SAPHYRE Cooperation among competitors
Offergelt, F.H.S.

SAPHYRE
Cooperation among competitors

Analysing sharing scenarios for mobile network operators using game theory

Master's Thesis - F.H.S. Offergelt

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TNO innovation for life

"The Best for the Group comes when everyone in the group does what's best for himself and the group."

JOHN NASH

Preface

This thesis is the result of seven months of research performed in order to obtain the M.Sc. degree in Applied mathematics from Leiden university. I had a wonderful six years at the Mathematical Institute, where I learned more than I could imagine. Everyone has always been kind and eager to help. I'll really miss the place, but I'm looking forward to start a new chapter of my life. I couldn't have come this far if it wasn't for my supervisor Floske Spieksma. I like to thank her for her continuous enthusiasm. I really appreciate all the support and advice she gave me, as well on the topics of my thesis as my future.

The opportunity for this research was offered by TNO Behavioural and Societal Sciences, at the department Strategic Business Analysis (SBA)[1]. TNO, Netherlands Organization for Applied Scientific Research, is an organization that focuses on applied science. The expertise group SBA focuses on research and modelling to support business decisions on innovations and make them more transparent and justifiable.

I really enjoyed my time at the department Strategic Business Analyses and I like to thank all people who helped me during my research. I like to say special thanks to my supervisors Frank Berkers and Gijs Hendrix. I thank Frank for letting me understand that there is a big difference between applied mathematics and actually applying mathematics. He made sure that I didn't lose my focus on the practical use. I thank Gijs for all his clever advice on mathematical topics. He quickly understood all the choices I made, and was always supportive.

Apart from the people coming from the academic and professional world, I would also like to thank all my friends. In particular I like to thank Anna and Carina. Without Anna, I would have been Lost. I like to thank her for all the hours of distraction she provided. I thank Carina for her inexhaustible optimism, which can even be sensed from the other side of the world.

Last but not least, I like to thank my family. I thank my sister for visiting me on regular occasions during the last six month to bring beer and beat me at 'de betoverde doolhof'. I'm eternally grateful to my parents for their unconditional moral and financial support. They always trusted me in making my own decisions and never pushed me to do otherwise.

Den Haag, June 2011

Fieke Offergelt

[1] I started my internship at TNO Information and Communication Technology. During my internship TNO reorganized, as a result SBA became part of the expertise centre Behavioural and Societal Sciences.

Executive summary

The mobile market is a fast emerging market. In the past twenty years mobile phones changed from devices that can only be used to make calls, to devices with text messaging, GPS, internet access and much more. Consumers can use these new services because of the fast development of technology for cellular networks. To handle continuously increasing data traffic, which is a result of the extra services, upgrades to better technologies are necessary. Long Term Evolution technology (LTE) and its successor LTE advanced are the latest technologies.

Mobile operators have to upgrade their networks to the latest technologies, to handle the increase in data traffic. But the upgrade is costly compared to the profits induced by the increased data volumes. The project SAPHYRE (Sharing Physical Resources), where TNO participates in, studies network sharing among mobile operators as a solution to the problem. In this way the costs for upgrading the shared network to the new technology will be divided among all operators and operational expenditures will decrease. While network sharing reduces costs, it also has influence on the possibility to differentiate on network level. Mobile operators will be forced to differentiate on other levels, which will bring extra expenses. The decision whether to make a (joint) investment in LTE technology is therefore nontransparent. Therefore it would be desirable to get more insights on what might happen.

The decision of one mobile operator influences the other operators in the same market. Game theory is a good tool to model situations where agents have to make decisions and their decisions are influenced by other agents. This leads to the following research question:

How can game theory give insights on the strategic decision making of mobile operators concerning the investment in a jointly or separately owned LTE network?

When mobile operators share their networks to make a joint investment, this can be seen as a coalition formation of these mobile operators. The question concerning which mobile operators will share their network and make a joint investment, translates therefore to the question: which mobile operators will form coalitions together? When answering this question, the influence of the presence of other coalitions on the decision of the mobile operators, has to be taken into account. Games that belong to the class of 'non-cooperative coalition formation games with spillovers' analyse which coalitions will form and takes the influence of the presence of other coalitions into account. These games are very useful to analyse coalition formation in a competitive market.

We model the situation in the mobile market as a simultaneous 'non-cooperative coalition formation game with spillovers'. We adjust existing models to the situation we are interested in. The method is formulated in such a way that it can be easily used to analyse examples of oligopolistic mobile markets, without very specific information about these markets.

To test the model, we use it to analyse the Dutch market. KPN, Vodafone and T-mobile are the three mobile operators in this market. The model gives as result the conclusion that it is in the best interest of all mobile operators to share their network with the three of them and jointly invest in LTE technology. Sensitivity analyses of this model show that the result is most sensitive to changes in the extra costs due to the necessity to differentiate on other levels. The result is very insensitive to the investment costs in integrating networks and LTE technology. Also the decrease in operational expenditures due to network sharing is of little influence.

The first chapter gives a more extensive explanation of the situation in the mobile market. The advantages and disadvantage of network sharing are clarified. The chapter ends with a formulation of the research objective. Chapter 2 starts with a literature overview concerning game theoretical models that could be suitable for the situation. Necessary properties that a model, that describes the situation in the mobile market should satisfy, are listed. These characteristics have led to a decision about which model should be explored. The third chapter describes the chosen models in more detail. Analyses are performed on non specific markets, to test the model. The last chapter shows how the Dutch market can be modelled with the chosen method. Here, sensitivity analyses show which factors influence the decision of the mobile operators the most.

Contents

1 Introduction

This chapter provides the necessary background. We start with a brief introduction of the mobile market and its recent developments. Based on the recent difficulties in this market the objectives of the Saphyre project are explained as well as some relevant research related to the project. The chapter concludes with the formulation of the research objective as part of the Saphyre project.

1.1 Mobile networks

Mobile telecommunication services play a crucial role in today's information society. Individuals, businesses and also governmental bodies are depending more and more on telecommunication networks. Mobile services constitute of a number of electronic services regarding communication such as voice, text and messaging and recently also advanced services such as internet access and mobile and wireless TV. These services are offered via mobile networks.

There are many companies that provide mobile phone services, but only few have their own network. The companies that have frequency allocation(s) and all required infrastructure to run an independent mobile network are Mobile Network Operators (MNOs). In the Netherlands there are three MNOs; KPN, Vodafone and T-Mobile. Providers who use the networks of the MNOs to offer their services are called Mobile Virtual Network Operators (MVNOs). There are a lot more MVNOs than MNOs, in the Netherlands there are currently 23 MVNOs. Examples are Tele2 mobiel, LycaMobile and Rabo Mobiel.

Figure (1.1) gives a schematic representation of a mobile network. The core network is the central part of a telecom network that provides various services to customers who are connected by the radio access network. One of the main functions is to route calls across the public switched telephone network. The public switched telephone network (PSTN) consists of telephone lines, cellular networks, communications satellites, and undersea telephone cables all inter-connected by switching centers which allows any telephone in the world to communicate with any other.
The radio access network (RAN) connects the user equipment (UE), e.g. mobile phone,

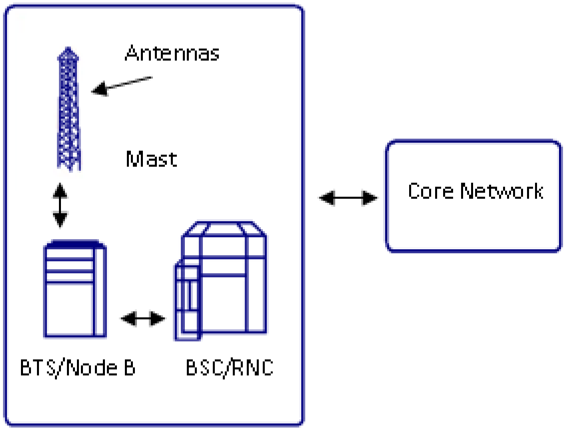

Figure 1.1: Schematic representation of a mobile network

to the core network. Part of the RAN are the masts with antennas, the base transceiver station (BTS) or Node B and the base station controller (BSC) or radio network controller (RNC). Whether we speak of a BTS or Node B and BSC or RNC depends on the technology, BTS and BSC apply to GSM, while Node B and RNC apply to UMTS. The BTS/Node B facilitates wireless communication between the UE and the rest of the network. The BSC/RNC has tens or even hundreds BTS/Nodes B under its control and handles among other things handovers from BTS/Node B to BTS/Node B.

The technology of mobile networks changed a lot over the years. The first generation of mobile networks where based on analogue technology (e.g. NMT) and supported voice services. In most countries, the incumbent telecom operator, which already operated a fixed analogue network deployed also this mobile network. In the Netherlands KPN was the MNO which deployed the analogue mobile network.
The second generation of mobile networks makes use of digital signals (e.g. GSM). With the introduction of this generation of networks the market changed, because governments also granted licenses for access to radio spectrum to other (new) network operators besides the incumbent operator. In Europe this resulted in a change from a monopolistic to an oligopolistic market.

In 1999 the International Telecommunications Union (ITU) developed the International Mobile Telecommunications (IMT) 2000 global standards for cellular communication. The ITU is the specialized agency of the United Nations which is responsible for information and communication technologies. Among other things ITU coordinates the shared global use of the radio spectrum and establishes worldwide standards. The IMT 2000 specifications are the standards for the third generation of mobile networks (3G) and combine features of mobile communications, internet access and mobile computing (e.g. UMTS). All networks that use technology that satisfies the IMT 2000 standards, are considered to be 3G networks.

In 2008 ITU specified the IMT-Advanced standards, which include enhanced peak data rates at 100 Mbps for high mobility communication (such as from trains and cars) and 1 Gbps for low mobility communication (such as pedestrians and stationary users). The fourth generation of mobile networks (4G) has to fulfill these IMT Advanced standards. The standards are based on the expectations of customers. The users expect reliable high-data rate services and ubiquitous access 24 hours a day, 7 days per week. Until now there is no technology that meets the IMT advanced standards. Long Term Evolution Technology (LTE) is a technology that was branded as '4G', but did not fully comply with the standards. However, the successor, LTE advanced, is expected to fulfill the standards. The envisaged deployment is between 2012 and 2015.

1.2 Objectives of the SAPHYRE project

Upgrades to technology are costly, but necessary because of the increasing demand of users with higher expectations. Users expect higher transmission rates and higher quality of service. But they also consider these services as a commodity and are not willing to pay more for their high expectations. So while data traffic volumes are exploding, the revenues of the mobile network operators show little increase [19]. The fact that the costs increase while the revenues lag behind, gives rise to a re-evaluation of the current mobile business model. In current wireless communications the use of radio spectrum and infrastructure is such that interference is avoided by exclusive allocation of frequency bands and employment of base stations. Resource sharing can change the business model in such a way that resources will be used more efficiently and production costs will be reduced.
The objective of the FP7 project SAPHYRE (Sharing Physical Resources), that is funded by the European Union [24], is to study resource sharing among mobile operators as a solution to the problem. TNO participates in the SAPHYRE project. Within this project the mechanisms as well as the implementations for resource sharing are studied. The objective of SAPHYRE is to demonstrate how equal-priority resource sharing in wireless networks improves spectral efficiency, enhances coverage, increases user satisfaction, leads to increased revenue for operators, and decreases capital and operating expenditures.

There are several aspects to this project. The focus of this thesis lies on one of the objectives of the fifth work package (WP5) of SAPHYRE named 'Spectrum policy and business models'. WP5 deals with issues related to policy, regulation, and business models, and will also address the pricing mechanism on which resources will be shared between the mobile operators. The result of this project will therefore be of interest regulators as well as MNOs. Resource sharing can be done on different levels. The first objective of WP5 is to identify different sharing scenarios that include radio spectrum sharing, network infrastructure sharing or complete sharing of resources. After identifying these scenarios, the next step will be to establish what costs and benefits they will bring. Several methods will be used to analyse the business scenarios [24].

1.3 Resource sharing

Resource sharing can be done on different levels, from radio spectrum sharing to network infrastructure sharing or even complete sharing of resources.

Radio spectrum is one of the most valuable resources; mobile services would be impossible without this. The radio spectrum allocation has always been controlled by governments, in order to avoid interference. But the spectrum utilization with the existing radio spectrum allocation regime is very low; less than 17 % in urban areas and less than 5 % elsewhere [2]. So sharing the spectrum among different players could give higher utilization and therefore economic benefits.

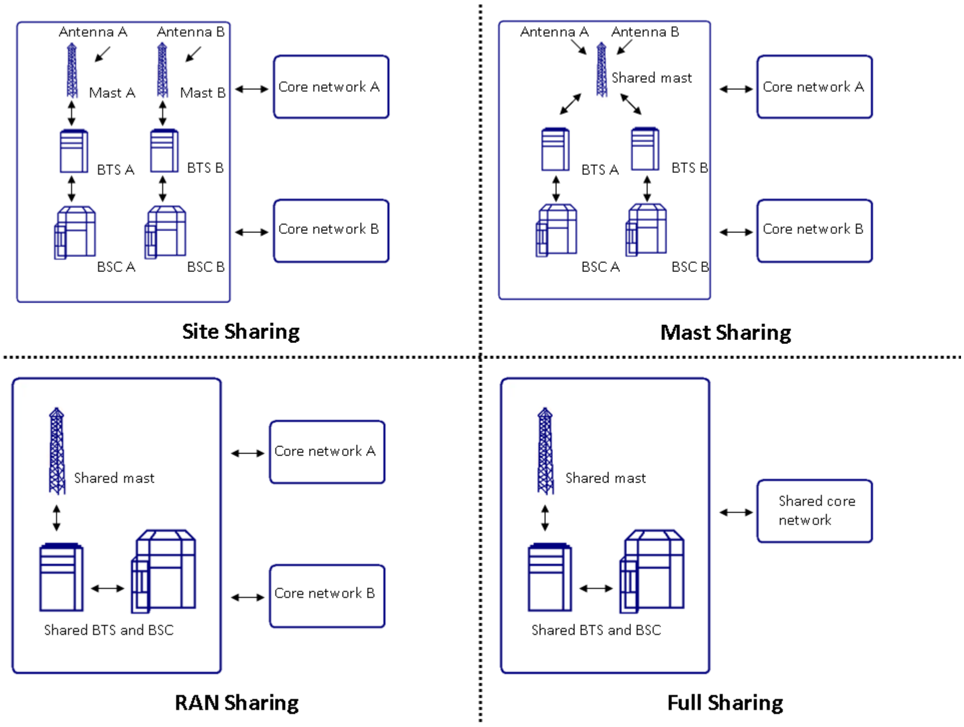

Figure 1.2: Schematic representation of infrastructure sharing

Infrastructure is another important resource that could be shared. Infrastructure sharing may take many forms, ranging from sharing sites and masts to sharing Radio Access Networks (RANs) or even the core network [14]. Figure (1.2) gives an overview of the different forms of infrastructure sharing. Generally, site sharing, mostly referred to as passive sharing, involves sharing of costs related to trading, leasing, acquisition of property items, contracts and technical facilities and the sharing of passive RAN infrastructure. Passive RAN infrastructure includes physical space on the ground, towers, roof tops and other premises, power supply, air conditioning and alarm installations. This form of sharing is often favoured in urban and suburban areas where there is a shortage of available sites or complex planning requirements. Site sharing allows MNOs to reduce both capital and operational expenditure by reducing their investments in passive network infrastructure and in network operating costs.

The other forms of sharing are referred to as active sharing. When a mast is shared, the MNOs involved can all install antennas on this mast. This clearly saves costs, because fewer masts are necessary. The next level of sharing is the sharing of the RAN. The investment in RAN sharing is high compared to lower levels of sharing, but the cost reduction due to RAN sharing is significantly larger. The Radio Access Network is the part that has the most influence on the quality of a network. Quality refers to the size of the coverage area and the reception in this area. RAN sharing will therefore have a big influence on the quality of the network of the participating MNOs.

At last, it could also be possible to share the complete network. Combining complete networks is extremely costly. Network sharing on this level will therefore be preferred with new entrants, i.e. parties without a network. The possibility to share with new entrants is out of scope of this thesis.

The focus in this thesis will be on RAN sharing combined with radio spectrum sharing, however the general model discussed in chapter 3 can be used for other forms of sharing.

RAN sharing can result in significant cost reduction on operational level as well as on future investments, but also reduces the possibility to differentiate on the level of network quality (see next section). Therefore it is important to understand more about the decision process of MNOs concerning RAN sharing. From now on when we speak of sharing we refer to RAN sharing combined with radio spectrum sharing.

1.4 Drivers and Barriers for network sharing

Research about the advantages and disadvantages of RAN and spectrum sharing for MNOs shows that several aspects influence the decision to share [3]. The most important driver for MNOs to share is the cost reduction. Operation expenditures of the RAN will decrease and costs of future investments will be divided and will therefore be less. Sharing the risk of these future investments is also an important driver. Although network sharing reduces costs, the investment to integrate networks is high, so these cost reduction should measure up to the integration costs.
Another important driver is reduced environmental, health and aesthetic concerns. Reducing these concerns has a positive influence on the reputation of mobile operators.
There are also downsides to RAN sharing. One important barrier is the greater inter-relatedness on one or more network partners. Higher inter-relatedness means higher exposure of each network operator's financial situation to his partners. If one partner is not able to meet his cost sharing commitments then the position of the other partner will be weakened. Another barrier for MNOs is the fact that sharing can lead to long term loss of their competitive advantage and their ability to differentiate services. The possibility to differentiate on network level will fall away, which can be a disadvantage for MNOs. How much this will affect the MNO, depends on his strategy. Treacy and Wiersema define three value disciplines, which a company could follow as strategy [29]. Table (1.1) gives an overview of these values.

Operational Excellence. The operations and execution are superb. A reasonable quality is provided at a very low price. A task-oriented vision towards personnel is applied. The focus is on efficiency, streamlined operations, Supply Chain Management and no frills. Volume is very important and there is extremely limited variation in product assortment. Most large international corporations are operating out of this discipline.
Product Leadership. Innovation and brand marketing is very strong. The focus is on development, innovation, design, time to market and high margins in a short time frame. The company operates in dynamic markets.
Customer Intimacy. The company excels in customer attention and customer service. It tailors its products and services to individual or almost individual customers. It has large variation in product assortment. Focus is on: customer relationship management (CRM), delivering products and services on time and above customer expectations and reliability.

Table 1.1: Value disciplines [29]

By dividing MNOs in these categories, one can derive quickly how much an MNO will be affected, due to loss of differentiation. An MNO that pursues operational excellence will be limitedly affected. Because RAN costs will be the same for all parties, the option to

differentiate on cost and process efficiency reduces. But overall profitability depends on the total of their activities, on customer base and service portfolio, and these activities will not be affected.
An MNO that pursues product leadership differentiation will be significantly affected, because network quality disappears as an important component. The development and innovations in this area will be the same for all cooperating parties.
An MNO that focuses on customer intimacy is affected the least. Customer relationships will not be affected by RAN sharing. However, it could the case that more MNOs will pursue this strategy, because of the limited options to differentiate on other levels.

All these factors are important when an MNO makes a decision about network sharing.

1.5 Research objective

When MNOs make an investment in LTE, network sharing with (an)other MNO(s) can be beneficial. So MNOs have to decide whether to invest in LTE technology or not and whether they will share their network with (an)other MNO(s) to reduce investment costs. The success of a decision made by one MNO depends on the decisions that other MNOs make. The investment in LTE network gives MNOs a competitive advantage. So if one MNO decides not to invest while his competitors upgrade to the new technology, it is possible that this MNO looses customers. Jointly investing can even give more competitive advantage because of cost reduction. If one MNO invests alone and his competitors invest jointly, his competitors could be in a better position to compete on price. So the MNOs have potentially conflicting objectives.

Game theory is a useful tool to analyse decision making by agents with potentially conflicting objectives. In game theory a distinction is made between cooperative and non-cooperative games. Because MNOs make a decision about cooperation with competitors, both categories could model the decision process. This leads to the research question:

How can game theory give insights on the strategic decision making of MNOs concerning the investment in a jointly or separately owned LTE network?

This question leads to the following sub-questions:

1. Which game theoretical models can be used to analyse coalition formation between competitors?
2. Which of these models is most suitable to analyse the decision process of the MNOs?
3. How should these models be adjusted for practical use?
4. What conclusion can be drawn from the performed analyses?

The answers to these questions will be of interest to both regulators and MNOs. So when answering these questions we have to keep in mind that both parties have to be able to make use of the results. The first two questions are answered in chapter 2 whereas the third question is answered in chapter 3. The last question is answered partly in the chapter 3, where the analyses are performed on non specific markets, and partly in chapter 4, where conclusions are drawn for the Dutch Market.

2
Defining the game

The aim of this chapter is to give insights into options to model the decision process of the MNOs. The chapter starts with an overview of literature about coalition formation games in a competitive environment. The second section elaborates on the basic properties that are necessary to model the decision process of MNOs as a game (from now referred to as the network sharing game). The basic elements of the network sharing game are defined, such as the set of players, the strategy sets and the payoff functions. Finally a decision is made about which models should be explored.

2.1 Endogenous coalition formation

The study of endogenous coalition formation has been a central topic in game theory, since the introduction by Von Neumann and Morgenstern in 1944 [18]. There are three main questions that arise concerning the subject:

1. Which coalitions will be formed?
2. How will the coalitional worth be divided among the members of the coalition?
3. How does the presence of other coalitions affect the incentives to cooperate?

Cooperative game theory has mostly focused on the second question. The answer to the first question has almost always been assumed to be the 'grand coalition', i.e. the coalition where all players are a member of. The third question, concerning competition between coalitions, is simply ignored in traditional game theory. But when studying the possibilities of for example joint ventures between competitors, all three questions are important. Joint ventures in a market with more than two players, will mostly not be between all competitors in this market. And the success of the joint venture depends on the competition in this market.
In recent years, the limitations of cooperative game theoretic solution concepts has led to the emerge of a new strand of literature describing the formation of coalitions as a

non-cooperative process. Literature refers to these models as 'non-cooperative models of coalition formation in games with spillovers'. A good overview is given by Bloch [6]. The proposed models deal with the second question by modelling the coalition formation as a non-cooperative process; each player simply announces the coalition he prefers. Which coalition will form after the announcements, depends on the rules of the game. Further on in this chapter a few models will be discussed.
To deal with the third question, the model takes spillover effects into account. Spillover effects are externalities of economic activity or processes affecting those who are not directly involved in it. There are positive and negative spillovers; positive spillovers occur when externalities have a positive effect on those not involved, while negative spillovers give a negative effect. Translated to the coalition formation game, positive spillovers occur when the formation of coalition increases the payoff of a player outside the coalition and negative spillovers occur when the formation decreases this payoff.

2.1.1 Coalitional worth

Because of the presence of spillovers, a new definition of coalitional worth is necessary. In cooperative game theory the worth of a coalition is given by a function $v(\cdot)$, the characteristic function, which assigns a value $v(S)$ to each coalition S. This function represents what a coalition S can get irrespective of the behaviour of other players. The earliest attempt to generalize this function to the case of externalities among coalitions is the introduction of partition function games by Thrall and Lucas [27]. To define the partition function, we need a definition of a coalitional structure π:

Definition 2.1 (Coalition structure)**.** A coalitional structure π is a partition of the set N of players $\pi = \{C_1, C_2, \dots, C_m\}$, representing all coalitions formed in the game.

Definition 2.2 (Partition function)**.** A partition function v is a mapping which associates to each coalition structure π a vector $\mathbb{R}^{|\pi|}$, representing the worth of all coalitions in π.

In this way the value of a coalition is not only determined by the coalition itself, but by all coalitions present. If all coalitions have the same worth in every coalitional structure, then is partition function equal to the characteristic function.
Besides the generalization of the characteristic function, there is another common way to deal with spillovers, called valuation. The valuation assigns to each coalitional structure not a vector of coalitional worth but a vector of individual payoffs. The term 'valuation' indicates that each player is able to evaluate directly the payoff she obtains in different coalition structures. For example, in case the rule of division of the payoffs between coalition members is fixed, valuation occurs. The formal definition is given by:

Definition 2.3 (Valuation)**.** A valuation v is a mapping which associates to each coalition structure π a vector of individual payoffs in $\mathbb{R}^n$.

These two ways to define the payoff in games with spillovers are used in non-cooperative coalition formation games. As in traditional game theory, a distinction is made between simultaneous and sequential games. The next two sections give an overview of some of the important models in these two categories.

2.1.2 Simultaneous games

In simultaneous games all players announce their decision to form coalitions at the same time. It is common to divide simultaneous games of coalition formation in two types:

- Open membership games
- Exclusive membership games

In open membership games, any player is free to join or leave a coalition. We give two examples of these kind of games. The first is called the cartel formation game, which was studied by d'Aspremont et al. [1]. In this game there are N players and each player decides to join the cartel or not. The coalition that is formed consists of all players that decide to join the cartel and all other players remain singletons. Formally, the coalition structure will be

$$\pi = C \cup \{j\}_{j \notin C},$$

where C is the set of players who decided to join the cartel. A big disadvantage of this game is that it only allows one coalition to form.
The second open membership game allows an arbitrary coalition structure to form and was introduced by Yi and Shin [31]. This game has a strategy set M that consists of messages m, where $|M| \geq N$. There have to be at least as many messages as players. All players simultaneously announce a message and every set of players that announces the same message forms a coalition. Formally

$$C(m) = \{i \in N : m_i = m\},$$

with m_i the message of player i, represents a coalition and the coalitional structure is given by

$$\pi = \{C(m)\}$$

for all m such that $C(m) \neq \emptyset$. This game is not a generalization of the cartel formation game because a player cannot decide to remain independent in the game. When another player announces the same message, this player is forced into a coalition with this player.

Exclusive membership games are a solution to this last disadvantage. Players are not free to join any coalition. The general idea is that players announce coalitions instead of a message. The first example of such game is called the game Γ, which was originally proposed by Von Neumann and Morgenstern and reintroduced by Hart and Kurz [10]. The strategy set for player i consists of all coalitions C where i is part of:

$$S_i = \{C \subset N : i \in C\}.$$

A coalition C is formed when all players that are a member of this coalition announced this coalition, i.e. all $i \in C$ have chosen $s_i = C$. If there is at least one member of this coalition that announced a different coalition, the coalition will not be formed. Instead all member of C, that announced C will become singletons, i.e. form a coalition alone.

Example 2.4 (The game Γ)*: Consider the game with 5 players* $N = \{1, 2, 3, 4, 5\}$*, where the strategy set of player* $i \in N$ *consists of all subsets of* $\{1, 2, 3, 4, 5\}$*, where player* i *is part of. Assume that the players choose the following strategies:*

Player 1: $\{1, 2, 3\}$
Player 2: $\{1, 2, 3\}$
Player 3: $\{3, 4\}$
Player 4: $\{3, 4\}$
Player 5: $\{1, 5\}$.

These strategies result in the formation of the following coalitions: $\{1\}, \{2\}, \{3, 4\}, \{5\}$. *Player* 1,2 *and* 5 *announced a coalition that was not unanimously announced by all its members, and therefore become singletons. Player* 3 *and* 4 *both announced the coalition* $\{3, 4\}$ *and will therefore form this coalition.*

Another exclusive membership game introduced by Hart and Kurz [10] is called the game Δ. The strategy set is the same as in the game Γ, but the outcome set is different. Coalitions are now formed by all players who announced the same coalition. If m denotes the proposed coalition then

$$C(m) = \{i \in N : s_i \in m\}$$

gives the coalition formed by all players that announced this coalition. So whenever a player deviates, the coalition will not break apart.

Example 2.5 (The game Δ)*: Consider the game from example 2.4. Playing the game* Δ*, the strategies will result in the formation of the following coalitions:* $\{1, 2\}, \{3, 4\}, \{5\}$. *Players* 1 *and* 2 *will not become singletons, but form a coalition together because they announced the same coalition.*

In the game Γ coalitions fall apart in singletons when a player deviates and in the game Δ the coalition stays together. This means that an assumption is made about the second choice of each player. In the game Γ this is assumed to be the singleton coalition and in the game Δ this is the subset of players that announced the same coalition. Because the game is simultaneous, these kind of assumptions have to be made.
More realistic would be, that players can individually react to the deviation of a player. In a sequential coalition formation game this is a possibility.

2.1.3 Sequential games

Bloch introduced a sequential game of coalition formation [5]. This game makes use of a fixed rule of payoff division, so that the underlying cooperating structure is represented by valuation. Players are ordered according to a fixed rule. The first player starts the game by proposing a coalition C. Each prospected member of the coalition C reacts in the order determined by the fixed rule. If a prospected member rejects the proposal, this player should give a counteroffer C'. If all members accept, a coalition is formed and the game continues with the players that are left.

Figure (2.1) gives an extensive form representation of a three player game, which is a common way to describe sequential games. The players are denoted with A, B and C, the strategies are written in italics and either represent a proposal (e.g. *AB*) of coalition or a yes or no to a proposal (y,n). The outcome of the game, i.e. coalitional structure, is denoted between brackets (e.g. {AB},{C}). The game doesn't have to be finite, an example is given in [5]. Another disadvantage is that the game depends on the ordering of players. Each ordering could give another course of the game.

Ray and Vohra generalized the game of Bloch by representing the underlying cooperating structure by a partition function instead of a valuation [23]. Players do not only propose a

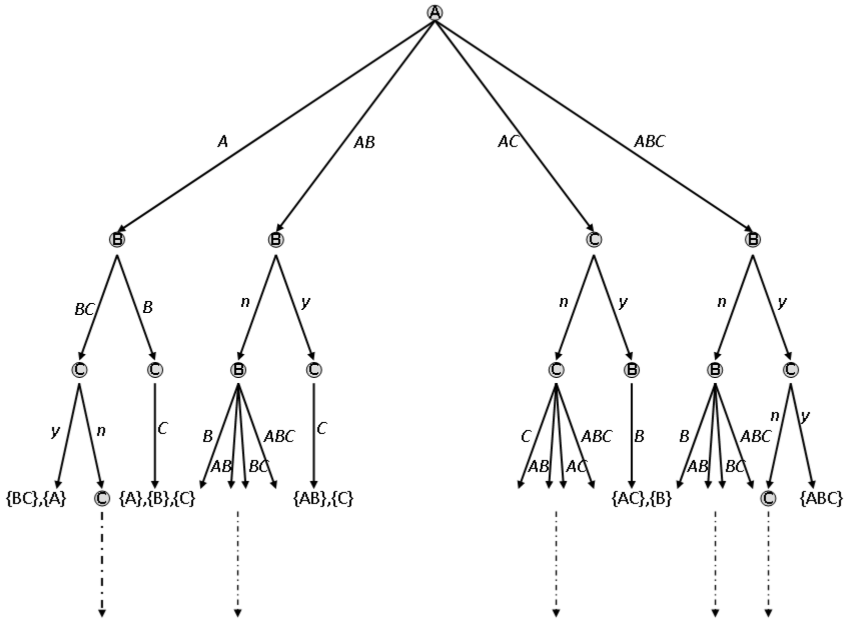

Figure 2.1: Sequential game of coalition formation

coalition, but also a division rule of the coalitional worth. This generalization was the first model that tackles simultaneously the three questions on the beginning of this chapter. Another adjustment was made by Montero [16]. He adjusted the game in such a way that the player who makes a proposal after a rejection is random.

2.2 Basics of the network sharing game

In order to model the network sharing game, we need to specify the set of players, the strategies available to each player and the payoff functions, which describes for each player the payoff for every combination of strategies.

The MNOs are the players in this game. In the Netherlands there are 3 MNOs, and in the rest of Europe the mobile market are also oligopolistic markets. The complexity of the model increases fast when the number of players increase. Reasons for this, is the general complexity of finding equilibria in games with multiple players (finding Nash equilibria is PPAD-complete, see [20]) and the fast increasing number of strategies and outcomes (see Section 2.2.1). Therefore we consider mainly sets of players consisting of 2 or 3 players.

To describe the set of strategies we should take several things into account. Each player should have the control over the decision to invest alone or not invest at all. On the other hand the decision to invest jointly can only be made if all joining players agree. Also, cooperating players should be able to refuse cooperation with another player, if they prefer. So no player can decide to join a cooperation, without permission of the players in the cooperation. The game is therefore an exclusive membership game, as defined in the previous section.
We divide the strategies in two categories of decisions:

1. Decisions to invest or not;
2. Decisions to propose a joint investment.

The coalition formation games discussed in Section 2.1, do not have this first option. These games simply model all different coalition structures possible. What the strategy set exactly looks like depends on which game is played. A sequential game can give different strategies than a simultaneous game. Why is explained in Section 2.2.1. At first we look at what the outcomes and payoff functions should look like, because this will not depend on the rules of the game.

The set of outcomes of the game should include all possible combinations of joint investments combined with separate investments and MNOs who do not invest. In a two player game, a market with only two MNOs, there are the following outcomes:

(u_1)	None of the MNOs invests .
(u_2)	Only MNO 1 invests.
(u_3)	Only MNO 2 invests.
(u_4)	Both MNO 1 and MNO 2 invest separately.
(u_5)	MNO 1 and MNO 2 invest jointly.

From these possible outcomes it is easy to see that the game should be modelled as a game with spillovers. The outcome u_2 describes the case where the coalition $\{1\}$ of MNO 1 is formed. If the worth of this coalition wouldn't be influenced by the presence of other coalitions in the market, u_4 would be the same outcome as this one, from the perspective of MNO 1. Since we do make a distinction between these situations, spillovers are taken into account.
In a three person game, the outcome space grows:

(u_1)	None of the MNOs invests.
(u_2)	Only MNO 1 invests.
(u_3)	Only MNO 2 invests.
(u_4)	Only MNO 3 invests.
(u_5)	MNO 1 and MNO 2 invest separately, MNO 3 doesn't invest.
(u_6)	MNO 1 and MNO 3 invest separately, MNO 2 doesn't invest.
(u_7)	MNO 2 and MNO 3 invest separately, MNO 1 doesn't invest.
(u_8)	All MNOs invest separately.
(u_9)	MNO 1 and MNO 2 invest jointly, MNO 3 doesn't invest.
(u_{10})	MNO 1 and MNO 3 invest jointly, MNO 2 doesn't invest.
(u_{11})	MNO 2 and MNO 3 invest jointly, MNO 1 doesn't invest.
(u_{12})	MNO 1 and MNO 2 invest jointly, MNO 3 invests alone.
(u_{13})	MNO 1 and MNO 3 invest jointly, MNO 2 invests alone.
(u_{14})	MNO 2 and MNO 3 invest jointly, MNO 1 invests alone.
(u_{15})	All MNOs invest jointly.

For games with more players, the outcomes can be stated in the same way. From the way the possible outcomes are stated one can see that the number of possible outcomes increases fast when another player is added. For a four player game the number of unique outcomes is 51 and for a five player game this number becomes 233.

The payoff functions should give a value to each of the outcomes for each MNO. This value should tell what the MNO wins or loses when the market changes to the scenario described by the outcome. To specify this value for each MNO and to each outcome, will be hard. The payoff functions depend on numerous factors such as the MNOs market position, customer relationship, strategy, resources etc (see Section 1.3). All these factors should be taken into account when calculating the value of a sharing scenario, while giving a value to the separate factors is already a rough guess. To avoid this valuation process we define the payoff function in a different way. If we play a non-cooperative game this is possible. In a non-cooperative game the objective is for each player to maximize his own payoff; i.e. to get the outcome with the highest value possible. The value of the outcome tells us two things:

1. which outcome is better,
2. and how much better.

If we decide to lose the second purpose of the value, it is possible to forget about the actual values. The only thing we need to know for each two outcomes, is which one is better, but not how much better. So for each player the payoff function is given by an order of preference on the different outcomes. In the two player game the payoff function for MNO 1 could look like this:

$$u_2 > u_5 > u_4 > u_1 > u_3.$$

MNO 1 perceives that he profits the most from the situation where he is the only one that invests in LTE technology, followed by the case where he jointly invests with MNO 2 in a network. The third best scenario is when they both invest separately, followed by the scenario where they both make no investment. The least valuable scenario for MNO 1 is when MNO 2 is the only one who invests. The function only tells that u_2 is more valuable than u_5, but not how much more.
By formulating the payoff functions in this way, we can avoid having to determine all values of the outcomes. As a start this is a good way to analyse the different scenarios without assuming too much about the value of outcomes. We will refer to these functions as preference functions.

Remark: *The preference functions can be seen as a partition function, as well as a valuation. This depends on the way that the players determine the preference order of the coalitional structures. If they determine the preference order based on the total value of each coalitional structure, the game will be in partition function form. If they are able to determine the individual value they receive in each coalitional structure, the preference function is a valuation.*

2.2.1 Strategy set

As stated before, the sequential game is played with a different strategy set than the simultaneous game. In a simultaneous game all players make a decision as the same time, without knowing anything about the decisions of the other players. In a sequential game there are multiple stages and players react to each other. In order to model the situation in a realistic way there are a few things that have to be taken into account.
For instance, when a player proposes the coalition of his first choice and a prospective coalition member refuses the proposal, the player should have the option to try to get his second choice. In a sequential game this is easy to model, because the player can propose

his second choice in a later stage of the game. But in a simultaneous game we have to be more careful. In the game Γ, the result will be that the player will always end up alone in a coalition and in the game Δ the player ends up with all other players that proposed the same coalition (2.1.2). But this doesn't have to be the second choice of the player. In order to make sure that a player has more influence on getting his second choice when the first choice is not possible, players can also make a decision about what happens when a proposal is rejected. This will be a combination of the game Γ and Δ. Instead of determining beforehand what will happen after a proposal is rejected, the players can choose what will happen. So a player will not only propose a coalition, but will also make a choice between Γ, Δ and X, where X stands for making no investment. When a player chooses X, and one member of the proposed coalition deviates from this coalition, the player will end up not investing in LTE. When a player chooses Γ, and one member of the proposed coalition deviates from this coalition, the player will end up in a singleton coalition, i.e. investing alone. When a player chooses Δ, and one member of the proposed coalition deviates from this coalition, the player will end up in a coalition with all members that proposed the same coalition and chose Δ.

To explain the strategies in more detail, we give the example of the three player game. We call the players in this game player A, B and C and their strategy sets S_A, S_B resp. S_C. These sets are given by

$$\begin{aligned} S_A &= \{X, A, AB(X), AC(X), AB(\Gamma), AC(\Gamma), ABC(X), ABC(\Gamma), ABC(\Delta)\}, \\ S_B &= \{X, B, AB(X), BC(X), AB(\Gamma), BC(\Gamma), ABC(X), ABC(\Gamma), ABC(\Delta)\}, \\ S_C &= \{X, C, AC(X), BC(X), AC(\Gamma), BC(\Gamma), ABC(X), ABC(\Gamma), ABC(\Delta)\}. \end{aligned}$$

Table (2.1) explains the definition of these strategies for player A. The definition can be stated in the same way for player B and C.

S_A	
X	Do not invest
A	Invest alone in the technology
$AB(X)$	Propose a joint investment with player B, else do not invest.
$AC(X)$	Propose a joint investment with player C, else do not invest.
$AB(\Gamma)$	Propose a joint investment with player B, else invest alone.
$AC(\Gamma)$	Propose a joint investment with player C, else invest alone.
$ABC(X)$	Propose a joint investment with player B and C, else do not invest.
$ABC(\Gamma)$	Propose a joint investment with player B and C, else invest alone.
ABC(Δ)	Propose a joint investment with player B and C, else invest with all players who made the same proposal.

Table 2.1: Strategies player A

So when a player proposes a coalition, he also proposes what will happen when a prospective member refuses. In this way the game will be a variation on the simultaneous exclusive membership games, explained in Section 2.1.2. In general the number of strategies, $|S|$ is given by

$$|S| = 2 + 2 \cdot \binom{n-1}{1} + 3 \sum_{i=2}^{n-1} \binom{n-1}{i}$$

for $n \geq 2$. This means that in a 4 player game, each player has 20 strategies. All combinations of these strategies give $20^4 = 160.000$ outcomes. For a 5 player game the number of strategies grows to 45, which will give $45^5 \approx 185 \cdots 10^6$ outcomes. In section

3.2 becomes clear how the fast growing number of strategies, and the even faster growing number of outcomes, causes difficulties in solving games with more than three players.

Remark: *The correct formulation requires also the strategies $AB(\Delta)$ and $AC(\Delta)$. However, these strategies are the same as $AB(\Gamma)$ respectively $AC(\Gamma)$. This is always the case when a player proposes a coalition with two players. When player A proposes $AB(\Delta)$ and player B refuses, then player A's second choice will be a coalition with all players that proposed the same coalition. Because player A is the only player who proposed AB, this means that his second choice is the singleton coalition. When player A proposes $AB(\Gamma)$ his second choice is also the singleton coalition. Therefore these two options are the same. In Appendix D on can find an example for the three player game that illustrates the difference between a game with these strategies and the game Δ and Γ.*

In Section 3.2 we will describe the simultaneous model in more detail.

In a sequential game the strategy set is different, and consists of two categories of strategies. The strategy set for the three player game is given by:

$$\begin{aligned} S_A &= \{X, A, AB, AC, ABC\} \cup \{y, n\} \\ S_B &= \{X, B, AB, BC, ABC\} \cup \{y, n\} \\ S_C &= \{X, C, AC, BC, ABC\} \cup \{y, n\}. \end{aligned}$$

Except for the strategy, X, doing nothing , this is the same strategy set that is used in the model from Bloch and Ray and Vohra [5], [23]. The set $\{y, n\}$ consists of the answers yes and no to a proposed coalition and the remaining sets consist of proposals. Section 3.1 describes the sequential game in more detail.

3

Non-cooperative games

This chapter describes the sharing game as a non-cooperative game. Non-cooperative games are solved by finding equilibria, i.e. solutions, were no player will deviate from. To determine these equilibria, it is necessary to assume that there is complete information. This means that all players know all payoff functions. This assumption doesn't reflect a realistic situation, because MNOs do not know payoff functions of other MNOs, (although the formulation as a preference function will make it easier to predict these functions). But the models used in the next sections are not supposed to describe the actual decision process, but to give some insights where the decision process can be based on.

The first section considers the sequential model. We will conclude that the sequential game isn't a good model. The second section explains the simultaneous model. In the last section several scenarios of the sharing game will be analysed using the simultaneous model.

3.1 Non-cooperative sequential game

In this chapter we show what the sequential game looks like. Intuitively this model will be more realistic, because decisions about sharing will not be made simultaneous, but players react to each other. In a sequential game, there are multiple stages in which players make decisions. The description is usually in the form of a tree, called extensive form representation. At first we consider the game where the number of players $N = 2$. Assume that the two players, player A and player B, have strategy sets $S_A = \{X, A, AB\} \cup \{y, n\}$ resp. $S_B = \{X, B, AB\} \cup \{y, n\}$. Here the sets $\{X, A, AB\}$ and $\{X, B, AB\}$ are proposed coalitions and the set $\{y, n\}$ consists of the reaction yes or no to a proposal. The strategy set is therefore the same as Bloch used in his sequential game [5], with the additional strategy X of doing nothing. The outcomes of the game are given by the following set: U = {{.}; {A}; {B}; {A}{B}; {AB} }, again using the same notation as Bloch. The additional outcome {.} means that none of the MNOs invests. The difficulty that it is not clear in which order players react to each other is of little importance in the two player game. Because there are only two players the order is only relevant in that is specifies

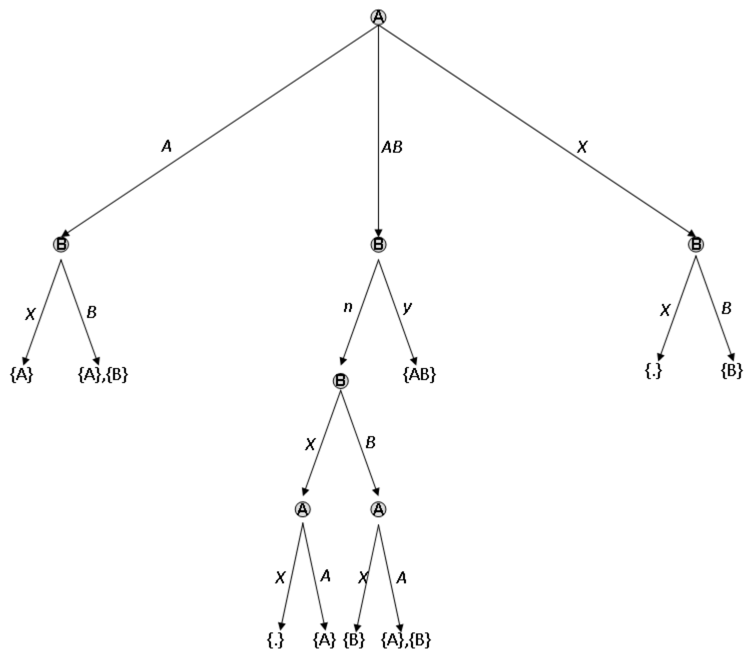

Figure 3.1: Decision tree for the 2 person game

which player makes the first proposal. Bloch [5] and Ray and Vohra [23] just assume a fixed order in which players make proposals. Following them we will assume that player A starts the game (the assumption that player B starts will give the same result). Figure 3.1 gives the corresponding decision tree for this game.

The payoff off functions are again given by preference functions. Take for example the preference functions p_A and p_B in table (3.1), where the preference order is from top to bottom.

p_A	p_B
{A}	{B}
{A}{B}	{AB}
{AB}	{.}
{.}	{A}{B}
{B}	{A}

Table 3.1: Preference functions two player extensive form game

Extensive form games can be solved with backward induction. By starting in the last stage, and walking back up, one can determine for every node which strategy the corresponding player will choose given the outcome of the next stage. In figure (3.2) the blue lines represent these strategies. In stage 4 player A prefers $\{A\}$ over $\{.\}$ and $\{A\},\{B\}$ over $\{B\}$. Based on this preference player B will play the strategy B in stage 3. Continuing the analysis will lead to the conclusion that player A should choose A and player B should react by playing B. The result of the game is {A}{B}; i.e. both MNOs will invest separately in the new technology.

It turns out that the two player game is the only game that can be solved easily with

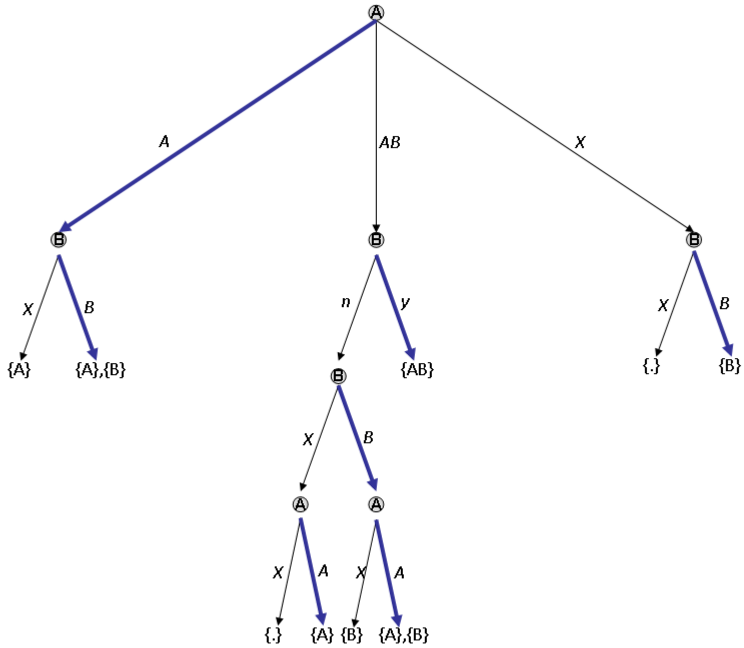

Figure 3.2: Solution for the 2 person game

a sequential model. When one tries to draw the decision tree for three players, the tree becomes very large. Figure (3.3) gives a sub tree of the tree. Again an assumption is made about the order in which the players react to each other: A first, B second and C last. When player A starts he has the option to propose the coalitions A,X,AB,AC and ABC. Figure (3.3) gives the part of the tree after player A proposed the coalition AC, so the total tree will be at least three times this big (proposals A and X will result in a smaller tree).

The common method to determine equilibria is solving the tree with backward induction. It will therefore be necessary to draw the tree. In fact one has to draw as many tree as there are different orders of players, if one wants to take all possibilities into account. We are looking for a model that is easy to use for regulators and MNOs. We conclude that the sequential game does not satisfy this requirement.

3.2 Non-cooperative simultaneous game

In this section we will explain the formulation as non-cooperative simultaneous game. A simultaneous non-cooperative game with complete information is called a normal form game. As stated before, the assumptions of a normal form game do not reflect the actual game that is played in practice. MNOs will not make their decisions simultaneously and independently and they do not have enough information about their competitors to know their payoff functions. However, the representation as a normal form game is very useful, because it gives information about the stable outcomes of the game, at least for the independent initial positions of the MNOs. Based on these outcomes MNOs can make decisions about cooperation with other MNOs. This last part will be illustrated in the

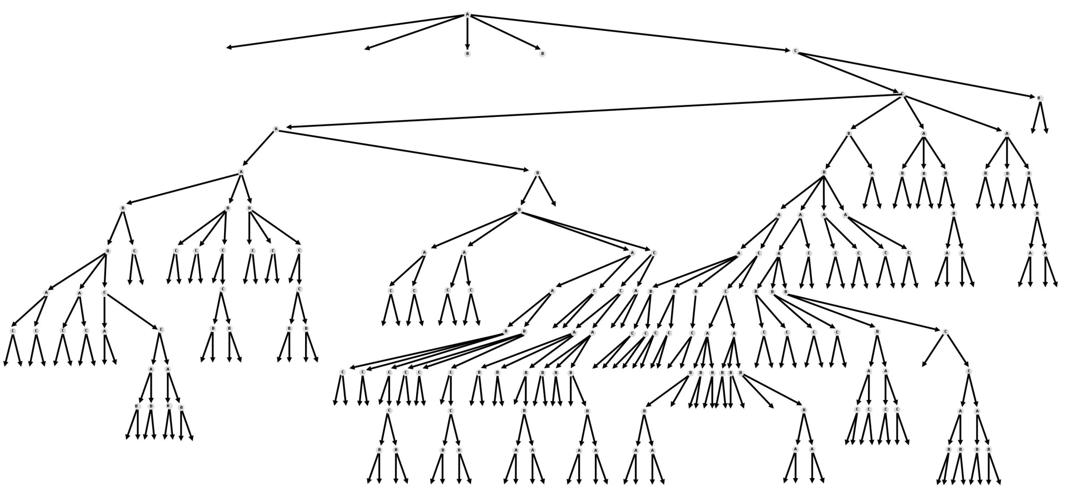

Figure 3.3: Sub tree of the 3 person game

last section of Chapter 3.3. The next two sections explain the model for the two and three player games respectively.

3.2.1 Two player game

The two person is given by player A and B, with strategy sets

$S_A = \{X, A, AB(X), AB(\Gamma)\}$ resp. $S_B = \{X, B, AB(X), AB(\Gamma)\}$.

Table (3.2) explains the different strategies.

Strategy A	Strategy B	
X	X	Do nothing.
A	B	Invest in the technology alone.
$AB(X)$	$AB(X)$	Propose a joint investment to the other player, if rejected do nothing
$AB(\Gamma)$	$AB(\Gamma)$	Propose a joint investment to the other player, if rejected invest alone.

Table 3.2: Strategies two player simultaneous game

The first two strategies are decisions whether to make an investment and the last two strategies are decisions to propose a joint investment, as defined before. The combinations of the strategies give the following set of outcomes of the game: U = $S_A \times S_B$ = {(X,X), (A,X), (X,B), (A,B), (A&B) }. Table (3.3) explains the different outcomes.

Outcome	
(X,X)	They both do nothing.
(A,X)	Player A invests and player B does nothing.
(X,B)	Player A does nothing and player B invests.
(A,B)	Both players invest separately.
(A&B)	They jointly invest in the technology.

Table 3.3: Outcome of two player extensive form game

Normal form games are represented by matrices, in the network sharing game this will give the following matrix representation:

$$\begin{array}{l|llll} & X & A & AB(X) & AB(\Gamma) \\ \hline X & (X,X) & (A,X) & (X,X) & (A,X) \\ B & (X,B) & (A,B) & (X,B) & (A,B) \\ AB(X) & (X,X) & (A,X) & (A\&B) & (A\&B) \\ AB(\Gamma) & (X,B) & (A,B) & (A\&B) & (A\&B), \end{array} \tag{3.1}$$

where can be seen which combination of strategies leads to which outcome. (Outcomes are denoted between brackets, in order to distinguish them from strategies.) As stated before, all outcomes have a different value for player A and B, given by their payoff functions. The game can be solved by determining Nash equilibria. With the following algorithm all equilibria can be calculated, once you know the preference functions for A and B.

Algorithm: Equilibria for 2 player game

```
For i = 1 To 4
   For j = 1 To 4
      Equilibrium(i, j) = TRUE
      For l = 1 To 4
         If l ≠ j Then
            If OutcomeA(i, j) < OutcomeA(i, l) Then
               Equilibrium(i, j)= FALSE
               Exit For
            End If
         End If
         If l ≠ i Then
            If OutcomeB(i, j) < OutcomeB(l, j) Then
               Equilibrium(i, j)= FALSE
               Exit For
            End If
         End If
      Next l
   Next j
Next i
```

The index i denotes the rows of the matrix (3.1), i.e. the strategies player B can choose and the index j denotes the columns of (3.1), player A's strategies. The algorithm compares an element (i, j) with other elements in the same row (i, l), based on the preference function of player A and with other elements in the same columns based on the preference function of player B. As long as the element (i, j) is better, the comparison continues, if not, the algorithm concludes that (i, j) is not an equilibrium and moves to the next element.
A logical assumption about the preference functions in a two person game is that for player A the order

$$(A, X) > (A, B) > (X, B)$$

has to be respected and for player B the order

$$(X, B) > (A, B) > (A, X),$$

because owning LTE technology gives a competitive advantage and each player prefers less competition. With this assumption, the number of preference functions for each player is reduced to 20. See appendix A for an overview of these functions. We used the algorithm to calculate all equilibria for all combinations of the preference functions in the appendix. After analysing the equilibria we can conclude the following:

- All combinations of preference functions lead to the equilibrium (A,B). If one looks at the second row and column in the matrix (3.1) this is easy to see. Both players will never deviate from the equilibrium (2,2) = (A,B), because player A can only deviate to the solution (X,B) and player B can only deviate to (A,X). All preference functions of player A respect the order (A,B) > (X,B) and all functions of B respect the order (A,B) > (A,X).
- Only when (A&B) is preferred over (A,X) by player A and (X,B) by player B, this outcome will be an equilibrium. Also this can be seen by looking at matrix (3.1). For all elements (A&B) in the matrix, there is an element (A,X) or (B,X) in the same row or in the same column.
- Only when (X,X) is preferred over (A,X) by player A and over (X,B) by player B, this outcome will be an equilibrium. By looking at the first row and column of matrix (3.1) this becomes clear. Only if this is the case both players will not deviate from the equilibrium (1,1) = (1,3) = (3,1) =(X,X).
- The outcomes (A,X) and (X,B) will never be equilibria. For these outcomes it holds that (A,B) is in the same row and column respectively. Because the preference functions are such that (A,B) > (X,B) for player A and (A,B) > (A,X) for player, it will always be the case that either player A or player B will deviate to (A,B).

From these notions we can conclude that in case the preference functions describe this realistic situation, the game is not very interesting. All equilibria can be determined directly by looking at the preference functions. Therefore we will focus on a game with three players in the next section.

3.2.2 Three player game

In the Netherlands as well as in other countries in Europe, there are three MNOs on the mobile market [15]. So the three player game will be applicable in several markets.
To describe the three person game we will call the players A, B and C. We assume that the players are not symmetric. This is a logical assumption, because there are no markets where all MNOs are the same (follow the exact same strategies, have the same resources, have the same amount of funds, etc). We rank the MNOs by the amount of funds they can invest in building a new network, so player A is the 'biggest' and player C is the 'smallest'. Assuming this, will make the formulation of the strategy set and preference functions more straightforward. As a reminder, the sets of available strategies in this game are given by

$$\begin{aligned}
S_A &= \{X, A, AB(X), AC(X), AB(\Gamma), AC(\Gamma), ABC(X), ABC(\Gamma), ABC(\Delta)\},\\
S_B &= \{X, B, AB(X), BC(X), AB(\Gamma), BC(\Gamma), ABC(X), ABC(\Gamma), ABC(\Delta)\},\\
S_C &= \{X, C, AC(X), BC(X), AC(\Gamma), BC(\Gamma), ABC(X), ABC(\Gamma), ABC(\Delta)\}.
\end{aligned}$$

where Table (3.4) explains the different strategies.

S_A	S_B	S_C	
X	X	X	Do not invest
A	B	C	Invest in the technology alone
$AB(X)$	$AB(X)$	$AC(X)$	Propose a joint investment to the biggest player, else do not invest
$AC(X)$	$BC(X)$	$BC(X)$	Propose a joint investment to the smallest player, else do not invest
$AB(\Gamma)$	$AB(\Gamma)$	$AC(\Gamma)$	Propose a joint investment to the biggest player, else invest alone
$AC(\Gamma)$	$BC(\Gamma)$	$BC(\Gamma)$	Propose a joint investment to the smallest player, else invest alone
$ABC(X)$	$ABC(X)$	$ABC(X)$	Propose a joint investment to both other players, else do not invest
$ABC(\Gamma)$	$ABC(\Gamma)$	$ABC(\Gamma)$	Propose a joint investment to both other players, else invest alone
ABC(Δ)	ABC(Δ)	ABC(Δ)	Propose a joint investment to both other players, else invest with all players who made the same proposal.

Table 3.4: Strategies three player simultaneous game

As stated before, the combination of these strategies leads to fifteen different outcomes. We use the notation given by:

$$\begin{aligned} U = S_A \times S_B \times S_C = \quad & \{(X,X,X),(A,X,X),(X,B,X),(X,X,C),(A,B,X),(A,X,C),\\ & (X,B,C),(A,B,C),(A\&B,X),(A\&C,X),(B\&C,X),(A\&C,B)\\ & (B\&C,A),(A\&B\&C)\}. \end{aligned} \tag{3.2}$$

Table (3.5) explains the different outcomes.

Outcome	
(X,X,X)	All three do not invest
(A,X,X)	Player A invests, B and C do not invest
(X,B,X)	Player B invests, A and C do not invest
(X,X,C)	Player C invests, A and B do not invest
(A,B,X)	Player A and B invest separately, C does not invest
(A,X,C)	Player A and C invest separately, B does not invest
(X,B,C)	Player B and C invest separately, A does not invest
(A,B,C)	All three players invest separately
(A&B,X)	Player A and B invest jointly, C does not invest
(A&C,X)	Player A and C invest jointly, B does not invest
(B&C,X)	Player B and C invest jointly, A does not invest
(A&B,C)	Player A and B invest jointly, C invests alone
(A&C,B)	Player A and C invest jointly, B invests alone
(B&C,A)	Player B and C invest jointly, A invests alone
(A&B&C)	All three players invest jointly

Table 3.5: Outcomes three player game

A three-player strategic game is usually presented as a collection of matrices, with one player choosing the row, the second the column, and the third the matrix. The representation can be found in appendix B.

Each outcome has a different value for each player, given by the payoff functions p_A, p_B, p_C. These have to be defined for each player. As before, these functions give a preference order on the possible outcome of the game. Because there are 15 possible outcomes, every player can have 15! different preference functions. It would be impossible to test all functions. Moreover this wouldn't be very interesting, because most of these preference functions do not describe a realistic situation. In order to describe realistic situations we assume that the following has to hold.

- Because of the assumption that the new technology gives a competitive advantage to the investors, a player doesn't prefer the situation where the competition (jointly or separately) invests in LTE technology, while he does not. For example, the situation where player A prefers (B&C,X) over (B&C,A) does not occur.
- Less competition is always better, therefore all players will prefer the situation where the smallest number of competitors invest. The players also prefer competition from weak over strong competitors. So for player A the order (A,X,X) > (A,X,C) > (A,B,X) > (A,B,C) has to be respected, as well as the orders : (A&B,X) > (A&B,C) and (A&C,X) > (A&C,B).

To determine all possible functions the strategies were divided into categories: alone, minimal sharing, maximal sharing and not investing at all. Minimal sharing means that the player prefers sharing with smaller players and with just one other player over sharing with big players and both other players. Maximal sharing means exactly the opposite. All logical combinations of these categories resulted in twelve different functions. The interpretation of the functions is the same for each player, and is given by:

1. Alone > Minimal Sharing > Not: The players' first choice is to invest in the network alone. Even if the other two players decide to share the investment, the player prefers investing alone over sharing with other players. The players' second choice is minimal sharing and his last choice is not to share at all.
2. Alone > Minimal Sharing > Others share > Not : This is the same preference function as the first one, accept for the fact that the player does not prefer investing in the network alone in case the other players share. Then he prefers sharing to be minimal sharing.
3. Alone > Maximal Sharing > Not: The players' first choice is to build the network alone. Even if the other players share, the player prefers building alone over sharing with other players. The players' second choice is maximal sharing and his last choice is not to share at all.
4. Alone > Maximal Sharing > Others share > Not: This is the same preference function as the third one, accept for the fact that the player does not prefer building the network alone in case the other players share. Then he prefers sharing to be maximal.
5. Minimal Sharing > Alone > Not: The players' first choice is to share minimally. His second choice is to build a network alone and his last choice is to do nothing.
6. Maximal Sharing > Alone > Not: The players' first choice is to share maximally. His second choice is to build a network alone and his last choice is to do nothing.
7. Minimal Sharing > Not > Alone > Others share: The players' first choice is to share minimal. His second choice is to do nothing, accept when the other two players decide to share, then he prefers building a network alone.
8. Maximal Sharing > Not > Alone > Others share: The players' first choice is maximal sharing. His second choice is to do nothing, accept when the other players decide to share, then he prefers building the network alone.

9. Not > Minimal Sharing > Others share > Alone: The player prefers to do nothing, accept when the other players share, then he prefers sharing minimal. His last choice is building the network alone.

10. Not > Maximal Sharing > Others share > Alone: The player prefers to do nothing, accept when the other players share, then he prefers sharing maximal. His last choice is building the network alone.

11. Not > Alone > Others do something > Minimal Sharing: The players' first choice is that nobody does anything. Accept when the other players decide to share, then he prefers to build a network alone. His last choice is sharing minimal.

12. Not > Alone > Others do something > Maximal: The players' first choice is that nobody does anything. His second choice is to build alone. His third choice is that he does nothing, but the others are building a network. After this he prefers sharing maximal over sharing minimal.

Appendix A gives the corresponding preference functions for the three players. The interpretation of the functions will be the same for each player, but the description of the order might be different due to the difference in size and preferences of the players.
The solution of the game is calculated with help of an algorithm, see appendix C for a detailed description. In basic terms the algorithm looks like this:

```
For i = 1 To 9
  For j = 1 To 9
    For k = 1 To 9
      Equilibrium(i, j, k) = TRUE
      For l = 1 To 9
        If l ≠ k Then
          If RankA(i, j, k) < RankA(i, j, l) Then
            Equilibrium(i, j, k)= FALSE
            Exit For
          End If
        End If
        If l ≠ j Then
          If RankB(i, j, k) < RankB(i, l, k) Then
            Equilibrium(i, j, k)= FALSE
            Exit For
          End If
        End If
        If l ≠ i Then
          If RankC(i, j, k) < RankC(l, j, k) Then
            Equilibrium(i, j, k)= FALSE
            Exit For
          End If
        End If
      Next l
    Next k
  Next j
Next i
```

The index i gives the index of the matrix, j of the row and k of the column. Every element (i, j, k) is compared with other elements in the same column (i, j, l) according to the preference of player A, in the same row (i, l, k) according to the preference of player B and with elements at the same place in another matrix (l, j, k) according to the preference of player C. As long as the element (i, j, k) is better, the comparison continues, else is concluded that element (i, j, k) is not an equilibrium (Equilibrium(i, j, k) = False). If there is no element that is better, Equilibrium (i, j, k) remains true and the algorithm concludes that (i, j, k) is a Nash-Equilibrium.

Remark: *To find an equilibrium, one has to compare a large number of all the outcomes with each other. The number of outcomes grows fast when more players are added (see Section 2.2.1). Therefore the computational time will also grow fast.*

3.3 Analysis of the three player game

In this section it is shown how the model can be used to analyse situations. With the algorithm the stable situations (equilibria) in a market with three different MNOs, given their sharing preferences, can be determined. These stable situations will not tell much about what will happen to the business model of the mobile market, because of the assumptions that are necessary to play a normal form game. However, the algorithm makes it possible to quickly analyse many different market structures and many different scenarios with respect to outcome preferences. This gives good insights into the initial positions of the MNOs towards sharing. These insights can be helpful to determine a good strategy and make the decision process more efficient. Also, some of the equilibria are of regulatory interest as they induce 'conspiratorial' behaviour, or suggest end-states that are easily improved if cooperation is stimulated or forced. The algorithm is helpful in identifying such situations.

The section begins with an example to explain that in contrary to the two player game, the game cannot be solved by just examining the preference functions. After this a short overview is given of some conclusions that can be deduced immediately after running the algorithm for all combinations of the preference function given in (3.2.2). The three examples that follow illustrate how the model can be used. To do this we sketch possible combinations of different MNOs in the same market and examine what will be the outcome when the MNOs have to make a decision about investing in LTE and network sharing.

In the two player game most of the times it was easy to see by just looking at the preference functions what the equilibrium/equilibria should be. This isn't straightforward for the three player game. The following example will illustrate this.

Example 3.1 (Unexpected Equilibria)*: Assume that the players have the following preference functions*

Running the algorithm gives the results stated in Table 3.7. This includes all equilibria and all corresponding strategies that each player should play in order to reach this equilibrium. For example, the first row states that the equilibrium (A&B&C) can be reached when all players play a combination of the strategies $ABC(X)$, $ABC(\Gamma)$ and $ABC(\Delta)$. The strategies corresponding to the last equilibrium (X,X,C) are divided over three rows, because not all combinations of these strategies will lead to the equilibrium (X,X,C). The combination where players A and B play $ABC(X)$ and player C plays $ABC(\Gamma)$ will lead to the equilibrium (A&B&C).

p_A	p_B	p_C
(A&B&C)	(A&B&C)	(B&C,X)
(A&B,X)	(A&B,X)	(A&C,X)
(A&C,X)	(B&C,X)	(B&C,A)
(A&B,C)	(A&B,C)	(A&C,B)
(A&C,B)	(B&C,A)	(A&B&C)
(X,X,X)	(X,X,X)	(X,X,C)
(X,X,C)	(X,X,C)	(X,B,C)
(X,B,X)	(A,X,X)	(A,X,C)
(X,B,C)	(A,X,C)	(A,B,C)
(A,X,X)	(X,B,X)	(A&B,C)
(A,X,C)	(X,B,C)	(X,X,X)
(A,B,X)	(A,B,X)	(X,B,X)
(A,B,C)	(A,B,C)	(A,X,X)
(B&C,A)	(A&C,B)	(A,B,X)
(B&C,X)	(A&C,X)	(A&B,X)

Table 3.6: Preference functions example (3.1)

The columns titled Rank A, B and C give the position of the equilibrium outcome in the preference ordering for each player. So if Rank A is 3 for a certain equilibrium, this means that this equilibrium is player A's third favoured outcome.

Equilibria	RankA	RankB	RankC	Strategy A	Strategy B	Strategy C
(A&B&C)	1	1	5	ABC(X), ABC(Γ), ABC(Δ)	ABC(X), ABC(Γ), ABC(Δ)	ABC(X), ABC(Γ), ABC(Δ)
(A&B,C)	4	4	10	AB(X), AB(Γ)	AB(X), AB(Γ)	C, AC(Γ), BC(Γ), ABC(Γ), ABC(Δ)
(A&C,B)	5	14	4	AB(X), AB(Γ)	B, ABC(Γ)	AC(X), AC(Γ)
(B&C,A)	14	5	3	A, AC(Γ), ABC(Γ)	BC(X), BC(Γ)	BC(X), BC(Γ)
(X,X,C)	7	7	6	X	X, ABC(X)	C, ABC(Γ)
				X, ABC(X)	X	C, ABC(Γ)
				X, ABC(X)	X, ABC(X)	C

Table 3.7: Result (8,8,5)

One could see from the preference function that (A&B&C) should be an equilibrium. But the other four are not that obvious. There are even two equilibria where a player gets his last but one choice. One wouldn't be able to predict this by just looking at the preference function.

The algorithm was used to calculate the Nash-equilibria for every combination of the twelve preference functions. Some results are worth taking a closer look at. Other results appear to be trivial. For example, almost every combination of the first four preference functions will result in the equilibrium (A,B,C); i.e. every MNO invests in the technology alone. This is of course trivial, because in these preference functions each MNO prefers to invest alone over everything else. It is questionable if this situation will arise in real life, because this would assume that for all three players the business case of building alone is better than the case for sharing. Particularly for a player with a smaller market share this seems unlikely.
Another trivial situation is when all MNOs prefer to cooperate with all other MNOs (preference functions 6 or 8). The outcome will then of course be that they will all

cooperate: (A&B&C). Finally when all MNOs prefer to do nothing (preference functions 9, 10, 11 or 12) the result will be that they all do nothing (X,X,X).
Besides these trivial situations, there are also a lot of situations that are very much alike. Therefore it is not useful to analyse each and every situation.

In the following subsections, a few examples are discussed that give interesting results or describe a recognizable situation. Because there is more than one equilibrium in most scenarios, it is necessary to analyse the results given by the algorithm. If you look at the outcome of the game as a starting point for negotiation, it is important to take a closer look at the different equilibria. The following examples show how this works.

3.3.1 Example 1: Market leader without influence

In this example we look at situation where the players have the following preference functions:

Player A: (1) Alone > Minimal > Nothing

Player B: (5) Minimal > Alone > Nothing

Player C: (8) Maximal > Nothing > Alone > Others share

These preference functions could for instance describe the following mobile market. The market has three mobile operators that have to make a decision about investing in LTE. Player A could be the market leader that pursues operational excellence (for the definition of this discipline see Table 1.1) and wants to maintain full control while requiring the new technology. This MNO is perfectly able to invest in the technology alone and to compete with the other MNOs in case they are investing separately in the technology as well. His second choice is to cooperate with another MNO, preferably just one, the smaller of the two. In this way, he eliminates one competitor and could continue to pursue operational excellence. His least favourite choice will be to do nothing; the MNO has to find other ways to adjust to the latest technology in order to stay market leader. The second MNO (player B) is also capable of investing in LTE-technology alone, but has fewer resources to do this and to compete in the mobile market. This MNO will therefore need a little help from another MNO; he will prefer to share investment costs with another MNO (a small one). In this case he will be able to compete with the market leader. Cooperation will give him a better perspective than investing in a LTE alone, but he will still prefer investing over doing nothing. If he does not adjust to the latest technology, he will lose market share. The smallest MNO prefers sharing in a maximal way. This MNO pursues customer intimacy (1.1) and could focus more on this if his competitors wouldn't be able to differentiate in another way. Besides that, this MNO has little resources to invest alone and needs as much help as possible.

Table (3.8) gives an overview of the result of the game played by these MNOs.

In this game there are two Nash-Equilibria: (A,B,X) and (B&C,A). The first equilibrium is the situation where the two biggest MNOs invest alone and the smallest MNO does nothing and the second is the situation where the biggest MNO invests alone and the smallest MNOs cooperate. In these situations none of the MNOs will change their decision (individually), because none of them can do better. Because there are two equilibria, this does not give an exclusive outcome of this sharing scenario. We have to take a closer look at the actual valuation of the outcomes and the company strategies to determine what might happen here.

Equilibria	RankA	RankB	RankC	Strategy A	Strategy B	Strategy C
(B&C,A)	5	3	4	A, AC(Γ), AB(Γ)	BC(X),BC(Γ), ABC(Δ)	BC(X),BC(Γ), ABC(Δ)
				A,ABC(Γ), AC(Γ), AB(Γ)	BC(X),BC(Γ)	BC(X),BC(Γ), ABC(Δ)
				A,ABC(Γ), AC(Γ), AB(Γ)	BC(X),BC(Γ), ABC(Δ)	BC(X),BC(Γ)
(A,B,X)	3	8	9	A	B,AB(Γ), ABC(Γ)	X,AC(X), ABC(X)
				A,ABC(Γ)	B,AB(Γ)	X,AC(X), ABC(X)
				A,ABC(Γ)	B,AB(Γ), ABC(Γ)	X,AC(X)

Table 3.8: Results for $(1, 5, 8)$

In both equilibria player A chooses the strategy A, i.e. investing alone. Therefore we can say that player A has no influence on which equilibrium the result will be. Player B either plays B, i.e. invests alone or plays BC, i.e. chooses to propose a joint investment to player C. This means that player B has a big influence on the outcome of the game. If he plays B, the equilibrium will be (A, B, X) and neither player A or C can do anything about this. If he plays BC player C still has the choice to reject the proposal of B, but he will never do this, because this will only give a worse outcome. Therefore player B is the one who decides which equilibrium will be the result. Because (A, B&C) will give B the best value, player B will choose to play BC.

In a market with these kinds of MNOs a stable situation will be when the biggest MNO invests alone and the smaller MNOs share their network and make joint investment. The MNOs that cooperate will get their third respectively fourth choice and the market leader gets his fifth choice. So the market leader will have the worse result (in terms of ranks).

Again, this is not a reflection of what will actually happen. But knowing the outcome of the situation, MNOs can decide to change the discipline they are pursuing, change their perspective towards sharing, etc..

3.3.2 Example 2: Use of threats to force cooperation

In this example we look at situation where the players have the following preference functions:

Player A: (4) Alone > Maximal Sharing > Others share > Nothing

Player B and C: (6) Maximal > Alone > Nothing.

This could describe a market, where the market leader (player A) pursues product leadership (1.1). Therefore he prefers to invest alone and focuses on the quality of the network. If the other MNOs in this market (player B and C) decide to make a joint investment, it will be harder for the market leader to remain product leader. Therefore in this case he would a joint investment with the other MNOS. By combining their resources the all MNOs will become strong competitors in this area. It is likely that the market leader will change his discipline and pursue customer intimacy (1.1), in order to stay competitive.
The 'smaller' MNOs in this market might already pursue customer intimacy. By sharing a network and therefore sharing costs, they can focus more on this strategy. They would

Equilibria	RankA	RankB	RankC	Strategy A	Strategy B	Strategy C
(A&B&C)	5	1	1	ABC(X),ABC(Γ), ABC(Δ)	ABC(Δ)	ABC(Δ)
(B&C,A)	10	5	4	A, ABC(Γ), AC(Γ)	BC(X), BC(Γ)	BC(X), BC(Γ)
(A,B,C)	4	9	9	A	B,AB(Γ), ABC(Γ)	C, AC(Γ), ABC(Γ)
				A, ABC(Γ)	B,AB(Γ)	C, AC(Γ), ABC(Γ)
				A, ABC(Γ)	B,AB(Γ), ABC(Γ)	C, AC(Γ)

Table 3.9: Results for $(4, 6, 6)$

therefore prefer to share with as many other MNOs as possible. If this isn't possible they would still prefer to build a network, in order to stay competitive in the area of network quality. Table (3.9) gives the results of the game played by these players.

There are three equilibria, (A&B&C) , (B&C,A) and (A,B,C). These equilibria correspond to the situations 'all players share their network an invest', 'the market leader invests alone and the other MNOs share their network and make an investment' and 'all players invest separately'. This is again a game that does not result in a unique equilibrium.

The market leader (player A) prefers the first equilibrium. His corresponding strategy would be to start a network alone: *A*. But with this strategy he risks to end up in equilibrium (B&C,A), his #10 rank. This depends on the strategies of the other MNOs (player B and C). Because the other MNOs prefer (B&C,A) over (A,B,C) they will ensure that this will be the result of the game by choosing the strategy 'proposing *BC*'. But all three MNOs prefer the last equilibrium, (A&B&C), over (B&C,A) and would be better off by playing the corresponding strategies. Player A still prefers the equilibrium (A,B,C) over (A&B&C), but he knows that player B and C can use the threat of playing *BC* against A. If player A decides to play A, player B and C make sure that the outcome of the game becomes (B&C,A). Player A will therefore not deviate from the equilibrium (A&B&C). The result of this situation is they will all cooperate with each other.

So the market leader will not be able to keep his preferred equilibrium. The result will be a market where all MNOs share the network and focus on customer intimacy.

Remark: *It turns out that all combinations where one player prefers to build alone and the others prefer sharing maximally, will result in the equilibrium (A&B&C). This can be seen as the power of uniting. However it is unclear how this type of collaboration is considered by regulators. Depending on the difference between rank #4 and rank #5, this could be viewed as a 'conspiracy' to weaken the position of A. There is one exception, in case the player, who wants to build a network alone, has preference function (1) (Alone > Minimal Sharing > Not). Then he also prefers building a network alone, when the others are cooperating to build a network. Therefore this game will result in the equilibrium (B&C,A). It is not clear at this point under what circumstances the market will have a player with such a preference structure.*

3.3.3 Example 3: A game without results

In the last example the players have the following preference functions:

Player A: (5) Minimal > Alone > Nothing

Equilibria	RankA	RankB	RankC	Strategy A	Strategy B	Strategy C
(A&B&C)	5	1	5	ABC(X),ABC(Γ), ABC(Δ)	ABC(X),ABC(Γ), ABC(Δ)	ABC(X),ABC(Γ), ABC(Δ)
(A&B,C)	4	4	14	AB(X), AB(Γ)	AB(X), AB(Γ)	C,BC(Γ),ABC(Γ)
(A&C,B)	3	10	4	AC(X), AC(Γ)	B,ABC(Γ), AB(Γ)	AC(X), AC(Γ)
(B&C,A)	10	5	3	A, ABC(Γ), AC(Γ)	BC(X), BC(Γ)	BC(X), BC(Γ)
(A,B,X)	8	8	9	A	B, ABC(Γ)	X, ABC(X)
				A, ABC(Γ)	B	X, ABC(X)
				A, ABC(Γ)	B, ABC(Γ)	X

Table 3.10: Results for $(5, 6, 7)$

Player B: (6) Maximal > Alone > Nothing

Player C: (7) Minimal > Nothing > Alone > Others share.

This could describe the situation where market leader (player A) pursues operational excellence (1.1). In order to keep up this strategy he needs to upgrade to the next next generation network but his funding is such that could use a little help of another MNO to realise this. The second largest MNO in this market (player B) has customer intimacy as strategy (see Table 1.1). In order to keep up his good services this MNO would like to upgrade as well. He prefers to share the new technology with all other MNOs. In this way the quality of the network will be optimal and he could keep pursuing customer intimacy. The smallest MNO (player C) has little resources to upgrade to the new network, without cooperating with other parties. He would therefore prefer to share the costs. In order to keep up his current strategy of product leadership (1.1), he will prefer to cooperate with just one other MNO. Cooperating with both MNOs will reduce his option to differentiate on product level.

Table (3.10) gives the results of this game. There are five equilibria, (A,B,X), (A&B,C), (A&C,B), (B&C,A) and (A&B&C). Player A prefers (A&C,B), but player B and player C prefer to switch to (B&C,A). They have the power to do this by both proposing BC. From this point, player B can do even better by switching to (A&B,C), player A would also prefer this over (B&C,A) and so both will propose AB. But in this situation both A and C can do better than (A&B,C) by switching to (A&C,B), by both proposing AC. But this is the same situation as the first. So there is no optimal equilibrium.

This game does not lead to a unique equilibrium. Therefore it would be better to look at the best equilibrium for all players together. Without any further information we would break this tie by suggesting equilibrium (A&B&C), because the sum of the ranks for each player is minimum for this equilibrium. This equilibrium has a total rank value of 11, while the other four solutions have total values of 25, 18, 17, and 22.
The players can only come to this solution by cooperating. In markets and situations like this we foresee a role for the regulator. Player B has to give player A and C some kind of incentive, such that these players prefer (A&B&C), over (A&C,B). There are methods in cooperative game theory to calculate such an incentive (such as the Shapley value). To model this, however, you need a payoff function with actual values, instead of a function that only gives an order of preference.

The examples show that even with the restriction to payoff functions without values, you can get more insights on the possible sharing scenarios in different situations. In order to see how good these insights are, in the next chapter we will try to determine payoff functions with actual values for the three MNOs in the Dutch market.

4

Modelling the Dutch market

In this chapter we will model the situation in the Dutch market. To use our model, we will need to know the preference functions for KPN, Vodafone and T-mobile. In order to determine these, we will estimate the value of all different coalition structures in this market for each MNO. There are different factors that influence the value of a structure. We measure this influence by means of costs. Section 4.1 will explain how this works.

The preference functions can be used to draw conclusions about the coalitional structure that will most likely be formed. The model of Chapter 3 is used to calculate which coalitional structures are equilibria. Based on the set of equilibria can be argued which coalitional structure will most likely be the result in the Dutch market.

To determine the quality of the results, a sensitivity analysis is performed. This analysis will also give more insights on which factors that influence the decision making are most important. We will increase and decrease the costs of these factors one by one with different amounts and recalculate the set of equilibria. In this way we can see in which situation we will get a different result.

4.1 Valuation of coalition structures

The valuation of coalitional structures will be based on the costs of these structures for each player. For the valuation we discard all factors that remain equal in all structures and concentrate on the factors that change. This leads to following costs that have to be considered:

(i) Investment in Long Term Evolution technology;

(ii) Operational expenditures of the Radio Access Network (RAN-OPEX);

(iii) Network integration costs;

(iv) Subscriber acquisition and retention costs (SAC/SRC).

The investment in LTE technology is an expense that depends on the coalition that makes this expense. When a coalition makes no investment, these costs do not have to be taken into account. Section 4.1.1 will explain how these investment costs can be calculated.
Network integration costs are only relevant when MNOs share their network, these therefore also depend on the coalition that is formed. In section 4.1.2 these costs are estimated.
RAN-OPEX differs in each coalition, because these expenditures reduce when MNOs share their network. An MNO pays less in coalition with two MNOs than when he is alone in a coalition. Section 4.1.2 explains how these expenditures can be determined and how they change due to network sharing.
The investment in integrating networks and LTE technology as well as RAN-OPEX are factors that are already given as costs. But we also have to take into account that LTE technology gives a competitive advantage and network sharing a competitive disadvantage. These factors are not simply given as an expense. So in order to valuate these factors, we also take subscriber acquisition and retention costs into account. These are the costs that an MNO pays in order to acquire or retain customers. These costs can increase or decrease due to changes in competitive advantage. In contrary to previous factors, these costs do not only depend on the coalition itself, but on the whole coalitional structure. Section 4.1.3 explains in more detail how changes in competitive advantages are valuated by these costs.

Remark: *The investment in LTE technology and the network integration costs are one time investments. The RAN-OPEX and SAC/SRC have to be paid on annual basis. The LTE investment costs and the network integration costs have to be spread over a number of years to compare them with the annual costs. There are several ways to do this, we will divide these costs in equal increments over a certain period.*

4.1.1 Investment in Long Term Evolution technology

The investment in LTE technology is calculated per site that has to be upgraded to the new technology. These costs are estimated by experts to be 100.000 euro per site. We distinguish between coverage sites and capacity sites. Coverage sites are necessary to make sure that a customer is able to make get access to the network in every part of the Netherlands. Capacity sites are necessary to make sure that all customers that want to get access at the same time in a certain area are able to do this. In Amsterdam, for example, it might be enough to have 50 sites to make sure that a customer can get access to the network in every part of the city. But if 100.000 customers want to get access at the same time, 50 sites are not enough to handle the amount of traffic. The extra capacity sites make sure that this will be possible.
In rural parts of the Netherlands, coverage sites are sufficient to serve all customers, while in the urban area, extra sites are necessary. We assume that each MNO wants to upgrade enough sites to have full coverage in the Netherlands. With the Mason BULRIC model [7] we can calculate how many sites are necessary to have full coverage and be able to serve all customers in the Netherlands. LTE technology can be used on different frequencies. We assume that the frequency is 800MHz; the low frequency for LTE. This is the most interesting situation, because for higher frequencies more sites are necessary. So if sharing is interesting for 800MHz, it will also be interesting for higher frequencies. The following table gives the number of sites necessary to serve 100% for 800mHz:

	Coverage	Capacity
Urban	212	318
Suburban	808	105
Rural	447	0

Because of the assumption that all MNOs want full coverage in the Netherlands, the number of coverage sites is independent of the percentage of the market that an MNO has to serve. Even if an MNO only serves 10% of the market, he still needs all coverage sites.

It is estimated that KPNs network serves 50% of the market, Vodafones network serves 30% and T-mobile 20% [26]. The assumption is that all different parties will try to maintain serving the same market share with LTE technology, as they serve now. Using the previous table and these percentages, we made an estimation about the number of sites that are necessary for all coalitions (Table 4.1).

Sites necessary for KPN (K), 50% of the market.

	Coverage	Capacity	Total
Urban	212	53	1520
Suburban	808	0	
Rural	447	0	

Sites necessary for Vodafone (V), 30% of the market.

	Coverage	Capacity	Total
Urban	212	0	1467
Suburban	808	0	
Rural	447	0	

Sites necessary for T-mobile (T), 20% of the market.

	Coverage	Capacity	Total
Urban	212	0	1467
Suburban	808	0	
Rural	447	0	

Sites necessary for coalition KPN-Vodafone (K&V), 80% of the market.

	Coverage	Capacity	Total
Urban	212	212	1679
Suburban	808	0	
Rural	447	0	

Sites necessary for coalition KPN-T-mobile (K&T), 70% of the market.

	Coverage	Capacity	Total
Urban	212	159	1626
Suburban	808	0	
Rural	447	0	

Sites necessary for coalition T-mobile-Vodafone (T&V), 50% of the market

	Coverage	Capacity	Total
Urban	212	53	1520
Suburban	808	0	
Rural	447	0	

Table 4.1: Necessary sites per coalition

To show how this works, we will determine how many sites are necessary for the coalition KPN-Vodafone (serves 80% of the market). Because there are $212 + 318 = 530$ sites necessary to serve 100% of the market in urban areas of the Netherlands, KPN and Vodafone need $530 * 0,80 = 424$ sites to serve 80% of the market in this area. From these sites there are 212 coverage sites, which leaves 212 capacity sites. In the suburban area there will be $(808 + 105) * 0,80 \approx 731$ sites necessary to serve 80% of the market. Because there are at least 808 sites necessary for coverage in this area, we can conclude 731 sites is not enough. So KPN and Vodafone will need 808 sites in the suburban area, which will give some overcapacity. For the rural case, the calculation is similar.

The grand coalition will need the same number of sites to serve 100% of the market, which is a total of 1890. With the estimation of 100.000 euro per site the LTE investment costs for each coalition are calculated (see Table 4.2). From these values we can already conclude that investment costs will decrease a lot when MNOs cooperate.

Coalition	LTE costs (millions)
K	152
V	146,7
T	146,7
K&V	167,9
K&T	162,6
T&V	152
K&T&V	189

Table 4.2: LTE technology investment costs

4.1.2 Network integration costs and RAN OPEX

When MNOs want to share their network, they have to integrate their networks. The costs of integrating networks depends on current technology used by the MNOs. Based on an estimate T-mobile made for the UK market, we estimated the following integration costs for the different coalitions (see Table 4.3).

Coalition	Integration costs (millions)
K&V	110,2
K&T	106,7
V&T	99,7
K&V&T	124

Table 4.3: Network integration costs

Integrating networks will also save costs. It is estimated that operation expenditures of the radio access network will decrease by 25 % when two MNOs integrate their network [12]. Based on this estimate, we assume that the RAN-OPEX will decrease by 35% when three MNOs integrate their network. In annual reports the current RAN-OPEX can be found [13], [30], [8]. With these values, Table (4.4) gives the RAN-OPEX in the three different situations. Because these costs will not be shared, they are given separately for every MNO.

MNO	Alone	Integrated with 1 other	Integrated with 2 others
KPN	125	93,75	81,25
Vodafone	115	86,25	74,75
T-Mobile	110	82,5	71,5

Table 4.4: RAN-OPEX for the MNOs (millions)

4.1.3 Subscriber acquisition and retention costs

MNOs invest money in acquiring and retaining customers. For example, by offering mobile phones with subscriptions. Both cooperation with other MNOs and investment in LTE will influence the amount an MNO has to invest in order to maintain his market share. When an MNO does not invest in LTE and his competition does, his competition has a competitive advantage. In order to acquire and retain customers, the MNO has to invest more money to differentiate in a different area. To compensate the fact that this MNO does not offer LTE technology, the MNO has to offer something else. Else consumers have less incentive to be a customer of this MNO. When two MNOs cooperate, customers will see less difference between these MNOs and will therefore have less loyalty to an MNO. Also in this case MNOs have to invest more money in acquiring and retaining customers.

Annual reports state that SAC/SRC are currently around 150 euro per customer per year. We assume that this amount will increase by some percentage that depends on the coalition structure formed. The following tables give the increase in terms of percentages due to network sharing for each coalitional structure. These percentages are a rough forecast, based on increases in SAC/SRC due to other factors (such as the introduction of the iPhone 4) found in annual reports of these MNOs.

	(K,X,X)	(X,V,X)	(X,X,T)	(K,V,X)	(K,X,T)	(X,V,T)	(K,V,T)	(K&V,X)
KPN	0%	0%	0%	0%	0%	0%	0%	3%
Vodafone	0%	0%	0%	0%	0%	0%	0%	3%
T-mobile	0%	0%	0%	0%	0%	0%	0%	0%

	(K&T,X)	(V&T,X)	(K&V,T)	(K&T,V)	(V&T,K)	(K&V&T)	(X,X,X)
KPN	3%	0%	3%	3%	0%	5%	0%
Vodafone	0%	3%	3%	0%	3%	5%	0%
T-mobile	3%	3%	0%	3%	3%	5%	0%

The following tables give the increase due to the advantage of owning LTE.

	(K,X,X)	(X,V,X)	(X,X,T)	(K,V,X)	(K,X,T)	(X,V,T)	(K,V,T)	(K&V,X)
KPN	0%	7%	7%	1%	1%	12%	2%	0%
Vodafone	6%	0%	6%	1%	11%	1%	2%	0%
T-mobile	5%	5%	0%	10%	1%	1%	2%	13%

	(K&T,X)	(V&T,X)	(K&V,T)	(K&T,V)	(V&T,K)	(K&V&T)	(X,X,X)
KPN	0%	15%	1%	1%	10%	0%	4%
Vodafone	14%	0%	1%	9%	1%	0%	4%
T-mobile	0%	0%	8%	1%	1%	0%	4%

We assume that when an MNO owns LTE technology, that this will cause an increase in SAC/SRC of competing MNOs. When more MNOs share their network, they reduce costs, and this will therefore cause a bigger increase of SAC/SRC of competing MNOs. We also assume that not investing at all will increase SAC/SRC. In this case, the increase in data traffic cannot be handled and customers will search for alternatives.

Tables (4.5) and (4.6) give an overview of the total increase in terms of percentages, due to the changes in the market.

Note that these costs, in contrary to the previous costs, do not only depend on the coalition an MNO is part of, but also on the other coalitions formed in the market.

These percentages multiplied with 150 and the number of subscribers each MNO has, gives the extra SAC/SRC for each MNO to each corresponding coalition structure. KPN has

	(K,X,X)	(X,V,X)	(X,X,T)	(K,V,X)	(K,X,T)	(X,T,V)	(K,T,V)	(K&V,X)
KPN	0%	7%	7%	1%	1%	12%	2%	3%
Vodafone	6%	0%	6%	1%	11%	1%	2%	3%
T-mobile	5%	5%	0%	10%	1%	1%	2%	13%

Table 4.5: Total increase SAC/SRC in terms of percentages (1)

	(K&T,X)	(V&T,X)	(K&V,T)	(K&T,V)	(V&T,K)	(K&T&V)	(X,X,X)
KPN	3%	15%	4%	4%	10%	5%	4%
Vodafone	14%	3%	4%	9%	4%	5%	4%
T-mobile	3%	3%	8%	4%	4%	5%	4%

Table 4.6: Total increase SAC/SRC in terms of percentages (2)

approximately 10 million subscribers, Vodafone 6 million and T-mobile 4 million. This gives the following costs in millions:

	(K,X,X)	(X,V,X)	(X,X,T)	(K,V,X)	(K,X,T)	(X,T,V)	(K,T,V)	(K&V,X)
KPN	-	105	105	15	15	180	30	45
Vodafone	54	-	54	9	99	9	18	27
T-mobile	30	30	-	60	6	6	12	78

	(K&T,X)	(V&T,X)	(K&V,T)	(K&T,V)	(V&T,K)	(K&T&V)	(X,X,X)
KPN	45	225	60	60	150	75	60
Vodafone	126	27	36	81	36	45	36
T-mobile	18	18	48	24	24	30	24

4.1.4 Total coalition structure costs

The investment in LTE technology and the network integration costs are costs that have to be divided among the members of the coalition, while the RAN-OPEX and SAC/SRC are chargeable to each MNO separately. In which way this amount is divided, depends on the division rule. This division rule can be determined beforehand, or proposed by one of the players. Section 4.1.5 gives some examples of possible division rules.
As stated before, LTE technology and the network integration costs should be depreciated over several years, the depreciation period. Because we depreciate these costs in equal increments, the fraction of costs paid each year is just the cost divided by the depreciation period. This will give the following formula to describe the costs for each MNO per coalition structure:

$$Divisionrule \cdot \frac{LTEcosts + Integrationcosts}{depreciationperiod} + RAN - OPEX + SAC/SRC$$

where 'Divisionrule' gives the percentage of LTE and integration costs that the MNO has to pay. To illustrate how this works, we will consider the situation where KPN and Vodafone are investing together and T-mobile is investing alone. We will calculate the resulting costs for KPN. The investment in LTE technology for KPN and Vodafone together costs 167,9 million and the integration costs of their networks are 110,2 million. The RAN-OPEX for KPN, when cooperating with one other MNO is 93,75 million and the SAC/SCR costs will increase by 4%. This will give a total of $10.000.000 \cdot 150 \cdot 0,04 = 60.000.000$ i.e. 60 million. If we assume a division rule that divides the costs equally and a 10 years depreciation period, then KPN has to pay

$$0,50 \cdot \frac{167,9+110,2}{10} + 93,75 + 60 \approx 182$$

million a year. Before calculating the total costs for each coalitional structure, we give some examples of division rules.

4.1.5 Division Rules

The most obvious division rule, used in the example in the previous section, is dividing the costs equally. Table (4.7) gives for each coalitional structure what each player should pay in this situation.

		Percentage			Division of costs		
Structure	Joint investment	KPN	Vodafone	T-mobile	KPN	Vodafone	T-mobile
(K&V,X)	27.805.661	50%	50%	0%	13.902.831	13.902.831	0
(K&T,X)	26.927.937	50%	0%	50%	13.463.968	0	13.463.968
(V&T,X)	25.172.487	0%	50%	50%	0	12.586.243	12.586.243
(K&V,T)	27.805.661	50%	50%	0%	13.902.831	13.902.831	0
(K&T,V)	26.927.937	50%	0%	50%	13.463.968	0	13.463.968
(V&T,K)	25.172.487	0%	50%	50%	0	12.586.243	12.586.243
(K&V&T)	31.300.000	33%	33%	33%	10.433.333	10.433.333	10.433.333

Table 4.7: Equally division of the costs

It seems fair to split the cost. However the possibilities to participate in different coalitions can give a different perspective on this. Cooperative game theory offers methods to divide the coalitional worth among its members, taking the presence of alternatives into account. The class of games these methods apply to are called transferable utility (TU) games. This refers to the possibility of transferring value (or costs) from one member of a coalition to another. The best known methods are the Shapley value, compromise value and the nucleolus. The compromise value τ for transferable utility games can only be calculated for so called compromise admissible games (for more insights on this subject see [28]). The worth of the coalitions does not represent a compromise admissible game. This excludes the value τ from the possible division rules. Because there is no straightforward method to calculate the nucleolus, this isn't the best value neither (see [22]).

Shapley Value

The only reasonable alternative is the Shapley value. This value exists for all transferable utility games (TU-games) and can be calculated in a straightforward way. There are several ways to define the Shapley value. We use the definition where the Shapley value is given by the average marginal contribution of each player. For a TU-game with N players (TU^N) this gives the following formal definition:

Definition 4.1 (Shapley value)**.** The Shapley value $\Phi : TU^N \rightarrow \mathbb{R}^N$ is defined by

$$\Phi(v) = \frac{1}{|N|!} \sum_{\sigma \in \Pi(N)} m^{\sigma}(v).$$

where $\Pi(N) := \{\sigma : \{1, \ldots, |N|\} \rightarrow N | \sigma \text{ is bijective}\}$ is the set of all orders on N and the marginal vector $m^\sigma(v) \in \mathbb{R}^N$, for $\sigma \in \Pi(N)$, is defined by

$$m^\sigma_{\sigma(k)}(v) = v(\{\sigma(1), \ldots, \sigma(k-1), \sigma(k)\}) - v(\{\sigma(1), \ldots, \sigma(k-1)\})$$

for all $k \in \{1, \ldots, |N|\}$.

The marginal vector $m^\sigma(v)$ depends on the order $\sigma(1), \sigma(2), \ldots, \sigma(|N|)$, in which players enter the game. The marginal contribution of a player is the additional value that this player creates by entering the coalition. This is the value of the coalition containing this player compared to the value of the coalition without this player.
The Shapley value is determined by summing the marginal contribution of a player for all possible orders and dividing this sum by the number of orders. The Shapley value can therefore be seen as the average of marginal contributions. We give an example 4.2 of the calculation of the Shapley value following the definition of the marginal contributions.

Example 4.2: Consider the TU game with three players and the following coalitional values:

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$v(S)$	0	1	0	2	3	3	4

The following table gives the marginal contribution of each player for all orders:

σ	1	2	3
123	0	2	2
132	0	1	3
213	1	1	2
231	1	1	2
312	3	1	0
321	1	3	0
sum	6	9	9

To get the average marginal contribution we divide the sum for all players by 6, which results in the following vector representing the Shapley value $(1, 3/2, 3/2)$. *Player 1 receives* 1 *and player 2 and 3 receive* $3/2$.

The Shapley value is a vector representing for each player the amount of the total value, he should receive. This can be converted to percentages. In the example player 1 gets 25% and player 2 and 3 both get $37,5\%$ of the value of the grand coalition.

The Shapley value is used to divide the value of the grand coalition for games without spillovers. Our game differs from this on three points:

- We want to divide the value of an arbitrary coalition;
- The coalitional worth is influenced by the presence of other coalitions, there are spillovers;
- Apart from shared investment costs, each player has personal costs that differ in each coalitional structure.

To adjust the Shapley value in such a way that it can also be used to divide coalitional worth for other coalitions besides the grand coalition, we divide the game into subgames. The coalition of which the worth has to be divided is assumed to be the 'grand coalition'.

To take the second aspect into account, the Shapley value of a coalition has to be calculated for each possible coalition structure separately. For example, if the coalitional worth of the coalition K&V has to be divided and the coalitional structure is (K&V,T), then the worth of the grand coalition is determined by the values for KPN and Vodafone for the coalitional structures (K&V,T) and (K,V,T). If the coalitional structure is (K&V,X) the worth of the coalition is determined by the value for KPN and Vodafone for the coalitional structures (K&V,X) and (K,V,X).

We deal with the third aspect by adding the individual costs to the total. In this way also the individual costs are part of the coalitional worth that has to be divided. To avoid that an MNO ends up paying for individual costs of a different MNO, these individual costs are subtracted again when the coalitional worth is divided. It could be the case that after the subtraction an MNO is left with a negative amount. When that happens, this MNO should pay 0% of the joint costs. In this way no MNO will pay individual costs for another MNO. However this situation does not occur with the values that we determined.

Example (4.3) gives an example of the calculation of the Shapley value for the sharing game.

Example 4.3 (Adjusted Shapley value)*: To calculate the Shapley value of the coalition KPN-Vodafone (K&V), where T-mobile is investing in LTE-technology separately, the values of the coalition structures (K&V,T) and (K,V,T) are used for each MNO. Table (4.8) gives these values. The entry in the row (K&V,T) and column K gives KPNs personal costs in case that the coalitional structure is (K&V,T), the entry in the same row and column V gives these costs for Vodafone and column $K+V$ gives the costs they have to share. The last column is the sum of these costs, and represents the total value of the coalition K&V, when T is present. The second row gives the costs for KPN and Vodafone respectively, when the coalitional structure is (K,V,T).*

	K	V	K+V	Total
(K&V,T)	153,75	122,25	27,81	303,81
(K,V,T)	170,20	147,67		

Table 4.8: Coalitional values (millions)

Officially the costs need to be translated to values (for instance, by multiplying with -1), because costs represent a value that has to be paid instead of a value that a player receives. However since our goal is to calculate a percentage, this will not be necessary. Following the marginal value definition, Table (4.9) calculates the marginal values for the two orders K,V and V,K for both players. The last row gives the average marginal value (i.e. the sum divided by two).

Order	K	V
K,V	170,20	303,81 - 170,20 = 133,61
V,T	303,81 -147,67 = 156,14	147,67
Total/2	163,17	140,64

Table 4.9: Marginal values (millions)

When KPN and Vodafone start to cooperate, from the perspective of KPN Vodafone contributes a value of $303,81 - 170,20$ *million euro compared to the situation where KPN would invest alone. Looking at the situation from Vodafone's point of view, KPN contributes* $303,81 - 147,67$*. This leads to the average marginal costs of* $163,17$ *million euro for KPN and* $140,64$ *for Vodafone. Because this still includes the individual costs, these have to be subtracted again. This determines the amount that each MNO should contribute to the shared costs. Table (4.10) gives this amount and calculates the corresponding percentage of the total shared costs for both MNOs.*

	Costs	Percentage
K	163,17 - 153,75 = 9,42	9,42/27,81 = 34 %
V	140,64 - 122,25 = 18,39	18,39/27,81 = 66 %

Table 4.10: Percentages according to the Shapley value for structure (K&V,T)

We can conclude that KPN should pay 34% *and Vodafone* 66% *of the shared costs.*

For all other coalitions the Shapley value can be calculated in the same way. Dividing the value of the coalition (K&V&T) will be a bit more work, because there are six orders and the values of more coalitional structures ((K&V,T), (K&T,V), (V&T,K) and (K,V,T)) are part of the calculation. Table (4.11) gives for all coalition structures the corresponding joint costs, the percentage that each player has to pay of the joint costs and the precise amount for all coalitional structures.

		Percentage			Division of costs		
Structure	Joint investment	KPN	Vodafone	T-mobile	KPN	Vodafone	T-mobile
(K&V,X)	27.805.661	34%	66%	0%	9.417.831	18.387.831	0
(K&T,X)	26.927.937	25%	0%	75%	6.603.968	0	20.323.968
(V&T,X)	25.172.487	0%	41%	59%	0	10.211.243	14.961.243
(K&V,T)	27.805.661	34%	66%	0%	9.417.831	18.387.831	0
(K&T,V)	26.927.937	25%	0%	75%	6.603.968	0	20.323.968
(V&T,K)	25.172.487	0%	41%	59%	0	10.211.243	14.961.243
(K&V&T)	31.300.000	10%	38%	52%	3.216.437	11.823.713	16.259.850

Table 4.11: Division of the costs according to the Shapley value

Saving Value

Another way to look at the situation is to base the division rule on how much personal cost each MNO saves by cooperating. A logical assumption is that an MNO that saves more by cooperating than another MNO, profits more from the situation and should therefore invest more. The last division rule divides the value in such a way that an MNO pays what he saves in proportion to what the other members of the coalition save. We will denote this value by 'saving value' and we will illustrate how to calculate it in example (4.4).

Example 4.4*:* Saving Value *Assume that the coalitional structure is given by (K&V,T). Table (4.12) gives the values for this situation.*

KPN would have $170,2$ *million personal costs. If he makes the investment alone and* $153,7$ *million if he cooperates with Vodafone. KPN therefore saves* $170,2 - 153,7 = 16,5$ *million. Similarly, Vodafone saves* $25,4$ *million. Based on these numbers, KPN should pay a percentage of*

	K	V
(K&V,T)	153,7	122,3
(K,V,T)	170,2	147,7
Saving	16,5	25,4

Table 4.12: Saving value (millions)

$$\frac{16,5}{16,5+25,4} = 39\%$$

of the joint costs and Vodafone should pay the rest, i.e.

$$\frac{25,4}{16,5+25,4} = 61\%.$$

The saving values for other coalitional structures are given in Table (4.13).

		Percentage			Division of costs		
Structure	Joint investment	KPN	Vodafone	T-mobile	KPN	Vodafone	T-mobile
(K&V,X)	27.805.661	39%	61%	0%	10.924.364	16.881.297	0
(K&T,X)	26.927.937	35%	0%	65%	9.501.599	0	17.426.337
(V&T,X)	25.172.487	0%	46%	54%	0	11.510.786	13.661.700
(K&V,T)	27.805.661	39%	61%	0%	10.924.364	16.881.297	0
(K&T,V)	26.927.937	35%	0%	65%	9.501.599	0	17.426.337
(V&T,K)	25.172.487	0%	46%	54%	0	11.510.786	13.661.700
(K&V&T)	31.300.000	18%	36%	46%	5.667.640	11.343.406	14.288.954

Table 4.13: Division of costs according to the saving value

4.2 Results

Structure	KPN	Vodafone	T-Mobile
(K,X,X)	140.200.000	169.000.000	140.000.000
(X,V,X)	230.000.000	129.670.000	140.000.000
(X,X,T)	230.000.000	169.000.000	124.670.000
(K,V,X)	155.200.000	138.670.000	170.000.000
(K,X,T)	155.200.000	214.000.000	130.670.000
(X,T,V)	305.000.000	138.670.000	130.670.000
(K,T,V)	170.200.000	147.670.000	136.670.000
(K&V,X)	152.652.831	127.152.831	188.000.000
(K&T,X)	152.213.968	241.000.000	113.963.968
(V&T,X)	350.000.000	125.836.243	113.086.243
(K&V,T)	167.652.831	136.152.831	172.670.000
(K&T,V)	167.213.968	210.670.000	119.963.968
(V&T,K)	290.200.000	134.836.243	119.086.243
(K&V&T)	166.683.333	130.183.333	111.933.333
(X,X,X)	185.000.000	151.000.000	134.000.000

Table 4.14: Coalitional structure costs with equal splitting division rule (euro)

Table 4.14 contains the total cost per coalitional structure for each MNO, when the division rule splits the costs equally and the depreciation period is 10 years. With these values one can determine the preference order on the different coalition structures for each MNO, by assuming that lower costs are preferred. Table (4.15) gives the functions for this situation.

KPN			Vodafone			T-mobile		
140	(A,X,X)	(K,X,X)	126	(B&C,X)	(V&T,X)	112	(A&B&C)	(K&V&T)
152	(A&C,X)	(K&T,X)	127	(A&B,X)	(K&V,X)	113	(B&C,X)	(V&T,X)
153	(A&B,X)	(K&V,X)	130	(X,B,X)	(X,V,X)	114	(A&C,X)	(K&T,X)
155	(A,B,X)	(K,V,X)	130	(A&B&C)	(K&V&T)	119	(B&C,A)	(V&T,K)
155	(A,X,C)	(K,X,T)	135	(B&C,A)	(V&T,K)	120	(A&C,B)	(K&T,V)
167	(A&B&C)	(K&V&T)	136	(A&B,C)	(K&V,T)	125	(X,X,C)	(X,X,T)
167	(A&C,B)	(K&T,V)	139	(A,B,X)	(K,V,X)	131	(A,X,C)	(K,X,T)
168	(A&B,C)	(K&V,T)	139	(X,B,C)	(X,V,T)	131	(X,B,C)	(X,V,T)
170	(A,B,C)	(K,V,T)	148	(A,B,C)	(K,V,T)	134	(X,X,X)	(X,X,X)
185	(X,X,X)	(X,X,X)	151	(X,X,X)	(X,X,X)	137	(A,B,C)	(K,V,T)
230	(X,B,X)	(X,V,X)	169	(A,X,X)	(K,X,X)	140	(A,X,X)	(K,X,X)
230	(X,X,C)	(X,X,T)	169	(X,X,C)	(X,X,T)	140	(X,B,X)	(X,V,X)
290	(B&C,A)	(V&T,K)	211	(A&C,B)	(K&T,V)	170	(A,B,X)	(K,V,X)
305	(X,B,C)	(X,V,T)	214	(A,X,C)	(K,X,T)	173	(A&B,C)	(K&V,T)
350	(B&C,X)	(V&T,X)	241	(A&C,X)	(K&T,X)	188	(A&B,X)	(K&V,X)

Table 4.15: Preferred order of coalition structures

Here each second column contains the notation used in previous chapters, each third column contains the corresponding coalitional structure for the present case. Here KPN has the role of player A, Vodafone of player B and T-mobile of player C. With the model from Chapter 3.2 one can determine the equilibria. Table (4.16) gives the results of the game with the players KPN (player A), Vodafone (player B) and T-mobile (player C).

Equilibria	RankA	RankB	RankC	Strategy A	Strategy B	Strategy C
(A&B&C)	6	4	1	ABC(X), ABC(Γ), ABC(Δ)	ABC(Γ), ABC(Δ)	ABC(Γ), ABC(Δ)
(A&B,C)	8	6	14	AB(X), AB(Γ)	AB(Γ)	C, ABC(Γ)
(A&C,B)	7	14	5	AC(X), AC(Γ)	B,AB(Γ), ABC(Γ)	AC(Γ)
(B&C,A)	13	5	4	A, AB(Γ), AC(Γ), ABC(Γ)	BC(X), BC(Γ)	BC(X), BC(Γ)
(A,B,C)	9	9	11	A	B, ABC(Γ)	C, ABC(Γ)
				A, ABC(Γ)	B	C, ABC(Γ)
				A, ABC(Γ)	B, ABC(Γ)	C

Table 4.16: Results for the Dutch Market

At first we can notice that there are many equilibria. As a consequence it is harder to draw conclusions. The equilibria are all combinations of ‘cooperation among two’ combined with the third MNO alone, ‘cooperation among all MNOs’ and ‘everyone for themselves’. In spite of the fact that there are many equilibria, all MNOs rank the equilibrium ‘cooperation among all MNOs’ the highest of all equilibria, this is a strong Nash-equilibrium. KPN, Vodafone and T-mobile should therefore try to make an agreement to cooperate with each other. KPN is the player that ranks the equilibrium the lowest. This situation clearly might change when the division rule is different, as we shall discuss next.

Table (4.17) illustrates the difference in coalitional worth, when the Shapley value or the saving value are used. The coalitional worth differs when a player is member of a coalition with at least two players. KPN will have lower costs in all cases when the division rule is

	KPN			Vodafone			T-mobile		
Structure	50/50	Shapley	Saving	50/50	Shapley	Saving	50/50	Shapley	Saving
(K,X,X)	140	140	140	169	169	169	140	140	140
(X,V,X)	230	230	230	130	130	130	140	140	140
(X,X,T)	230	230	230	169	169	169	125	125	125
(K,V,X)	155	155	155	139	139	139	170	170	170
(K,X,T)	155	155	155	214	214	214	131	131	131
(X,V,T)	305	305	305	139	139	139	131	131	131
(K,V,T)	170	170	170	148	148	148	137	137	137
(K&V,X)	153	148	150	127	132	130	188	188	188
(K&T,X)	152	145	148	241	241	241	114	121	121
(V&T,X)	350	350	350	126	123	125	113	115	115
(K&V,T)	168	163	165	136	141	139	173	173	173
(K&T,V)	167	160	163	211	211	211	120	127	127
(V&T,K)	290	290	290	135	132	134	119	121	121
(K&V&T)	167	159	162	130	132	131	112	118	118
(X,X,X)	185	185	185	151	151	151	134	134	134

Table 4.17: Different division rules

changed to the Shapley value as well as the saving value. Vodafone will have lower costs in case he cooperates with only T-mobile, while all forms of cooperation that include KPN will increase the costs. T-mobile will have to pay more in all cases when the division rule is changed. Compared to Vodafone, in general T-mobile will profit more from sharing and Vodafone will profit more than KPN from sharing. Both the Shapley value and the saving value, take this into account. Hence, KPN has to pay less of the joint investments than Vodafone and T-mobile and Vodafone has to pay less than T-mobile.
These changes lead in both cases to a change in the preference order of the coalitional structures for all players. With these new preference functions we used our model to calculate the new equilibria. In both cases these will turn out to remain the same.
Changing the division rule to the Shapley value or the saving value has no influence on the outcome of the game. In a negotiation process however, KPN would certainly profit more if the division rule were changed. We will explain this.

When making a decision about network sharing and investing, T-mobile should try to make an agreement with both KPN and Vodafone to integrate their networks and invest jointly in LTE technology. T-mobile proposes his first choice, so little advice is necessary for this MNO. however, for KPN and Vodafone the situation is different. KPN should not try to form the coalitional structures of his first five choices. This would result in coalitional structures he prefers less. The same holds for Vodafone's first three choices. The reason is that all these choices are coalitional structures where one or more other players refrain from making any investment. None of the players has coalitional structures where they make no investment at all high on their preference list, and so they will all try to avoid these.
Both KPN and Vodafone should agree to the proposal of T-mobile, and negotiate about the division of the shared costs. Because T-mobile profits the most of all from this agreement compared to the other possible coalitional structures, KPN and Vodafone could use this to negotiate their share of costs. Here KPN has a stronger position than Vodafone to decrease this amount.

To see what these results are worth, we will perform a sensitivity analysis in the next section.

4.3 Sensitivity Analyses

Clearly, we have an educated guess of the costs. Hence, the outcome of the game might change when the costs change. Increasing or decreasing a cost factor will cause a change in the total costs of a coalitional structure for one or more players. As a result the preference functions of these players might change, as well as the set of equilibria. In the next section we will analyse the effect of changing the cost of one factor only[1].

4.3.1 Sensitivity of integration costs

The first factor to be analysed are the integration costs. The other costs are kept the same as in Section 4.1. First we assume that the division rule is the equal division rules, that the depreciation period is 10 years and the integration costs are twice as high (see Table 4.18).

	KPN			Vodafone			T-mobile		
Structure	Before	After	Increase	Before	After	Increase	Before	After	Increase
(K,X,X)	140	140	0,0%	169	169	0,0%	140	140	0,0%
(X,V,X)	230	230	0,0%	130	130	0,0%	140	140	0,0%
(X,X,T)	230	230	0,0%	169	169	0,0%	125	125	0,0%
(K,V,X)	155	155	0,0%	139	139	0,0%	170	170	0,0%
(K,X,T)	155	155	0,0%	214	214	0,0%	131	131	0,0%
(X,V,T)	305	305	0,0%	139	139	0,0%	131	131	0,0%
(K,V,T)	170	170	0,0%	148	148	0,0%	137	137	0,0%
(K&V,X)	153	158	3,6%	127	133	4,3%	188	188	0,0%
(K&T,X)	152	158	3,5%	241	241	0,0%	119	114	4,7%
(V&T,X)	350	350	0,0%	126	131	4,0%	118	113	4,4%
(K&V,T)	168	173	3,3%	136	142	4,1%	173	173	0,0%
(K&T,V)	167	173	3,2%	211	211	0,0%	125	120	4,5%
(V&T,K)	290	290	0,0%	135	140	3,7%	124	119	4,2%
(K&V&T)	167	171	2,5%	130	134	3,2%	116	112	3,7%
(X,X,X)	185	185	0,0%	151	151	0,0%	134	134	0,0%

Table 4.18: Coalition structure costs twice integration costs (millions)

The first column for each MNO gives the total costs in the normal situation and the second column gives these cost for the situation where the integration costs are doubled. The third column calculates the percentage of cost increase. The percentage of cost increase varies from 0% to 4,7% , where 0% increase occurs when there are no integration costs. From these low percentages we can conclude that the integration costs have little influence in this case. But when we study the impact on the set of equilibria, we have to conclude that the integration costs have some influence. The corresponding preference order changes so much that our algorithm gives a different set of equilibria (4.19).

Although the set of equilibria is different, it will still be in the best interest of all players to make sure that the result is to cooperate with all players. This is because Table 4.19 describes the same situation as in example (2) Section 3.3.2, where player B and C can force player A to cooperate by using threats.

Remark: *When the LTE costs and integration costs are depreciated over 5 years, this influence will be a bit more. The percentage of cost increase varies from 0% to 8,4%, which is still a small increase. The result of the game with the same integration costs and*

[1]Because there is no obvious dependence between one or more factors, we have no reason to assume that more factors change at the same time. Analysis where two or more factors change at the same time are therefore considered to be out of scope of this thesis.

Equilibria	RankA	RankB	RankC	Strategy A	Strategy B	Strategy C
(A&B&C)	7	4	1	ABC(Γ), ABC(Δ)	ABC(Δ)	ABC(Δ)
(B&C,A)	13	7	4	A, AB(Γ), AC(Γ), ABC(Γ)	BC(X), BC(Γ)	BC(Γ)
(A,B,C)	6	9	10	A	B,AB(Γ), ABC(Γ)	AC(Γ), ABC(Γ)
				A, ABC(Γ)	B,AB(Γ)	AC(Γ), ABC(Γ)
				A, ABC(Γ)	B,AB(Γ), ABC(Γ)	AC(Γ)

Table 4.19: Results double investment costs

a depreciation period of 5 years, has the same equilibria as the game with a depreciation period of 10 years. Also the game with double integration costs and a depreciation period of 5 years, consists of the same equilibria as the same game with a depreciation period of 10 years.

Multiplying the integration costs by three will still give the same results as Table 4.19, but multiplying by four or more result in a different outcome. The equilibrium (B&C,A) is no longer an equilibrium. Player B and C can no longer use this equilibrium as a threat, therefore the result will be that all MNOs invest separately: (A,B,C).

Decreasing the integration costs turns out to have no influence on the set of equilibria. Even when the integration costs are all equal to zero, this set remains the same. Integration costs are costs that form a barrier to cooperate, and so this result is not unexpected.

4.3.2 Sensitivity of LTE technology investment costs

The second factor to analyse is the investment in LTE technology. First we study the effect of increasing these costs. Table 4.20 gives the change in coalitional worth when these costs are doubled (second column for each MNO). The third column gives the increase in percentages. The increase in coalitional cost ranges from 0% (MNO makes no investment in

	KPN			Vodafone			T-mobile		
Structure	Before	After	Increase	Before	After	Increase	Before	After	Increase
(K,X,X)	140	155	10,8%	169	169	0,0%	140	140	0,0%
(X,V,X)	230	230	0,0%	130	144	11,3%	140	140	0,0%
(X,X,T)	230	230	0,0%	169	169	0,0%	125	139	11,8%
(K,V,X)	155	170	9,8%	139	153	10,6%	170	170	0,0%
(K,X,T)	155	170	9,8%	214	214	0,0%	131	145	11,2%
(X,V,T)	305	305	0,0%	139	153	10,6%	131	145	11,2%
(K,V,T)	170	185	8,9%	148	162	9,9%	137	151	10,7%
(K&V,X)	153	161	5,5%	127	136	6,6%	188	188	0,0%
(K&T,X)	152	160	5,3%	241	241	0,0%	114	122	7,1%
(V&T,X)	350	350	0,0%	126	133	6,0%	113	121	6,7%
(K&V,T)	168	176	5,0%	136	145	6,2%	173	187	8,5%
(K&T,V)	167	175	4,9%	211	225	7,0%	120	128	6,8%
(V&T,K)	290	305	5,2%	135	142	5,6%	119	127	6,4%
(K&V&T)	167	173	3,8%	130	136	4,8%	112	118	5,6%
(X,X,X)	185	185	0,0%	151	151	0,0%	134	134	0,0%

Table 4.20: Coalition structure costs twice LTE costs (millions)

LTE) to 11.77%. While the increase is higher on average compared to the integration costs, the influence on the outcome of the game is lower. The reason is that almost all coalitional structure costs increase, instead of only a few. So while the change in integration costs does influence the preference order for all players a lot, the preference functions remain almost the same when we change LTE technology costs. The equilibria of the game with double LTE costs therefore remain the same.

Multiplying LTE investment costs by three influences the results a bit more. The equilibrium (A&B,C) is replaced with (A&B,X), because the costs are too high for T-mobile to profit from the investment when players KPN and Vodafone jointly invest. Still the equilibrium (A&B&C) is preferred most. When the investment costs are four times as high, the equilibrium (A&C,B) is replaced with (A&C,X), for the same reasons as before. Also the outcome (X,X,X) becomes an equilibrium. Nevertheless, in this case (A&B&C) remains the best outcome for all. Multiplying the LTE investment costs by five will result in a different outcome: (X,X,X). The costs are then too high to profit from the investment.

When the LTE costs decrease there might be less of an incentive to cooperate. It turns out that decreasing the costs by 25% will still give the same results. When the costs decrease by 50%, the set of equilibria changes. Both (A&B,C) and (A&C,B) are no longer equilibria, but (A&B&C) will still be the equilibrium that is preferred among all. This situations remains the same when we decrease these costs even more, in particular when the LTE investment costs become zero.

4.3.3 Sensitivity of RAN-OPEX

While it could be the case that investment costs are twice as high as estimated, this is not likely for operational expenditures. The increase in these expenditures is less. To determine the sensitivity of the RAN-OPEX we will first increase these costs by 25%.

	KPN			Vodafone			T-mobile		
Structure	Before	After	Increase	Before	After	Increase	Before	After	Increase
(K,X,X)	140	171	22,3%	169	198	17,0%	140	168	19,6%
(X,V,X)	230	261	13,6%	130	158	22,2%	140	168	19,6%
(X,X,T)	230	261	13,6%	169	198	17,0%	125	152	22,1%
(K,V,X)	155	186	20,1%	139	167	20,7%	170	198	16,2%
(K,X,T)	155	186	20,1%	214	243	13,4%	131	158	21,1%
(X,V,T)	305	336	10,3%	139	167	20,7%	131	158	21,1%
(K,V,T)	170	201	18,4%	148	176	19,5%	137	164	20,1%
(K&V,X)	153	176	15,4%	127	149	17,0%	188	216	14,6%
(K&T,X)	152	176	15,4%	241	270	12,0%	114	135	18,1%
(V&T,X)	350	381	8,9%	126	147	17,1%	113	134	18,2%
(K&V,T)	168	191	14,0%	136	158	15,8%	173	200	15,9%
(K&T,V)	167	191	14,0%	211	239	13,7%	120	141	17,2%
(V&T,K)	290	321	10,8%	135	156	16,0%	119	140	17,3%
(K&V&T)	167	187	12,2%	130	149	14,4%	112	130	16,0%
(X,X,X)	185	216	16,9%	151	180	19,0%	134	162	20,5%

Table 4.21: Coalition structure costs 25% increase RAN-OPEX (millions)

Table (4.21) shows the effect on the coalitional structure costs. The first thing we can notice is that all costs increase, with a percentage ranging from $8,9\%$ to $22,3\%$. The increase is much higher than the increase caused by the change in integration and/or LTE costs. In contrary to these costs, RAN-OPEX is an annual expenditure, and will therefore have more influence on the total costs. However, the influence on the equilibrium set will be less. The reason for this is, that all coalitional structure costs increase. Integration costs only increase for coalitional structures where coalitions are formed and LTE costs for coalitional structures where an investment in LTE is made. As a result, the preference functions change less by increasing the RAN-OPEX, as compared to other costs. In fact, increasing RAN-OPEX by 25% will keep all preference functions the same, except for the

one from Vodafone. But the latter has no influence on the set of equilibria. The outcome of the game will be that all MNOs share their network and jointly invest in LTE.
When the RAN-OPEX is 50% higher, the equilibria remain the same. It turns out that the higher the RAN-OPEX, the more incentive to cooperate. This is what one should expect, since the higher these expenditures, the higher the reduction of the expenditures due to cooperation.

Decreasing RAN-OPEX does cause a change in the result of the game. Decreasing RAN-OPEX with 25% will give the same result, but when the RAN-OPEX is 50% lower the outcome changes. Table (4.22) contains the equilibria of the game, in case the RAN-OPEX decrease by 50%.

Equilibria	RankA	RankB	RankC
(A&B&C)	10	8	6
(A,B,C)	6	6	10

Table 4.22: Results decrease of 50% of RAN-OPEX

The game has the two equilibria: (A,B,C) and (A&B&C). Because both KPN as Vodafone prefer the equilibrium (A,B,C), they will make sure that this will be the outcome of the game. When we decrease the RAN-OPEX by more than 50%, also T-Mobile will prefer (A,B,C).

4.3.4 Sensitivity of SAC/SRC

The subscriber acquisition and retention costs are determined based on annual reports. The percentage that these costs will increase due to LTE technology or sharing are just a prediction based on intuition. Therefore this is the most import post to analyse. Table (4.23) gives the costs of the coalitional structures when we increase all percentages by 1%.

	KPN			Vodafone			T-mobile		
Structure	Before	After	Increase	Before	After	Increase	Before	After	Increase
(K,X,X)	140	155	10,7%	126	178	41,5%	112	146	30,4%
(X,V,X)	152	245	61,0%	127	139	9,1%	113	146	29,1%
(X,X,T)	153	245	60,5%	130	178	37,3%	114	131	14,7%
(K,V,X)	155	170	9,7%	130	148	13,4%	119	176	47,8%
(K,X,T)	155	170	9,7%	135	223	65,4%	120	137	13,9%
(X,V,T)	167	320	92,0%	136	148	8,5%	125	137	9,6%
(K,V,T)	167	185	10,8%	139	157	13,0%	131	143	9,2%
(K&V,X)	168	168	0,0%	139	136	-1,8%	131	194	48,5%
(K&T,X)	170	167	-1,8%	148	250	69,3%	134	120	-10,5%
(V&T,X)	185	365	97,3%	151	135	-10,7%	137	119	-12,9%
(K&V,T)	230	183	-20,6%	169	145	-14,1%	140	179	27,6%
(K&T,V)	230	182	-20,8%	169	220	30,0%	140	126	-10,0%
(V&T,K)	290	305	5,2%	211	144	-31,7%	170	125	-26,4%
(K&V&T)	305	182	-40,4%	214	139	-35,0%	173	118	-31,7%
(X,X,X)	350	200	-42,9%	241	160	-33,6%	188	140	-25,5%

Table 4.23: Coalition structure costs when SAC/SRC increase is 1% higher

The coalition structure costs change a lot, ranging from a decrease of $42,9\%$ to an increase up to $97,3\%$. Nevertheless, the preference functions of all players remain exactly the same. The preference functions turn out to remain the same in every case where we increase or decrease all percentages of increase in SAC/SRC with the same amount. We will therefore examine these costs in a different way. Since SAC and SRC are describing the influence of two factors, we will analyse these separately. The first factor is the influence of network sharing on the SAC and SRC. The second factor is the influence of owning LTE technology in the SAC and SRC.

Influence of network sharing on SAC/SRC

First we look what happens if network sharing has a different influence on the increase percentage of SAC/SRC, while the influence of owning LTE-technology remains the same. If the increase in SAC/SRC due to network sharing is less, all players have more incentive to cooperate. Since all forms of cooperation are already an equilibrium, this will probably not give a different outcome. As an example we look at the situation where sharing networks gives no competitive disadvantages at all; the increase in SAC/SRC due to network sharing is 0%. One could argue that consumers do not base their choice of MNO on network quality, but on other factors, so this is not an unlikely situation. The percentages for this situation are given by first two tables of Section 4.1.3. Percentages for players that are a member of a coalition when there are no other coalitions present become 0. If there is another coalition present this percentage becomes 1. This 1% represents the extra SAC/SCR due to the advantage of LTE technology of the other coalition present. Table (4.24) gives the change in coalition structure costs.

	KPN			Vodafone			T-mobile		
Structure	Before	After	Increase	Before	After	Increase	Before	After	Increase
(K,X,X)	140	140	0,0%	169	169	0,0%	140	140	0,0%
(X,V,X)	230	230	0,0%	130	130	0,0%	140	140	0,0%
(X,X,T)	230	230	0,0%	169	169	0,0%	125	125	0,0%
(K,V,X)	155	155	0,0%	139	139	0,0%	170	170	0,0%
(K,X,T)	155	155	0,0%	214	214	0,0%	131	131	0,0%
(X,V,T)	305	305	0,0%	139	139	0,0%	131	131	0,0%
(K,V,T)	170	170	0,0%	148	148	0,0%	137	137	0,0%
(K&V,X)	153	108	-29,5%	127	100	-21,2%	188	188	0,0%
(K&T,X)	152	107	-29,6%	241	241	0,0%	114	96	-15,8%
(V&T,X)	350	350	0,0%	126	99	-21,5%	113	95	-15,9%
(K&V,T)	168	123	-26,8%	136	109	-19,8%	173	173	0,0%
(K&T,V)	167	122	-26,9%	211	211	0,0%	120	102	-15,0%
(V&T,K)	290	290	0,0%	135	108	-20,0%	119	101	-15,1%
(K&V&T)	167	92	-45,0%	130	85	-34,6%	112	82	-26,8%
(X,X,X)	185	185	0,0%	151	151	0,0%	134	134	0,0%

Table 4.24: Coalition structure costs when sharing has no influence

The variation in costs ranges from -45% to 0%, where only costs for coalition structures where a coalition of more than two players is present change. These costs will decrease for the players that are a member of this coalition. The result will be that all scenarios where players share are preferred even more. All players have the equilibrium (A&B&C) ranked number 1, so this will be the result of the game.

A more interesting situation is the case where the increase of SAC/SRC due to sharing is higher. We want to determine the case where sharing is no longer attractive. When we increase the percentages by one percent, the outcome remains the same: (A&B&C). But the set of equilibria is different in this situation. The equilibria describe the same situation as in example (2) in Section 3.3.2. If we increase the percentages by two percent, the set of equilibria is given by

Equilibria	RankA	RankB	RankC
(A&B&C)	8	7	1
(A,B,C)	4	6	10

In this situation sharing will no longer be the result, because both KPN and Vodafone would prefer otherwise. While increasing the percentages has no influence, decreasing by only two percent will already give a different outcome.

Influence of owning LTE technology on SAC/SRC

The investment in LTE technology of competitors could also have a different influence on the increase in SAC/SRC. First we look at the extreme case, where consumers see no added value in LTE technology (see the second pair of tables of Section 4.1.3). Owning LTE technology will therefore give no increase in SAC/SCR of competitors. We assume that network sharing still has influence on SAC/SCR. Therefore there will be an increase in SAC/SCR for players that are a member of a coalition of at least to players.

As a consequence the preference functions change, as can be seen from Table (4.25).

p_A	p_B	p_C
(X,X,X)	(X,X,X)	(X,X,X)
(X,X,C)	(X,X,C)	(X,B,X)
(X,B,X)	(A,X,X)	(A,X,X)
(X,B,C)	(A,X,C)	(A,B,X)
(B&C,X)	(A&C,X)	(A&B,X)
(A,X,X)	(B&C,X)	(A&B&C)
(A,B,X)	(B&C,A)	(B&C,X)
(A,X,C)	(A&B,X)	(B&C,A)
(A,B,C)	(A&B,C)	(A&C,X)
(B&C,A)	(X,B,X)	(A&C,B)
(A&C,X)	(A,B,X)	(X,X,C)
(A&C,B)	(X,B,C)	(A,X,C)
(A&B,X)	(A,B,C)	(X,B,C)
(A&B,C)	(A&C,B)	(A,B,C)
(A&B&C)	(A&B&C)	(A&B,C)

Table 4.25: Preference functions when LTE technology gives no advantage

It can be seen immediately that these functions lead to one equilibrium: (X,X,X).
Even when we decrease the influence of the competitive advantage of LTE technology by only one percent the result will be different. Tables (4.26) and (4.27) gives the percentages in this case.

	(K,X,X)	(X,V,X)	(X,X,T)	(K,V,X)	(K,X,T)	(X,V,T)	(K,V,T)	(K&V,X)
KPN	0%	6%	6%	0%	0%	11%	1%	3%
Vodafone	5%	0%	5%	0%	10%	0%	1%	3%
T-mobile	4%	4%	0%	9%	0%	0%	1%	12%

Table 4.26: Percentage increasing SAC/SRC when LTE technology has 1% less influence (1)

The resulting preference functions give the following equilibria:

	(K&T,X)	(V&T,X)	(K&V,T)	(K&T,V)	(V&T,K)	(K&V&T)	(X,X,X)
KPN	3%	14%	3%	3%	9%	5%	3%
Vodafone	13%	3%	3%	8%	3%	5%	3%
T-mobile	3%	3%	7%	3%	3%	5%	3%

Table 4.27: Percentage increasing SAC/SRC when LTE technology has 1% less influence (2)

Equilibria	RankA	RankB	RankC
(A&B,C)	7	4	14
(A&C,B)	5	13	5
(B&C,A)	13	2	3
(A&B&C)	9	8	1
(A,B,C)	8	9	10

One can see immediately that both Vodafone and T-mobile prefer (B&C,A). Because they do not need KPN to accomplish an agreement that results in this equilibrium, this will be the outcome of the game. Decreasing these factors by 2% gives a different set of equilibria; (A&B,C) and (A&C,B) are no longer equilibria. However, the outcome of the game remains the same: both Vodafone and T-mobile have a preference for (B&C,A). Decreasing by 3% adds another equilibrium: (X,X,X). This equilibrium is ranked first for both KPN and Vodafone, and T-mobile has no choice but to adjust. Decreasing the percentages more than 3%, also results in the equilibrium (X,X,X).

When the competitive advantage of owning LTE technology increases, the results do not change. In fact, all players prefer the outcome (A&B&C) even more. When we increase all percentage KPN and Vodafone have (A&B&C) as their second choice and T-mobile has this equilibrium as his first choice. Therefore this will result in the outcome (A&B&C). Increasing the influence of owning LTE technology will only give MNOs more incentive to invest in technology, but it does not influence the choice to make the investment alone or jointly. Because the set of equilibria did not contain an equilibrium where an MNO made no investment, the set of equilibria remains the same.

4.3.5 Conclusions sensitivity analyses

The integration costs have little influence on the result of the game. Increasing will only give a different result when these costs are four times as high. Then the result will be (A,B,C) instead of (A&B&C). The integration costs are then too high to benefit from sharing. Decreasing these costs makes no difference at all, the result remains (A&B&C).

The investment in LTE technology also has an insignificant influence. If these costs are five times as high, the result will be different: (X,X,X). In this case the LTE is too costly to benefit from. If the costs are less than five times the original the result remains (A&B&C), also in the case where we decrease these costs.

The result is more sensitive to changes in the RAN-OPEX, although increasing this expense will not give a different result. This is because the higher the RAN-OPEX, the higher the amount that can be saved by network sharing. All players prefer cooperation more in this case. Decreasing RAN-OPEX will already give a different result when the decrease is more than 50%. In this case, the incentive to cooperate does not compensate the expenses for both KPN and Vodafone. The equilibrium will then be (A,B,C).

The result is very sensitive for changes in the increase of SAC/SRC. We made a distinction between the influence of network sharing and of owning LTE technology on SAC/SRC. Decreasing the influence of network sharing has no influence on the results. Decreasing this influence means that there is less competitive disadvantage of network sharing, so cooperation remains the preferred result. If the influence is increased, the result changes. If there is only an increase in SAC/SRC of 2% more than in the original situation, the result will be (A,B,C). So if the increase in SAC/SRC is 5% (or 6% when there is also competition) for coalitions of two MNOs and 7% for coalitions of all MNOs, the competitive disadvantage of sharing networks is so high that MNOs can no longer benefit from cooperation.
Increasing the influence of owning LTE technology on SAC/SCR, does not change the results. MNOs will have more incentives to invest in technology, but it does not influence the choice to make the investment alone or jointly. Because the set of equilibria did not contain an equilibrium where an MNO made no investment, the set of equilibria remains the same.
Decreasing the influence changes the results quickly. If owning LTE technology influences the SAC/SRC 1% less, the result will change to (B&C,A). And if this influence is 3% less, the result will be (X,X,X). When the influence of owning LTE technology on SAC/SCR decreases, it will be easier for an MNO to compete with other MNOs. Therefore MNOs prefer cooperation less. If the influence decreases too much, the investment is no longer valuable, and MNOs prefer not to invest at all.

Compared to the influence of the increase in SAC/SCR the influence of the other factors is next to nothing. The SAC/SRC is the most important factor that determines the results. Only a small change can make a difference. Drawing conclusions based on uncertain estimations of SAC/SRC can give the wrong image. Therefore one should try to estimate these costs very carefully.

5
Conclusions

To draw the right conclusions from the previous chapters, we look back at the main research question.

How can game theory give insights on the strategic decision making of MNOs concerning the investment in a jointly or separately owned LTE network?

To answer this question we answer the sub questions formulated in the first chapter.

1. *Which game theoretical models can be used to analyse coalition formation between competitors?*

   Traditional game theory has little focus on questions concerning which coalitions are formed and how the influence of the presence of other coalitions can be taken into account. In recent years these limitations led to formulation of 'non-cooperative coalition formation games with spillovers'. This is a category of games that describes the coalition formation as a non-cooperative process. Players propose the coalition they want to be part of, without considering the other players in the proposed coalition. Here players do not base their choice merely on the coalition they want to be part of, but also on which other coalitions will be formed. Their choice is based on the total coalitional structure that is formed. Games that belong to this class of games are very useful to analyse coalition formation in a competitive market.

2. *Which of these models is most suitable to analyse the decision process of the MNOs?*

   Within the class of 'non-cooperative coalition formation games with spillovers' usually is made a distinction between open and exclusive membership games. Open membership games allow players to join a coalition whenever they please. This is not a desirable property for the model we need. Exclusive membership games are therefore a better candidate. There are sequential and simultaneous exclusive membership games. Although a sequential game reflects the actual negotiation process better, working with this model is time consuming and almost impossible for situations with more than 2 players. The simultaneous exclusive membership game is the best candidate to model the decision process of the MNOs.

3. *How should these models be adjusted?*

   In simultaneous exclusive membership games the strategy set consists of proposing coalitions, where the singleton coalition can be proposed as well. MNOs have an extra option next to forming a singleton coalition, namely 'doing nothing'. The strategy 'doing nothing' has to be added to set of strategies.
   A disadvantage of a simultaneous game that there is only one stage to make the decision. An important question is therefore; what happens if a member of the proposed coalition deviates from the coalition? The game called Γ assumes that the coalition will fall apart in singletons. While in the game Δ all players which proposed this coalition stay together. Both games assume what will happen beforehand. A better solution would be to let players have influence on what happens when a member deviates. Adjusting the strategy set such that players do not only propose a coalition, but also what happens when a member of the coalition deviates, is a way to do this. Next to the coalition players propose either Γ, Δ or X. When a player proposes Γ, this player prefers to form a singleton coalition if the proposal is not unanimously accepted. When a player proposes Δ, his second choice will be to form a coalition with the members that proposed the same coalition. And when a player proposes X, he prefers to do nothing if a member deviates.
   Because of the fact that it is hard to valuate coalitional structures for each MNO, a different payoff function was necessary. Instead of using actual values, we used functions that only require a preference order on the different coalitional structures.

4. *What conclusion can be drawn from the performed analyses?*

   At first we analysed how useful the model is, when using preference functions. We can conclude that the preference functions are a useful tool when there is little information available about the value of the coalitional structures. With very little assumptions and details on the market structure and the strategies of the players involved we are able to identify equilibria and corresponding strategies for each of the players. The implication of this is that we can quickly analyse many different market structures and many different scenarios with respect to outcome preferences. For players this approach helps not only to evaluate different strategies; it also provides useful indications on which situations further detailed analysis (e.g. in the form of a numerical business case) is needed, therefore it helps to speed up and make the decision process more efficient.

   The second part of the analysis was to see how the model could be used analysing the Dutch market. To analyse this market we used an estimation of the costs of the coalitional structures, for KPN, Vodafone and T-Mobile. Based on these costs, we could see which preference function each of the MNOs has. Playing the game with these function led to the conclusion that it is in the best interest of all MNOs to share their network and jointly invest in LTE.
   From the sensitivity analyses we can conclude the following:

   - If the integration costs are four times as high as estimated or more, 'network sharing among all' is no longer attractive. In this case each MNO should invest separately. But in all other cases the result remains the same.
   - If the investment in LTE technology is five times as high as estimated or more, MNOs should not make an investment jointly or separately. In all other cases, the joint investment remains the most attractive option.

- If RAN-OPEX is less than half the amount that we estimated, the savings in operational expenditures due to sharing can no longer compete with the integration costs. In this case MNOs should invest separately. A higher RAN-OPEX will give more incentives to cooperate.
- The result is very sensitive to changes in the increase of subscriber acquisition and retention costs. A change of 1% can already give a different outcome. One can therefore doubt the quality of the solution. If there would be more certainty about the SAC/SRC, the model will give a strong result, because other factors that determine the costs have little influence on the results.

Summarizing the sub questions gives the following answer on the main research question: if one is able to determine in a certain market which preference order on all possible coalitional structures each MNO has, the simultaneous exclusive membership game can be used to draw conclusion about which coalitional structure will be formed in this market.

5.1 Reflections and Recommendations

The proposed model can be used in situations where similar decisions have to be made. In general decisions about joint investments or joint research projects with competitors can be analysed using the proposed model. The biggest advantages are that only little information is necessary and that the model provides a quick result. The set of equilibria has to be seen as a result where the negotiation process can be based on, not as the actual outcome of the game. To conclude I list some possible improvements and recommendations for future research relating to the topics in my thesis.

- A disadvantage of the model is that even though players have a choice in what will happen if a member of the coalition deviates, this choice is limited. A possibility for future research could therefore be to adjust the model according to Hafalir [9]. Here non deviating players either try to minimize the payoff of the deviating coalition, they merge, or they try to maximize their own payoff. Another alternative would be to assume that players are farsighted. Players are said to be farsighted if they anticipate that any action by a group of players may generate a further chain of actions by some other groups. These players consider the possibility that once they act, another coalition might react, a third coalition might in turn react, and so on, without limit. Herings, Mauleon and Vannetelbosch study coalition formation games with spillovers where players are farsighted [11]. The solution concept in this game is called the farsighted stable set. A disadvantage of this solution is that this set can be hard to calculate.

- We used Nash-equilibria to solve the game. There are several extensions of the concept equilibrium, which might be useful to solve the game (see Bloch [6]).

- The models works for games with two and three players. A four player game could be modelled with the same method, but with more players the model will become too complex. For situations with more than four players, a different model will be necessary. One can question whether game theory should be used in this case, because of the general complexity of solving games with more than three players.

- In Chapter 4 the Shapley value was adjusted in such a way that it could be used for our model. In our model the value had to be adjusted because there were personal costs, spillovers and an arbitrary coalition was formed instead of the grand coalition. These personal costs are usually not considered analysing games. Some further research can be done in this area. There are formulations of the Shapley value for games with spillovers, without these personals costs. These games are in partition function form (see for example [17], [4] and [21]).

- The valuation for the coalitional structures in the Dutch Market is a rough estimation. This estimation has to be much more precise if one wants to draw good conclusions by using the game. Especially the factor SAC/SCR has to be estimated with more accuracy, because of the sensitivity of the results to this factor. But, if one is able to make a good valuation, the model gives new insights where MNOs can base their decisions on.

- Performing the sensitivity analyses, we saw that there are many cases where changing one cost factor causes a big change in the worth of coalitional structures, but where the outcome of the game doesn't change. The results of the sensitivity analyses give therefore a good view on which factors influence the coalition formation and which not. Coalitional worth might be sensitive to changes in a certain factor, while the result of the game is not.

- The sensitivity analyses were only performed to determine the influence of the separate factors. It might be useful to determine what will happen if more than one factor changes at the same time of such a situation is likely to happen.

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A

Payoff functions

Table (A.1) gives all preference functions for player A, where the order $(A, X) > (A, B) > (X, B)$ is respected. Table (A.2) gives all preference functions for player B, where the order $(X, B) > (A, B) > (A, X)$ is respected.

1	2	3	4	5	6	7	8	9	10
(A,X) (A,B) (X,B) (A&B) (X,X)	(A,X) (A,B) (X,B) (X,X) (A&B)	(A,X) (X,X) (A,B) (X,B) (A&B)	(A,X) (A&B) (A,B) (X,B) (X,X)	(A,X) (A&B) (X,X) (A,B) (X,B)	(A,X) (X,X) (A&B) (A,B) (X,B)	(A,X) (A,B) (A&B) (X,B) (X,X)	(A,X) (A,B) (X,X) (X,B) (A&B)	(A,X) (A,B) (A&B) (X,X) (X,B)	(A,X) (A,B) (X,X) (A&B) (X,B)
11	12	13	14	15	16	17	18	19	20
(A,X) (X,X) (A,B) (A&B) (X,B)	(A,X) (A&B) (A,B) (X,X) (X,B)	(X,X) (A,X) (A,B) (X,B) (A&B)	(A&B) (A,X) (A,B) (X,B) (X,X)	(X,X) (A,X) (A,B) (A&B) (X,B)	(A&B) (A,X) (A,B) (X,X) (X,B)	(X,X) (A,X) (A&B) (A,B) (X,B)	(A&B) (A,X) (X,X) (A,B) (X,B)	(X,X) (A&B) (A,X) (A,B) (X,B)	(A&B) (X,X) (A,X) (A,B) (X,B)

Table A.1: Preference functions player A in a two player game

1	2	3	4	5	6	7	8	9	10
(X,B) (A,B) (A,X) (A&B) (X,X)	(X,B) (A,B) (A,X) (X,X) (A&B)	(X,B) (X,X) (A,B) (A,X) (A&B)	(X,B) (A&B) (A,B) (A,X) (X,X)	(X,B) (A&B) (X,X) (A,B) (A,X)	(X,B) (X,X) (A&B) (A,B) (A,X)	(X,B) (A,B) (A&B) (A,X) (X,X)	(X,B) (A,B) (X,X) (A,X) (A&B)	(X,B) (A,B) (A&B) (X,X) (A,X)	(X,B) (A,B) (X,X) (A&B) (A,X)
11	12	13	14	15	16	17	18	19	20
(X,B) (X,X) (A,B) (A&B) (A,X)	(X,B) (A&B) (A,B) (X,X) (A,X)	(X,X) (X,B) (A,B) (A,X) (A&B)	(A&B) (X,B) (A,B) (A,X) (X,X)	(X,X) (X,B) (A,B) (A&B) (A,X)	(A&B) (X,B) (A,B) (X,X) (A,X)	(X,X) (X,B) (A&B) (A,B) (A,X)	(A&B) (X,B) (X,X) (A,B) (A,X)	(X,X) (A&B) (X,B) (A,B) (A,X)	(A&B) (X,X) (X,B) (A,B) (A,X)

Table A.2: Preference functions player B in a two player game

Table (A.3) gives the preference functions for player A corresponding to the twelve functions for the three player game described in Subsection 3.2.2. The order of preference is from top to bottom. Table (A.4) give these functions for player B.

1	2	3	4	5	6
(A,X,X)	(A,X,X)	(A,X,X)	(A,X,X)	(A&C,X)	(A&B&C)
(A,X,C)	(A,X,C)	(A,X,C)	(A,X,C)	(A&B,X)	(A&B,X)
(A,B,X)	(A,B,X)	(A,B,X)	(A,B,X)	(A&C,B)	(A&C,X)
(A,B,C)	(A,B,C)	(A,B,C)	(A,B,C)	(A&B,C)	(A&B,C)
(B&C,A)	(A&C,X)	(B&C,A)	(A&B&C)	(A&B&C)	(A&C,B)
(A&C,X)	(A&B,X)	(A&B&C)	(A&B,X)	(A,X,X)	(A,X,X)
(A&B,X)	(A&C,B)	(A&B,X)	(A&C,X)	(A,X,C)	(A,X,C)
(A&C,B)	(A&B,C)	(A&C,X)	(A&B,C)	(A,B,X)	(A,B,X)
(A&B,C)	(A&B&C)	(A&B,C)	(A&C,B)	(A,B,C)	(A,B,C)
(A&B&C)	(B&C,A)	(A&C,B)	(B&C,A)	(B&C,A)	(B&C,A)
(X,X,X)	(X,X,X)	(X,X,X)	(X,X,X)	(X,X,X)	(X,X,X)
(X,X,C)	(X,X,C)	(X,X,C)	(X,X,C)	(X,X,C)	(X,X,C)
(X,B,X)	(X,B,X)	(X,B,X)	(X,B,X)	(X,B,X)	(X,B,X)
(X,B,C)	(X,B,C)	(X,B,C)	(X,B,C)	(X,B,C)	(X,B,C)
(B&C,X)	(B&C,X)	(B&C,X)	(B&C,X)	(B&C,X)	(B&C,X)
7	8	9	10	11	12
(A&C,X)	(A&B&C)	(X,X,X)	(X,X,X)	(X,X,X)	(X,X,X)
(A&B,X)	(A&B,X)	(X,X,C)	(X,X,C)	(A,X,X)	(A,X,X)
(A&C,B)	(A&C,X)	(X,B,X)	(X,B,X)	(A,X,C)	(A,X,C)
(A&B,C)	(A&B,C)	(X,B,C)	(X,B,C)	(A,B,X)	(A,B,X)
(A&B&C)	(A&C,B)	(A&C,X)	(A&B&C)	(A,B,C)	(A,B,C)
(X,X,X)	(X,X,X)	(A&B,X)	(A&B,X)	(B&C,A)	(B&C,A)
(X,X,C)	(X,X,C)	(A&C,B)	(A&C,X)	(X,X,C)	(X,X,C)
(X,B,X)	(X,B,X)	(A&B,C)	(A&B,C)	(X,B,X)	(X,B,X)
(X,B,C)	(X,B,C)	(A&B&C)	(A&C,B)	(X,B,C)	(X,B,C)
(A,X,X)	(A,X,X)	(B&C,X)	(B&C,X)	(B&C,X)	(B&C,X)
(A,X,C)	(A,X,C)	(A,X,X)	(A,B,C)	(A&C,X)	(A&B&C)
(A,B,X)	(A,B,X)	(A,X,C)	(A,X,C)	(A&B,X)	(A&B,X)
(A,B,C)	(A,B,C)	(A,B,X)	(A,B,X)	(A&C,B)	(A&B,C)
(B&C,A)	(B&C,A)	(A,B,C)	(A&B,C)	(A&B,X)	(A&C,X)
(B&C,X)	(B&C,X)	(B&C,A)	(B&C,A)	(A&B&C)	(A&C,B)

Table A.3: Preference functions player A in a three player game

1	2	3	4	5	6
(X,B,X)	(X,B,X)	(X,B,X)	(X,B,X)	(B&C,X)	(A&B&C)
(X,B,C)	(X,B,C)	(X,B,C)	(X,B,C)	(A&B,X)	(A&B,X)
(A,B,X)	(A,B,X)	(A,B,X)	(A,B,X)	(B&C,A)	(B&C,X)
(A,B,C)	(A,B,C)	(A,B,C)	(A,B,C)	(A&B,C)	(A&B,C)
(A&C,B)	(B&C,X)	(A&C,B)	(A&B&C)	(A&B&C)	(B&C,A)
(B&C,X)	(A&B,X)	(A&B&C)	(A&B,X)	(X,B,X)	(X,B,X)
(A&B,X)	(B&C,A)	(A&B,X)	(B&C,X)	(X,B,C)	(X,B,C)
(B&C,A)	(A&B,C)	(B&C,X)	(A&B,C)	(A,B,X)	(A,B,X)
(A&B,C)	(A&B&C)	(A&B,C)	(B&C,A)	(A,B,C)	(A,B,C)
(A&B&C)	(A&C,B)	(B&C,A)	(A&C,B)	(A&C,B)	(A&C,B)
(X,X,X)	(X,X,X)	(X,X,X)	(X,X,X)	(X,X,X)	(X,X,X)
(X,X,C)	(X,X,C)	(X,X,C)	(X,X,C)	(X,X,C)	(X,X,C)
(A,X,X)	(A,X,X)	(A,X,X)	(A,X,X)	(A,X,X)	(A,X,X)
(A,X,C)	(A,X,C)	(A,X,C)	(A,X,C)	(A,X,C)	(A,X,C)
(A&C,X)	(A&C,X)	(A&C,X)	(A&C,X)	(A&C,X)	(A&C,X)
7	8	9	10	11	12
(B&C,X)	(A&B&C)	(X,X,X)	(X,X,X)	(X,X,X)	(X,X,X)
(A&B,X)	(A&B,X)	(X,X,C)	(X,X,C)	(X,B,X)	(X,B,X)
(B&C,A)	(B&C,X)	(A,X,X)	(A,X,X)	(X,B,C)	(X,B,C)
(A&B,C)	(A&B,C)	(A,X,C)	(A,X,C)	(A,B,X)	(A,B,X)
(A&B&C)	(B&C,A)	(B&C,X)	(A&B&C)	(A,B,C)	(A,B,C)
(X,X,X)	(X,X,X)	(A&B,X)	(A&B,X)	(A&C,B)	(A&C,B)
(X,X,C)	(X,X,C)	(B&C,A)	(B&C,X)	(X,X,C)	(X,X,C)
(A,X,X)	(A,X,X)	(A&B,C)	(A&B,C)	(A,X,X)	(A,X,X)
(A,X,C)	(A,X,C)	(A&B&C)	(B&C,A)	(A,X,C)	(A,X,C)
(X,B,X)	(X,B,X)	(A&C,X)	(A&C,X)	(A&C,X)	(A&C,X)
(X,B,C)	(X,B,C)	(X,B,X)	(X,B,X)	(B&C,X)	(A&B&C)
(A,B,X)	(A,B,X)	(X,B,C)	(X,B,C)	(A&B,X)	(A&B,X)
(A,B,C)	(A,B,C)	(A,B,X)	(A,B,X)	(B&C,A)	(A&B,C)
(A&C,B)	(A&C,B)	(A,B,C)	(A,B,C)	(A&B,C)	(B&C,A)
(A&C,X)	(A&C,X)	(A&C,B)	(A&C,B)	(A&B&C)	(B&C,X)

Table A.4: Preference functions player B in a three player game

Table (A.5) gives the preference functions for player C.

1	2	3	4	5	6
(X,X,C)	(X,X,C)	(X,X,C)	(X,X,C)	(B&C,X)	(A&B&C)
(X,B,C)	(X,B,C)	(X,B,C)	(X,B,C)	(A&C,X)	(A&C,X)
(A,X,C)	(A,X,C)	(A,X,C)	(A,X,C)	(B&C,A)	(B&C,X)
(A,B,C)	(A,B,C)	(A,B,C)	(A,B,C)	(A&C,B)	(B&C,A)
(A&B,C)	(B&C,X)	(A&B,C)	(A&B&C)	(A&B&C)	(A&C,B)
(B&C,X)	(A&C,X)	(A&B&C)	(A&C,X)	(X,X,C)	(X,X,C)
(A&C,X)	(B&C,A)	(A&C,X)	(B&C,X)	(X,B,C)	(X,B,C)
(B&C,A)	(A&C,B)	(B&C,X)	(B&C,A)	(A,X,C)	(A,X,C)
(A&C,B)	(A&B&C)	(B&C,A)	(A&C,B)	(A,B,C)	(A,B,C)
(A&B&C)	(A&B,C)	(A&C,B)	(A&B,C)	(A&B,C)	(A&B,C)
(X,X,X)	(X,X,X)	(X,X,X)	(X,X,X)	(X,X,X)	(X,X,X)
(X,B,X)	(X,B,X)	(X,B,X)	(X,B,X)	(X,B,X)	(X,B,X)
(A,X,X)	(A,X,X)	(A,X,X)	(A,X,X)	(A,X,X)	(A,X,X)
(A,B,X)	(A,B,X)	(A,B,X)	(A,B,X)	(A,B,X)	(A,B,X)
(A&B,X)	(A&B,X)	(A&B,X)	(A&B,X)	(A&B,X)	(A&B,X)
7	8	9	10	11	12
(B&C,X)	(A&B&C)	(X,X,X)	(X,X,X)	(X,X,X)	(X,X,X)
(A&C,X)	(A&C,X)	(X,B,X)	(X,B,X)	(X,X,C)	(X,X,C)
(B&C,A)	(B&C,X)	(A,X,X)	(A,X,X)	(X,B,C)	(X,B,C)
(A&C,B)	(B&C,A)	(A,B,X)	(A,B,X)	(A,X,C)	(A,X,C)
(A&B&C)	(A&C,B)	(B&C,X)	(A&B&C)	(A,B,C)	(A,B,C)
(X,X,X)	(X,X,X)	(A&C,X)	(A&C,X)	(A&B,C)	(A&B,C)
(X,B,X)	(X,B,X)	(B&C,A)	(B&C,X)	(X,B,X)	(X,B,X)
(A,X,X)	(A,X,X)	(A&C,B)	(B&C,A)	(A,X,X)	(A,X,X)
(A,B,X)	(A,B,X)	(A&B&C)	(A&C,B)	(A,B,X)	(A,B,X)
(X,X,C)	(X,X,C)	(A&B,X)	(A&B,X)	(A&B,X)	(A&B,X)
(X,B,C)	(X,B,C)	(X,X,C)	(X,X,C)	(B&C,X)	(A&B&C)
(A,X,C)	(A,X,C)	(X,B,C)	(X,B,C)	(A&C,X)	(A&C,X)
(A,B,C)	(A,B,C)	(A,X,C)	(A,X,C)	(B&C,A)	(B&C,A)
(A&B,C)	(A&B,C)	(A,B,C)	(A,B,C)	(A&C,B)	(A&C,B)
(A&B,X)	(A&B,X)	(A&B,C)	(A&B,C)	(A&B&C)	(B&C,X)

Table A.5: Preference functions player C in a three player game

B
Matrix representation

Table (B.1) gives a representation of the 3 person normal form game. There are nine matrices, where the strategy in the upper left cell of each matrix gives the strategy for player C, in the corresponding matrix. The rows of the matrix give the strategies for player B and the columns give the strategy for player A. The elements in the matrix are just the outcomes of the combined strategy and do not give a value. Each of the outcomes had a different value for each player.

Table B.1: 3 player matrix game

X	X	A	AB(X)	AC(X)	AB(Γ)	AC(Γ)	ABC(X)	ABC(Γ)	ABC(Δ)
X	X, X, X	A, X, X	X, X, X	X, X, X	A, X, X	A, X, X	X, X, X	A, X, X	A, X, X
B	X, B, X	A, B, X	X, B, X	X, B, X	A, B, X	A, B, X	X, B, X	A, B, X	A, B, X
AB(X)	X, X, X	A, X, X	A&B, X	X, X, X	A&B, X	A, X, X	X, X, X	A, X, X	A, B, X
BC(X)	X, X, X	A, X, X	X, X, X	X, X, X	A, X, X	A, X, X	X, X, X	A, X, X	A, X, X
AB(Γ)	X, B, X	A, B, X	A&B, X	X, B, X	A&B, X	A, B, X	X, B, X	A, B, X	A, B, X
BC(Γ)	X, B, X	A, B, X	X, B, X	X, B, X	A, B, X	A, B, X	X, B, X	A, B, X	A, B, X
ABC(X)	X, X, X	A, X, X	X, X, X	X, X, X	A, X, X	A, X, X	X, X, X	A, X, X	A, X, X
ABC(Γ)	X, B, X	A, B, X	X, B, X	X, B, X	A, B, X	A, B, X	X, B, X	A, B, X	A, B, X
ABC(Δ)	X, B, X	A, B, X	X, B, X	X, B, X	A, B, X	A, B, X	X, B, X	A, B, X	A&B, X
C	X	A	AB(X)	AC(X)	AB(Γ)	AC(Γ)	ABC(X)	ABC(Γ)	ABC(Δ)
X	X, X, C	A, X, C	X, X, C	X, X, C	A, X, C	A, X, C	X, X, C	A, X, C	A, X, C
B	X, B, C	A, B, C	X, B, C	X, B, C	A, B, C	A, B, C	X, B, C	A, B, C	A, B, C
AB(X)	X, X, C	A, X, C	A&B, C	X, X, C	A&B, C	A, X, C	X, X, C	A, X, C	A, B, C
BC(X)	X, X, C	A, X, C	X, X, C	X, X, C	A, X, C	A, X, C	X, X, C	A, X, C	A, X, C
AB(Γ)	X, B, C	A, B, C	A&B, C	X, B, C	A&B, C	A, B, C	X, B, C	A, B, C	A, B, C
BC(Γ)	X, B, C	A, B, C	X, B, C	X, B, C	A, B, C	A, B, C	X, B, C	A, B, C	A, B, C
ABC(X)	X, X, C	A, X, C	X, X, C	X, X, C	A, X, C	A, X, C	X, X, C	A, X, C	A, X, C
ABC(Γ)	X, B, C	A, B, C	X, B, C	X, B, C	A, B, C	A, B, C	X, B, C	A, B, C	A, B, C
ABC(Δ)	X, B, C	A, B, C	X, B, C	X, B, C	A, B, C	A, B, C	X, B, C	A, B, C	A&B, C
AC(X)	X	A	AB(X)	AC(X)	AB(Γ)	AC(Γ)	ABC(X)	ABC(Γ)	ABC(Δ)
X	X, X, X	A, X, X	X, X, X	A&C, X	A, X, X	A&C, X	X, X, X	A, X, X	A, X, X
B	X, B, X	A, B, X	X, B, X	A&C, B	A, B, X	A&C, B	X, B, X	A, B, X	A, B, X
AB(X)	X, X, X	A, X, X	A&B, X	A&C, X	A&B, X	A&C, X	X, X, X	A, X, X	A, X, X
BC(X)	X, X, X	A, X, X	X, X, X	A&C, X	A, X, X	A&C, X	X, X, X	A, X, X	A, X, X
AB(Γ)	X, B, X	A, B, X	A&B, X	A&C, B	A&B, X	A&C, B	X, B, X	A, B, X	A, B, X
BC(Γ)	X, B, X	A, B, X	X, B, X	A&C, B	A, B, X	A&C, B	X, B, X	A, B, X	A, B, X
ABC(X)	X, X, X	A, X, X	X, X, X	A&C, X	A, X, X	A&C, X	X, X, X	A, X, X	A, X, X
ABC(Γ)	X, B, X	A, B, X	X, B, X	A&C, B	A, B, X	A&C, B	X, B, X	A, B, X	A, B, X
ABC(Δ)	X, B, X	A, B, X	X, B, X	A&C, B	A, B, X	A&C, B	X, B, X	A, B, X	A&B, X
BC(X)	X	A	AB(X)	AC(X)	AB(Γ)	AC(Γ)	ABC(X)	ABC(Γ)	ABC(Δ)
X	X, X, X	A, X, X	X, X, X	X, X, X	A, X, X	A, X, X	X, X, X	A, X, X	A, X, X
B	X, B, X	A, B, X	X, B, X	X, B, X	A, B, X	A, B, X	X, B, X	A, B, X	A, B, X
AB(X)	X, X, X	A, X, X	A&B, X	X, X, X	A&B, X	A, X, X	X, X, X	A, X, X	A, X, X
BC(X)	B&C, X	B&C, A	B&C, X	B&C, X	B&C, A	B&C, A	B&C, X	B&C, A	B&C, A
AB(Γ)	X, B, X	A, B, X	A&B, X	X, B, X	A&B, X	A, B, X	X, B, X	A, B, X	A, B, X
BC(Γ)	B&C, X	B&C, A	B&C, X	B&C, X	B&C, A	B&C, A	B&C, X	B&C, A	B&C, A
ABC(X)	X, X, X	A, X, X	X, X, X	X, X, X	A, X, X	A, X, X	X, X, X	A, X, X	A, X, X
ABC(Γ)	X, B, X	A, B, X	X, B, X	X, B, X	A, B, X	A, B, X	X, B, X	A, B, X	A, B, X
ABC(Δ)	X, B, X	A, B, X	X, B, X	X, B, X	A, B, X	A, B, X	X, B, X	A, B, X	A&B, X
AC(Γ)	X	A	AB(X)	AC(X)	AB(Γ)	AC(Γ)	ABC(X)	ABC(Γ)	ABC(Δ)
X	X, X, C	A, X, C	X, X, C	A&C, X	A, X, C	A&C, X	X, X, C	A, X, C	A, X, C
B	X, B, C	A, B, C	X, B, C	A&C, B	A, B, C	A&C, B	X, B, C	A, B, C	A, B, C
AB(X)	X, X, C	A, X, C	A&B, C	A&C, X	A&B, C	A&C, X	X, X, C	A, X, C	A, X, C
BC(X)	X, X, C	A, X, C	X, X, C	A&C, X	A, X, C	A&C, X	X, X, C	A, X, C	A, B, C
AB(Γ)	X, B, C	A, B, C	A&B, C	A&C, B	A&B, C	A&C, B	X, B, C	A, B, C	A, B, C
BC(Γ)	X, B, C	A, B, C	X, B, C	A&C, B	A, B, C	A&C, B	X, B, C	A, B, C	A, B, C
ABC(X)	X, X, C	A, X, C	X, X, C	A&C, X	A, X, C	A&C, X	X, X, C	A, X, C	A, X, C
ABC(Γ)	X, B, C	A, B, C	X, B, C	A&C, B	A, B, C	A&C, B	X, B, C	A, B, C	A, B, C
ABC(Δ)	X, B, C	A, B, C	X, B, C	A&C, B	A, B, C	A&C, B	X, B, C	A, B, C	A&B, C
BC(Γ)	X	A	AB(X)	AC(X)	AB(Γ)	AC(Γ)	ABC(X)	ABC(Γ)	ABC(Δ)
X	X, X, C	A, X, C	X, X, C	X, X, C	A, X, C	A, X, C	X, X, C	A, X, C	A, X, C
B	X, B, C	A, B, C	X, B, C	X, B, C	A, B, C	A, B, C	X, B, C	A, B, C	A, B, C
AB(X)	X, X, C	A, X, C	A&B, C	X, X, C	A&B, C	A, X, C	X, X, C	A, X, C	A, X, C
BC(X)	B&C, X	B&C, A	B&C, X	B&C, X	B&C, A	B&C, A	B&C, X	B&C, A	B&C, A
AB(Γ)	X, B, C	A, B, C	A&B, C	X, B, C	A&B, C	A, B, C	X, B, C	A, B, C	A, B, C
BC(Γ)	B&C, X	B&C, A	B&C, X	B&C, X	B&C, A	B&C, A	B&C, X	B&C, A	B&C, A
ABC(X)	X, X, C	A, X, C	X, X, C	X, X, C	A, X, C	A, X, C	X, X, C	A, X, C	A, X, C
ABC(Γ)	X, B, C	A, B, C	X, B, C	X, B, C	A, B, C	A, B, C	X, B, C	A, B, C	A, B, C
ABC(Δ)	X, B, C	A, B, C	X, B, C	X, B, C	A, B, C	A, B, C	X, B, C	A, B, C	A&B, X
ABC(X)	X	A	AB(X)	AC(X)	AB(Γ)	AC(Γ)	ABC(X)	ABC(Γ)	ABC(Δ)
X	X, X, X	A, X, X	X, X, X	X, X, X	A, X, X	A, X, X	X, X, X	A, X, X	A, X, X
B	X, B, X	A, B, X	X, B, X	X, B, X	A, B, X	A, B, X	X, B, X	A, B, X	A, B, X
AB(X)	X, X, X	A, X, X	A&B, X	X, X, X	A&B, X	A, X, X	X, X, X	A, X, X	A, X, X
BC(X)	X, X, X	A, X, X	X, X, X	X, X, X	A, X, X	A, X, X	X, X, X	A, X, X	A, X, X
AB(Γ)	X, B, X	A, B, X	A&B, X	X, B, X	A&B, X	A, B, X	X, B, X	A, B, X	A, B, X
BC(Γ)	X, B, X	A, B, X	X, B, X	X, B, X	A, B, X	A, B, X	X, B, X	A, B, X	A, B, X
ABC(X)	X, X, X	A, X, X	X, X, X	X, X, X	A, X, X	A, X, X	A&B&C	A&B&C	A&B&C
ABC(Γ)	X, B, X	A, B, X	X, B, X	X, B, X	A, B, X	A, B, X	A&B&C	A&B&C	A&B&C
ABC(Δ)	X, B, X	A, B, X	X, B, X	X, B, X	A, B, X	A, B, X	A&B&C	A&B&C	A&B&C
ABC(Γ)	X	A	AB(X)	AC(X)	AB(Γ)	AC(Γ)	ABC(X)	ABC(Γ)	ABC(Δ)
X	X, X, C	A, X, C	X, X, C	X, X, C	A, X, C	A, X, C	X, X, C	A, X, C	A, X, C
B	X, B, C	A, B, C	X, B, C	X, B, C	A, B, C	A, B, C	X, B, C	A, B, C	A, B, C
AB(X)	X, X, C	A, X, C	A&B, C	X, X, C	A&B, C	A, X, C	X, X, C	A, X, C	A, X, C
BC(X)	X, X, C	A, X, C	X, X, C	X, X, C	A, X, C	A, X, C	X, X, C	A, X, C	A, X, C
AB(Γ)	X, B, C	A, B, C	A&B, C	X, B, C	A&B, C	A, B, C	X, B, C	A, B, C	A, B, C
BC(Γ)	X, B, C	A, B, C	X, B, C	X, B, C	A, B, C	A, B, C	X, B, C	A, B, C	A, B, C
ABC(X)	X, X, C	A, X, C	X, X, C	X, X, C	A, X, C	A, X, C	A&B&C	A&B&C	A&B&C
ABC(Γ)	X, B, C	A, B, C	X, B, C	X, B, C	A, B, C	A, B, C	A&B&C	A&B&C	A&B&C
ABC(Δ)	X, B, C	A, B, C	X, B, C	X, B, C	A, B, C	A, B, C	A&B&C	A&B&C	A&B&C
ABC(Γ)	X	A	AB(X)	AC(X)	AB(Γ)	AC(Γ)	ABC(X)	ABC(Γ)	ABC(Δ)
X	X, X, C	A, X, C	X, X, C	X, X, C	A, X, C	A, X, C	X, X, C	A, X, C	A&C, X
B	X, B, C	A, B, C	X, B, C	X, B, C	A, B, C	A, B, C	X, B, C	A, B, C	A&C, B
AB(X)	X, X, C	A, X, C	A&B, C	X, X, C	A&B, C	A, X, C	X, X, C	A, X, C	A&C, X
BC(X)	X, X, C	A, X, C	X, X, C	X, X, C	A, X, C	A, X, C	B&C, X	A, X, C	A&C, X
AB(Γ)	X, B, C	A, B, C	A&B, C	X, B, C	A&B, C	A, B, C	X, B, C	A, B, C	A&C, B
BC(Γ)	X, B, C	A, B, C	X, B, C	X, B, C	A, B, C	A, B, C	B&C, X	A, B, C	A&C, B
ABC(X)	X, X, C	A, X, C	X, X, C	X, X, C	A, X, C	A, X, C	A&B&C	A&B&C	A&B&C
ABC(Γ)	X, B, C	A, B, C	X, B, C	X, B, C	A, B, C	A, B, C	A&B&C	A&B&C	A&B&C
ABC(Δ)	B&C, X	B&C, A	B&C, X	B&C, X	B&C, A	B&C, A	A&B&C	A&B&C	A&B&C

C

Algorithm: Equilibria in three person game

This algorithm finds all possible equilibria in a 3 person normal form game. The algorithm is written in the language Visual Basic as an application for Excel (VBA). In two excel sheets the matrices (appendix B) and the preference functions (appendix A) for the three player game are described. The program written in VBA searches for equilibria in this matrix, for one combination of the preference functions.
The function VindEquilibria is the main function. VindEquilibria walks through the matrices and compares each element with other elements as long as the element remains the best outcome compared to the other elements. As soon as there is a better element in the same row, same column or on the same place in a different matrix for one of the players, VindEquilibria concludes that this is not an equilibrium by changing the fontcolor from black to grey. If no such element is found, there will be concluded that the element is an equilibrium and the color of the background will turn red.
VindEquilibria compares the elements by calling the function PREFBoolean. This function compares two elements by looking at the preference function for the corresponding player. So if two elements in the same row have to be compared, PREFBoolean looks at the preference function for player A to decide which element is the best outcome.
The last function, Strategie, gives for each equilibrium the corresponding strategies that each player should play.

```
Public Sub VindEquilibriaSub()
    Call VindEquilibria
End Sub

Public Function VindEquilibria() As Variant
    Dim Alternatieven(9) As Integer
    Dim Equilibria() As Variant
    Dim AantalEquilibria As Integer
    Dim i, j, k, l As Integer
    Dim Equilibrium As Boolean
    Dim StrategieA() As Variant
    Dim StrategieB() As Variant
    Dim StrategieC() As Variant

  'Reset the color of the cells
```

```
Sheets("Equilibria").Cells.Font.ColorIndex = 1
Sheets("Equilibria").Cells.Interior.ColorIndex = 0
'Clear list of strategies and equilibria
Range(Sheets("Equilibria").Cells(100, 2), Sheets("Equilibria").Cells(150, 5)).Delete

AantalEquilibria = 0

For i = 1 To 9
   Alternatieven(i) = i
Next i

For l = 1 To 9
   For k = 1 To 9
      For j = 1 To 9
         If Sheets("Equilibria").Cells(k + 4 + 12 * (l − 1), j + 1).Font.ColorIndex <> 15 Then
            'Set up the equilibrium searcher
            Equilibrium = True
            For i = 1 To 9
               'Checks everything for player 1
               If Alternatieven(i) <> j Then
                  If PREFboolean(1, Sheets("Equilibria").Cells(k + 4 + 12 * (l - 1), i + 1),
                  Sheets("Equilibria").Cells(k + 4 + 12 * (l − 1), j + 1)) Then
         Sheets("Equilibria").Cells(k + 4 + 12 * (l − 1), j + 1).Font.ColorIndex = 15
         Equilibrium = False
         Exit For
                  End If
               End If
               'Checks everything for player 2
               If Alternatieven(i) <> k Then
                  If PREFboolean(2, Sheets("Equilibria").Cells(i + 4 + 12 * (l − 1), j + 1),
                  Sheets("Equilibria").Cells(k + 4 + 12 * (l − 1), j + 1)) Then
         Sheets("Equilibria").Cells(k + 4 + 12 * (l − 1), j + 1).Font.ColorIndex = 15
         Equilibrium = False
         Exit For
                  End If
               End If
               'Checks everything for player 3
               If Alternatieven(i) <> l Then
                  If PREFboolean(3, Sheets("Equilibria").Cells(k + 4 + 12 * (i − 1), j + 1),
                  Sheets("Equilibria").Cells(k + 4 + 12 * (l − 1), j + 1)) Then
                     Sheets("Equilibria").Cells(k + 4 + 12 * (l − 1), j + 1).Font.ColorIndex = 15
                     Equilibrium = False
                     Exit For
                  End If
               End If                          Next i
            If Equilibrium = True Then
               ' If equilibrium, change cellcolor
               Sheets("Equilibria").Cells(k + 4 + 12 * (l − 1), j + 1).Interior.ColorIndex = 3
               Sheets("Equilibria").Cells(k + 4 + 12 * (l − 1), j + 1).Font.ColorIndex = 1
               AantalEquilibria = AantalEquilibria +1
               ReDim Preserve Equilibria(AantalEquilibria)
               Equilibria(AantalEquilibria)= Sheets("Equilibria").Cells(k + 4 + 12 * (l − 1), j + 1)
               ReDim Preserve StrategieA(AantalEquilibria)
               StrategieA(AantalEquilibria) = j
               ReDim Preserve StrategieB(AantalEquilibria)
               StrategieB(AantalEquilibria) = k
               ReDim Preserve StrategieC(AantalEquilibria)
               StrategieC(AantalEquilibria) = l
            End If
         End If
      Next j
   Next k
```

```
    Next l

    'Determine Strategy
    If AantalEquilibria <> 0 Then
        StrategieA = Strategie(StrategieA, 1)
        StrategieB = Strategie(StrategieB, 2)
        StrategieC = Strategie(StrategieC, 3)
            For i = 1 To AantalEquilibria
            Sheets("Equilibria").Cells(99 + i, 2) = Equilibria(i)
            Sheets("Equilibria").Cells(99 + i, 3) = StrategieA(i)
            Sheets("Equilibria").Cells(99 + i, 4) = StrategieB(i)
            Sheets("Equilibria").Cells(99 + i, 5) = StrategieC(i)
        Next i
    End If

VindEquilibria = Equilibria

End Function


Public Function PREFboolean(Speler As Integer, Uitkomst1 As String, Uitkomst2 As String) As Boolean
    Dim Spelerrange As Range
    Dim LookupRow As Integer

    PREFboolean = False
    LookupRow = 5 - Speler

    Set Spelerrange = Range(Sheets("Payoff").Cells(5, Speler), Sheets("Payoff").Cells(19, 5))

    If Application.WorksheetFunction.VLookup(Uitkomst1, Spelerrange, LookupRow, False) <
    Application.WorksheetFunction.VLookup(Uitkomst2, Spelerrange, LookupRow, False) Then
        PREFboolean = True
    End If

End Function


Public Function Strategie(ArrayIn, Speler As Integer) As Variant
    Dim i As Integer
    Dim StrategieEq() As Variant
    Dim Element As Variant

    i = 1
    If Speler = 1 Then
    For Each Element In ArrayIn
        ReDim Preserve StrategieEq(i)
        If Element = 1 Then
            StrategieEq(i) = "𝒳"
        ElseIf Element = 2 Then
            StrategieEq(i) = "𝒜"
        ElseIf Element = 3 Then
            StrategieEq(i) = "𝒜&ℬ"
        ElseIf Element = 4 Then
            StrategieEq(i) = "𝒜&𝒞"
        ElseIf Element = 5 Then
            StrategieEq(i) = "𝒜&ℬ&𝒞"
        ElseIf Element = 6 Then
            StrategieEq(i) = "𝒜+"
        ElseIf Element = 7 Then
            StrategieEq(i) = "𝒜&(ℬ ∨ 𝒞)"
        ElseIf Element = 8 Then
            StrategieEq(i) = "𝒜&ℬ+"
        ElseIf Element = 9 Then
            StrategieEq(i) = "𝒜&𝒞+"
        End If
        i = i + 1
```

```
    Next Element
    ElseIf Speler = 2 Then
    For Each Element In ArrayIn
        ReDim Preserve StrategieEq(i)
        If Element = 1 Then
            StrategieEq(i) = "𝒳"
        ElseIf Element = 2 Then
            StrategieEq(i) = "ℬ"
        ElseIf Element = 3 Then
            StrategieEq(i) = "𝒜&ℬ"
        ElseIf Element = 4 Then
            StrategieEq(i) = "ℬ&𝒞"
        ElseIf Element = 5 Then
            StrategieEq(i) = "𝒜&ℬ&𝒞"
        ElseIf Element = 6 Then
            StrategieEq(i) = "ℬ+"
        ElseIf Element = 7 Then
            StrategieEq(i) = "ℬ&(𝒜 ∨ 𝒞)"
        ElseIf Element = 8 Then
            StrategieEq(i) = "𝒜&ℬ+"
        ElseIf Element = 9 Then
            StrategieEq(i) = "ℬ&𝒞+"
        End If
        i = i + 1
    Next Element
    ElseIf Speler = 3 Then
    For Each Element In ArrayIn
        ReDim Preserve StrategieEq(i)
        If Element = 1 Then
            StrategieEq(i) = "𝒳"
        ElseIf Element = 2 Then
            StrategieEq(i) = "𝒞"
        ElseIf Element = 3 Then
            StrategieEq(i) = "𝒜&𝒞"
        ElseIf Element = 4 Then
            StrategieEq(i) = "ℬ&𝒞"
        ElseIf Element = 5 Then
            StrategieEq(i) = "𝒜&ℬ&𝒞"
        ElseIf Element = 6 Then
            StrategieEq(i) = "𝒞+"
        ElseIf Element = 7 Then
            StrategieEq(i) = "𝒞&(𝒜 ∨ ℬ)"
        ElseIf Element = 8 Then
            StrategieEq(i) = "𝒜&𝒞+"
        ElseIf Element = 9 Then
            StrategieEq(i) = "ℬ&𝒞+"
        End If
        i = i + 1
    Next Element
    End If

    Strategie = StrategieEq
End Function
```

D

Comparing models

In this appendix we show that the formulation of our simultaneous game, give a different result than the game Δ and Γ. We will show this with an example for the game Δ. That it holds for the game Γ can be argued in the same way.

When the strategy ABC(Γ) is left out of the game, the game will only consist of the choice between Δ and X. If a player proposes a coalition with one other player, the choice for Γ is the same as the choice for Δ), (e.g. $AB(\Gamma) = AB(\Delta)$, see Section 2.2.1). A logical question is therefore: what is the added value of the strategy ABC(Γ). On first sight one might think that the strategy ABC(Γ) adds nothing to the game. In most cases this is true, however there are situations where this strategy changes the outcome of the game. This can be seen by looking at the matrix representation in appendix B. The only case when there is a difference between playing ABC(Γ) and ABC(Δ) is when there is one other player playing ABC(Δ). If not all players propose coalition ABC, playing ABC(Γ) will result in a singleton coalition, while playing ABC(Δ) results in a coalition with the other player that plays ABC(Δ). Example (D.1) illustrates a situation where the game with the strategy ABC(Γ) gives another outcome than the game without the strategy ABC(Γ).

Example D.1 (Game without strategy ABC(Γ))*: Consider the example where the preference function of the players are given by the functions p_A, p_B and p_C:*

p_A	p_B	p_C
(A&C,X)	*(B&C,X)*	*(A&C,X)*
(A&C,B)	*(A&B,X)*	*(A&C,B)*
(A,X,X)	*(B&C,A)*	*(A&B&C)*
(A,X,C)	*(A&B,C)*	*(B&C,X)*
(A,B,X)	*(X,B,X)*	*(B&C,A)*
(B&C,A)	*(X,B,C)*	*(X,X,C)*
(A&B,X)	*(A,B,X)*	*(A,X,C)*
(A&B,C)	*(A&C,B)*	*(X,B,C)*
(A&B&C)	*(A&B&C)*	*(A,B,C)*
(X,X,X)	*(X,X,X)*	*(A&B,C)*
(A,B,C)	*(A,B,C)*	*(X,X,X)*
(X,B,X)	*(A,X,X)*	*(A,X,X)*
(X,X,C)	*(X,X,C)*	*(X,B,X)*
(X,B,C)	*(A,X,C)*	*(A,B,X)*
(B&C,X)	*(A&C,X)*	*(A&B,X)*

Table (D.1) gives the results of the game with the strategy ABC(Γ) and Table (D.2) the results without. This includes all equilibria and all strategies that have to be played in order to reach the corresponding equilibrium.

Equilibria	Strategy A	Strategy B	Strategy C
(A&B&C)	ABC(Γ), ABC(Δ)	ABC(Γ)	ABC(Γ), ABC(Δ)
(A&B,C)	AB(X), AB(Γ)	AB(Γ)	C,ABC(Γ)
(A&C,B)	AC(X), AG(Γ), ABC(Δ)	B, AB(Γ), BC(Γ), ABC(Γ), ABC(Δ)	AC(X), AC(Γ), ABC(Δ)
(B&C,A)	A, AB(Γ), ABC(Γ)	BC(X), BC(Γ), ABC(Δ)	BC(X), BC(Γ), ABC(Δ)
(A,B,C)	A,ABC(Γ)	B, ABC(Γ)	C, ABC(Γ)

Table D.1: Equilibria with strategy ABC(Γ)

Equilibria	Strategy A	Strategy B	Strategy C
(A&B,C)	AB(X), AB(Γ)	AB(Γ)	C
(A&C,B)	AC(X), AG(Γ), ABC(Δ)	B, AB(Γ), BC(Γ), ABC(Δ)	AC(X), AC(Γ), ABC(Δ)
(B&C,A)	A,AB(Γ)	BC(X), BC(Γ), ABC(Δ)	BC(X), BC(Γ), ABC(Δ)
(A,B,C)	A	B	C

Table D.2: Equilibria without strategy ABC(Γ)

The results differ on one point; the outcome (A&B&C) is an equilibrium when the game is played with strategy ABC(Γ), while in this is not the case in the game without ABC(Γ).

The example shows that adding the strategy ABC(Γ) can make a difference in the results of the game. Next to this, the strategy ABC(Γ) reflects a realistic option. While the three player game, only differs on a small point when adding the strategy ABC(Γ), the generalization to a game with more players changes a lot. In this case there are more coalitions with at least two players. Therefore there are more situations where the choice for Δ gives a different outcome than the choice for Γ.