{Q}}(\frac{\sigma^a}{2})Q_L) + g_W D^a_G\bar{\tilde{Q}}(\frac{\sigma^a}{2})\tilde{Q}
\end{aligned}$$

$$
\begin{aligned}
&+\sqrt{2}g_B(\bar{Q}_L(\frac{1}{6})\tilde{Q}\lambda_{B,R}-\bar{\lambda}_{B,R}\bar{\tilde{Q}}(\frac{1}{6})Q_L)+g_B D_B\bar{\tilde{Q}}(\frac{1}{6})\tilde{Q}\\
&+\sqrt{2}g_C(\bar{Q}_L(\frac{\alpha}{6})\tilde{Q}\lambda_{C,R}-\bar{\lambda}_{C,R}\bar{\tilde{Q}}(\frac{\alpha}{6})Q_L)+g_C D_C\bar{\tilde{Q}}(\frac{\alpha}{6})\tilde{Q}\\
&-\nabla\tilde{u}_R\cdot\nabla\tilde{u}_R+i\overline{(u_R)^c}\gamma\cdot\nabla(u_R)^c+\bar{F}_{u_R}F_{u_R} \qquad \text{(A.29)}\\
&+\sqrt{2}g_S(\overline{(u_R)^c}(\frac{\lambda^a}{2})\tilde{u}_R\lambda^a_{G,R}-\bar{\lambda}_{G,R}\bar{\tilde{u}}_R(\frac{\lambda^a}{2})(u_R)^c+g_S D^a_G\bar{\tilde{u}}_R(\frac{\lambda^a}{2})\tilde{u}_R\\
&+\sqrt{2}g_B(\overline{(u_R)^c}(-\frac{2}{3})\tilde{u}_R\lambda_{B,R}-\bar{\lambda}_{B,R}\bar{\tilde{u}}_R(-\frac{2}{3})(u_R)^c+g_B D_B\bar{\tilde{u}}_R(-\frac{2}{6})\tilde{u}_R\\
&+\sqrt{2}g_C(\overline{(u_R)^c}(-\frac{2\alpha}{3}-\frac{1}{4})\tilde{u}_R\lambda_{C,R}-\bar{\lambda}_{C,R}\bar{\tilde{u}}_R(-\frac{2\alpha}{3}-\frac{1}{4})(u_R)^c)+g_C D_C\bar{\tilde{u}}_R(-\frac{2\alpha}{3}-\frac{1}{4})\tilde{u}_R\\
&-\nabla\tilde{d}_R\cdot\nabla\tilde{d}_R+i\overline{(d_R)^c}\gamma\cdot\nabla(d_R)^c+\bar{F}_{d_R}F_{d_R}\\
&+\sqrt{2}g_S(\overline{(d_R)^c}(\frac{\lambda^a}{2})\tilde{d}_R\lambda_{G,R}-\bar{\lambda}_{G,R}\bar{\tilde{d}}_R(\frac{\lambda^a}{2})(d_R)^c+g_S D^a_G\bar{\tilde{d}}_R(\frac{1}{3})\tilde{d}_R\\
&+\sqrt{2}g_B(\overline{(d_R)^c}(\frac{1}{3})\tilde{d}_R\lambda_{B,R}-\bar{\lambda}_{B,R}\bar{\tilde{d}}_R(\frac{1}{3})(d_R)^c+g_B D_B\bar{\tilde{d}}_R(\frac{1}{3})\tilde{d}_R\\
&+\sqrt{2}g_C(\overline{(d_R)^c}(\frac{\alpha}{3}+\frac{1}{4})\tilde{d}_R\lambda_{C,R}-\bar{\lambda}_{C,R}\bar{\tilde{d}}_R(\frac{\alpha}{3}+\frac{1}{4})(d_R)^c)+g_C D_C\bar{\tilde{d}}_R(\frac{\alpha}{3}+\frac{1}{4})\tilde{d}_R]
\end{aligned}
$$

The action for the leptons is given by:

$$
\begin{aligned}
S_{Lepton}=&\int d^4x[-\nabla\bar{\tilde{L}}\cdot\nabla\tilde{L}+i\bar{L}\gamma\cdot\nabla L+\bar{F}_L F_L\\
&+\sqrt{2}g_W(\bar{L}(\frac{\sigma^a}{2})\tilde{L}\lambda^a_{W,R}-\bar{\lambda}^a_{W,R}\bar{\tilde{L}}(\frac{\sigma^a}{2})L)+g_W D^a_W\bar{\tilde{L}}(\frac{\sigma^a}{2})\tilde{L}\\
&+\sqrt{2}g_B(\bar{L}(-\frac{1}{2})\tilde{L}\lambda_{B,R}-\bar{\lambda}_{B,R}\bar{\tilde{L}}(-\frac{1}{2})L)+g_B D_B\bar{\tilde{L}}(-\frac{1}{2})\tilde{L}\\
&+\sqrt{2}g_C(\bar{L}(-\frac{\alpha}{2})\tilde{L}\lambda_{C,R}-\bar{\lambda}_{C,R}\bar{\tilde{L}}(-\frac{\alpha}{2})L)+g_C D_C\bar{\tilde{L}}(-\frac{\alpha}{2})\tilde{L}\\
&-\nabla\bar{\tilde{e}}_R\cdot\nabla\tilde{e}_R+i\overline{(e_R)^c}\gamma\cdot\nabla(e_R)^c+\bar{F}_{e_R}F_{e_R} \qquad \text{(A.30)}\\
&+\sqrt{2}g_B(\overline{(e_R)^c}\tilde{e}_R\lambda_{B,R}-\bar{\lambda}_{B,R}\bar{\tilde{e}}_R(e_R)^c+g_B D_B\bar{\tilde{e}}_R\tilde{e}_R\\
&+\sqrt{2}g_C(\overline{(e_R)^c}(\alpha+\frac{1}{4})\tilde{e}_R\lambda_{C,R}-\bar{\lambda}_{C,R}\bar{\tilde{e}}_R(\alpha+\frac{1}{4})(e_R)^c)+g_C D_C\bar{\tilde{e}}_R(\alpha+\frac{1}{4})\tilde{e}_R\\
&-\nabla\bar{\tilde{\nu}}_R\cdot\nabla\tilde{\nu}_R+i\overline{(\nu_R)^c}\gamma\cdot\nabla(\nu_R)^c+\bar{F}_{\nu_R}F_{\nu_R}\\
&+\sqrt{2}g_C(\overline{(\nu_R)^c}(-\frac{1}{2})\tilde{\nu}_R\lambda_{C,R}-\bar{\lambda}_{C,R}\bar{\tilde{\nu}})R(-\frac{1}{2})(\nu_R)^c)+g_C D_C\bar{\tilde{\nu}}_R(-\frac{1}{2})\tilde{\nu}_R]
\end{aligned}
$$

The part of the action containing the terms for the Higgs doublets H_1 and H_2 along with the additional scalar fields ϕ_1 and ϕ_2 is found to be:

$$
S_{Higgs}=\int d^4x[-\nabla\bar{H}_1\cdot\nabla H_1+i\bar{\tilde{H}}_1\gamma\cdot\nabla\tilde{H}_1+\bar{F}_{H_1}F_{H_1}
$$

$$
\begin{aligned}
&+\sqrt{2}g_W(\bar{\tilde{H}}_1(\frac{\sigma^a}{2})H_1\lambda^a_{W,R}-\bar{\lambda}^a_{W,R}\bar{H}_1(\frac{\sigma^a}{2})\tilde{H}_1)+g_W D^a_W\bar{H}_1(\frac{\sigma^a}{2})H_1\\
&+\sqrt{2}g_B(\bar{\tilde{H}}_1(\frac{1}{2})H_1\lambda_{B,R}-\bar{\lambda}_{B,R}\bar{H}_1(\frac{1}{2})\tilde{H}_1)+g_B D_B\bar{H}_1(\frac{1}{2})H_1\\
&+\sqrt{2}g_C(\bar{\tilde{H}}_1(\frac{\alpha}{2}+\frac{1}{4})H_1\lambda_{C,R}-\bar{\lambda}_{C,R}\bar{H}_1(\frac{\alpha}{2}+\frac{1}{4})\tilde{H}_1)+g_C D_C\bar{H}_1(\frac{\alpha}{2}+\frac{1}{4})H_1\\
&-\nabla\bar{H}_2\cdot\nabla H_2+i\bar{\tilde{H}}_2\gamma\cdot\nabla\tilde{H}_2+\bar{F}_{H_2}F_{H_2} \qquad\qquad (A.31)\\
&+\sqrt{2}g_W(\bar{\tilde{H}}_2(\frac{\sigma^a}{2})H_2\lambda^a_{W,R}-\bar{\lambda}^a_{W,R}\bar{H}_2(\frac{\sigma^a}{2})\tilde{H}_2)+g_W D^a_W\bar{H}_2(\frac{\sigma^a}{2})H_2\\
&+\sqrt{2}g_B(\bar{\tilde{H}}_2(-\frac{1}{2})H_2\lambda_{B,R}-\bar{\lambda}_{B,R}\bar{H}_2(-\frac{1}{2})\tilde{H}_2)+g_B D_B\bar{H}_2(-\frac{1}{2})H_2\\
&+\sqrt{2}g_C(\bar{\tilde{H}}_2(-\frac{\alpha}{2}-\frac{1}{4})H_2\lambda_{C,R}-\bar{\lambda}_{C,R}\bar{H}_2(-\frac{\alpha}{2}-\frac{1}{4})\tilde{H}_2)+g_C D_C\bar{H}_2(-\frac{\alpha}{2}-\frac{1}{4})H_2]\\
&-\nabla\bar{\phi}_1\cdot\nabla\phi_1+i\bar{\tilde{\phi}}_1\gamma\cdot\nabla\tilde{\phi}_1+\bar{F}_{\phi_1}F_{\phi_1}\\
&+\sqrt{2}g_C(\bar{\tilde{\phi}}_1(\frac{1}{2})\phi_1\lambda_{C,R}-\bar{\lambda}_{C,R}\bar{\phi}_1(\frac{1}{2})\tilde{\phi}_1)+g_C D_C\bar{\phi}_1(\frac{1}{2})\phi_1\\
&-\nabla\bar{\phi}_2\cdot\nabla\phi_2+i\bar{\tilde{\phi}}_2\gamma\cdot\nabla\tilde{\phi}_2+\bar{F}_{\phi_2}F_{\phi_2}\\
&+\sqrt{2}g_C(\bar{\tilde{\phi}}_2(-\frac{1}{2})\phi_2\lambda_{C,R}-\bar{\lambda}_{C,R}\bar{\phi}_2(-\frac{1}{2})\tilde{\phi}_2)+g_C D_C\bar{\phi}_2(-\frac{1}{2})\phi_2
\end{aligned}
$$

Along with the action for the particles contained in the vector and chiral multiplets, there is the term of the action involving the superpotential:

$$
S_W=\int d^4x[\frac{\partial W}{\partial\phi^a}F^a+\frac{i}{2}\frac{\partial^2 W}{\partial\phi^a\partial\phi^b}\bar{\psi}^a_R\psi^b_L+\frac{\partial\bar{W}}{\partial\bar{\phi}^a}\bar{F}^a+\frac{i}{2}\frac{\partial^2\bar{W}}{\partial\bar{\phi}^a\partial\bar{\phi}^b}\bar{\psi}^a_L\psi^b_R]
$$

where the superpotential is given by equation [5.6]. In this formula the ϕ^a indicate all scalar fields contained in the chiral multiplets and the ψ^a_L indicate the fermions.

To the full action two Fayet-Iliopoulos terms are added, one for each $U(1)$ symmetry:

$$
S_{F.I.}=\int d^4x[g_B\xi D_B+g_C\zeta D_C]
$$

With all parts of the action written out, the scalar potential given by equation [5.13] can be derived after elimination of the auxiliary fields with their equations of motion.

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