# Confirmatory Factor Analysis for large datasets using Pairwise Maximum Likelihood

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# Foreword

The concept of this thesis project was set-up by Mariska Barendse in the past. Therefore, I would like to thank Mariska (my external supervisor) for sharing her ideas and enthusiasm during my thesis. Her motivational talks and support encouraged me to finish this thesis. Our weekly meetings helped me to understand the inner workings of the estimation method and how to set up a proper simulation study. Next to the content-related help, she also learned me how to deal with setbacks, for which I am grateful.

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# Contents

# Abstract

In the context of factor analysis, the most common estimation method for analysing discrete data is multiple step Diagonally Weighted Least Squares (DWLS). A novel estimation method is called Pairwise Maximum Likelihood (PML). PML calculates the product of bivariate likelihoods by only using a single step. PML estimation was found effective for small datasets with few discrete variables. In this study, we investigate how PML performs with large datasets and different types of data (e.g., discrete data, continuous data, and combinations thereof).

We conducted two different simulation studies to compare the performance of PML to the DWLS estimation method in terms of accuracy and efficiency. We thereby examined different experimental conditions; model sizes (small, medium, large, and huge), sample sizes (200, 400, and 800), and answer categories (two and four). In addition, we checked the robustness of PML by fitting a model without misspecifications (i.e., correctly specified model) and with misspecifications (i.e., misspecified model). ANOVAs were conducted to test whether the differences between PML and DWLS depend on the aforementioned design factors. Regarding the performance of PML and DWLS, our results indicate that the (relative) bias of both the parameter estimates and the standard errors remain very small among the varying experimental conditions for the correctly specified model and slightly increases in conditions with a misspecified model. Overall, our findings demonstrate that PML performed slightly better compared to DWLS in terms bias of both the parameter estimates and the standard errors.

# Chapter 1

# Introduction

Today's often used estimation method for analysing discrete data within the confirmatory factor analysis (CFA) context, is (diagonally) Weighted Least Squares (DWLS; Browne, 1984; Jöreskog, 1969; Muthén, 1984). However, less is known about the novel Pairwise Maximum Likelihood (PML) estimation method. The goal of the current study is to evaluate the performance of the PML estimates. DWLS estimation is a multi-step procedure in which the parameter estimates obtained in the first step do not reflect the sampling variability of the third step. In contrast, PML is able to estimate all parameters in one step simultaneously incorporating all sampling variability at once (Katsikatsou et al., 2012). In this study, two simulation studies will be conducted to compare the accuracy and efficiency of the estimated PML model parameters to the more common DWLS estimates. Following the proposed recommendation of Katsikatsou and colleagues (2012), this is the first study evaluating PML in large datasets.

In social and behavioural sciences, the researchers' interest often lies in examining (psychological) concepts, such as personality, intelligence, and motivation. In many cases, questionnaires or tests serve as a tool to measure such unobserved, latent variables. To identify the relationships between the observed and the latent variables in the data, structural equation modeling (SEM) is the standard analysis technique (Bollen, 1989). The measurement part of the model in factor analysis relates the latent variables to their observed indicators. In case there are no structural parameters (i.e., regressions) between the latent variables and the focus is only on the measurement part of the model, it is typically referred to as factor analysis (Thurstone, 1947).

Factor analysis is a classical statistical technique that can be divided into two types: exploratory factor analysis, which is data driven and mainly addresses the evaluation of a test's dimensionality, and CFA, which tests whether the observed variables form meaningful indicators to examine the latent variables that are inferred from the data (Jöreskog, 1969). In other words, CFA offers a more parsimonious and hypothesis-driven understanding of the dependencies among a set of those potentially meaningful observed variables and expresses those dependencies through the inferred latent responses. In this study, we will specifically focus on CFA. CFA was originally developed for analysing continuous data using the Maximum Likelihood (ML) fitting function (Brown and Moore, 2012). This method has been widely regarded as the standard iterative estimation approach in the SEM framework with optimal statistical properties if all the assumptions (e.g., normality) hold, and if the sample size is sufficiently large (Bollen, 1989). Throughout the years, SEM was developed to analyse different types of data (e.g., non-normal, discrete, multilevel data). In social sciences, discrete data are commonly collected using questionnaires in which subjects express their level of agreement, ranging from 'strongly disagree' to 'strongly agree'. Here, we will focus on the analysis of discrete data, such as data that stems from questionnaires with Likert point scales, or combinations of discrete and continuous data.

In analysing discrete data, we can distinguish two different approaches; full information approaches and limited information approaches. The full information methods use all information in the data. Two well-known full information approaches are Marginal Maximum Likelihood (MML; Bock and Aitkin, 1981) and Bayesian estimation method (e.g., Lee, 2007). However, the computational complexity of these methods increases rapidly with the number of latent and observed variables (Katsikatsou et al., 2012; Fox, 2010). Throughout this study, we will therefore focus on limited information methods which are mostly developed in the SEM framework. These limited information methods only use the summary of the data and won't be affected by the number of latent and observed variables. The most popular limited information method for analysing discrete data is multi-step Diagonally Weighted Least Squares (Browne, 1984; Muthén, 1984; DiStefano and Morgan, 2014; Sajid et al., 2021). Another limited information method, which is relatively unknown, is the so-called Pairwise Maximum Likelihood (PML) estimation method. This method was introduced inside the SEM framework by Jöreskog and Moustaki (2001) and has been examined by various researchers since (e.g., Varin, 2008; Varin et al., 2011; Katsikatsou et al., 2012; De Leon, 2005). The PML estimation calculates the product of bivariate (and sometimes univariate) likelihoods, as Jöreskog and Moustaki (2001) suggested. Within the SEM framework, PML already demonstrated good performance (i.e., low standard errors) in small datasets (Katsikatsou et al., 2012; Jöreskog and Moustaki, 2001; Barendse et al., 2016). Although Varin et al. (2011) suggested PML to be an effective alternative estimation method for large data, PML has not yet been thoroughly examined for large datasets.

Building on prior research of Katsikatsou and colleagues (2012) about the PML estimation method, we will perform two simulation studies to investigate this method in large datasets. Each of the simulation studies will compare the PML estimation method with DWLS. As DWLS is a well-known estimation method, DWLS will serve as benchmark to judge the performance of PML. The first simulation study addresses on the robustness of PML compared to DWLS using only discrete data, while the aim of the second simulation study is the same using a combination of discrete and continuous (i.e., mixed-type) data. Up until now, to the best of our knowledge, this is the first study that investigates the performance of the PML estimates based on large datasets with mixed-type data.

The outline of this thesis is organized as follows: Chapter 2 provides a theoretical framework consisting of a brief overview of CFA for continuous data followed by the mathematical fundamentals of the PML method and a brief explanation of the benchmark estimation method (i.e., DWLS). In Chapter 3, the method of the two simulation studies will be described to compare PML and DWLS in terms of accuracy and efficiency of the parameter estimates under different conditions. The results of the simulation studies will be discussed in Chapter 4. Chapter 5 covers the discussion and provides our future perspective on PML.

# Chapter 2

# Theoretical framework

## 2.1 Confirmatory Factor Analysis

The linear CFA relates the latent variables to their observed variables (i.e., indicators) and is defined as

$$\boldsymbol{y} = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\zeta} + \boldsymbol{\epsilon} \tag{2.1}$$

where $\boldsymbol{y}$ is a $p \times 1$ vector of observed variables ($\mathbf{Y} = \{\mathbf{y}_1, \mathbf{y}_2, \ldots \mathbf{y}_p\}$), $\boldsymbol{\nu}$ is a $p$-dimensional vector of intercepts. $\boldsymbol{\Lambda}$ is a $p \times m$ matrix of factor loadings, in which $m$ refers to the number of latent factors, $\boldsymbol{\zeta}$ is a vector of latent factor scores with $\zeta \sim N(0, \phi)$ and $\boldsymbol{\epsilon}$ is a $p$-dimensional vector of residual terms with $\epsilon \sim N(0, \Theta)$. The measurement errors are uncorrelated to the latent factor scores, i.e., $Cov(\zeta, \epsilon) = 0$. It follows that the variance–covariance matrix $\boldsymbol{\Sigma} = Var(\boldsymbol{y})$ and the model–implied mean vector $\mu = E(\boldsymbol{y})$ are given by:

$$\boldsymbol{\Sigma}(\boldsymbol{\theta}) = \boldsymbol{\Lambda}\boldsymbol{\Psi}\boldsymbol{\Lambda}^T + \boldsymbol{\Theta}, \tag{2.2}$$

$$\boldsymbol{\mu}(\boldsymbol{\theta}) = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\alpha}. \tag{2.3}$$

The variances and covariances of $\boldsymbol{\zeta}$ and $\boldsymbol{\epsilon}$ are denoted by $\boldsymbol{\Psi}$ and $\boldsymbol{\Theta}$, respectively. The model parameter vector $\boldsymbol{\theta}$ includes the free parameters in $\boldsymbol{\Lambda}$, $\boldsymbol{\Theta}$, $\boldsymbol{\Psi}$, and $\boldsymbol{\nu}$. For given values of $\boldsymbol{\theta}$, the model–implied statistics can be written as $\boldsymbol{\mu}(\boldsymbol{\theta})$ and $\boldsymbol{\Sigma}(\boldsymbol{\theta})$ which are functions of the free parameters. To achieve model identifiability, some parameters should be fixed. This can either be done by fixing the variance of the latent factor, by fixing a factor loading to unity and the mean of the factor, or by fixing the intercept to zero. Given a set of data vectors, we seek values of $\boldsymbol{\theta}$ that minimize a discrepancy function (e.g., maximum likelihood or (weighted) least squares) that reflects the distance between the model and the data.

## 2.2 Pairwise Maximum Likelihood Estimation

To estimate the parameters for a CFA, different estimation methods can be used to minimize a discrepancy function, i.e., the distance between the model and the data. In this study, we will examine the novel Pairwise Maximum Likelihood (PML) estimation method. When Jöreskog and Moustaki (2001) introduced PML in the SEM literature, they considered the product of the univariate as well bivariate likelihoods. Later, De Leon (2005) suggested the PML estimation method using only the bivariate information, because the univariate information has no additional value in parameter estimation. The results of Katsikatsou et al. (2012) also demonstrated that the sum of the univariate likelihoods did not change the estimated parameter accuracy.

The log-likelihood of the PML estimation method is calculated as the sum of $p^\star = p(p-1)/2$ components. Each component is the bivariate log-likelihood of two variables (i.e., $j$ and $k$) for individual $i$:

$$\begin{aligned}\log l_i &= \sum_{j=1}^{p-1} \sum_{k=j+1}^{p} [\log f(y_{ij}, y_{ik}; \boldsymbol{\theta})] \\ &= \sum_{j<k} [\log f(y_{ij}, y_{ik}; \boldsymbol{\theta})]. \end{aligned} \tag{2.4}$$

The sum of all individual contributions in a random sample determines the total log-likelihood that equals

$$\log L(\boldsymbol{\theta}; \mathbf{Y}) = \sum_{i=1}^{I} \log l_i. \tag{2.5}$$

The PML estimation method has been introduced for discrete data as the bivariate likelihood calculations are computationally easy to handle. Due to the natural order of these response categories, a discrete variable can be considered as the measured manifestation of a continuous underlying latent variable $y_i^*$. Consequently, the observed score $y_i$ on item $j$ with $C_j$ categories stems from an underlying continuous variable $y_{ij}^*$ with a normal distribution $N(y_{ij}^*; 0, \sigma_j^2)$ and $\tau_{j,c}$ values that refer to thresholds

$$y_{ij} = c_j \Longleftrightarrow \tau_{j,c-1} < y_{ij}^* < \tau_{j,c} \tag{2.6}$$

for categories $c_j = 1, 2, \ldots, C_j$, with $\tau_{j,0} = -\infty$ and $\tau_{j,C} = \infty$. The exact form of $f(y_{ij}, y_{ik}; \boldsymbol{\theta})$ in Equation (2.4) for discrete indicators $j$ and $k$ equals

$$\log f(y_{ij}, y_{ik}; \boldsymbol{\theta}) = \sum_{a=1}^{C_j} \sum_{b=1}^{C_k} I(y_{ij} = a, y_{ik} = b) \log \omega(y_{ij} = a, y_{ik} = b; \boldsymbol{\theta}), \tag{2.7}$$

where

$$\begin{aligned}\omega(y_{ij} = a, y_{ik} = b; \boldsymbol{\theta}) &= \int_{\tau_{j,a-1}}^{\tau_{j,a}} \int_{\tau_{k,b-1}}^{\tau_{k,b}} f(y_{ij}^*, y_{ik}^*; \boldsymbol{\theta}) dy_{ij}^* dy_{ik}^*, \\ &= \Phi(\tau_{j,a}, \tau_{k,b}; \rho_{jk}) - \Phi(\tau_{j,a-1}, \tau_{k,b}; \rho_{jk}) - \\ &\quad \Phi(\tau_{j,a}, \tau_{k,b-1}; \rho_{jk}) + \Phi(\tau_{j,a-1}, \tau_{k,b-1}; \rho_{jk}), \end{aligned} \tag{2.8}$$

where $\rho_{jk}$ is the model implied correlation between $y_{ij}^*$ and $y_{ik}^*$, and $\Phi(\tau_1, \tau_2; \rho)$ is the bivariate cumulative normal distribution with correlation $\rho$ evaluated at the point ($\tau_1, \tau_2$; Jöreskog and

Moustaki, 2001). For discrete data, the metric for $\boldsymbol{y}^*$ needs to be determined by using the delta- or the theta-parameterization. Within the SEM framework the delta parameterization is implemented, where $\boldsymbol{\Theta}$ is given by $\boldsymbol{\Theta} = \boldsymbol{\Delta}^{-2} - diag(\boldsymbol{\Sigma}^*)$ and in which $\boldsymbol{\Sigma}^*$ can be defined as $\boldsymbol{\Sigma}^* = \boldsymbol{\Lambda\Psi\Lambda}^T$.

Although the PML estimation method was originally developed for discrete data, Equations (2.4) and (2.4) can be more widely interpreted, thereby allowing us to incorporate more complex types of data into PML (e.g., Barendse and Rosseel, 2020). In case of a combination of a discrete and a continuous variable, we can estimate the association between those two variables using polyserial correlations (Olsson et al., 1982). For polyserial correlations, the likelihood is a conditional distribution for continuous variable $y_{ik}$ and discrete variable $y_{ij}$:

$$\log f(y_{ij} = a, y_{ik}; \boldsymbol{\theta}) = [log(y_{ik}) + log(y_{ij=a}; y_{ik})] \tag{2.9}$$

where

$$f(y_{ik}) = (2\pi\sigma_k)^{-\frac{1}{2}} \mathrm{e} \left[ -\frac{1}{2} \left( \frac{y_{ik} - \mu_k}{\sigma_k} \right)^2 \right]. \tag{2.10}$$

The log-likelihood is calculated with respect to $\mu_{y_k}$, $\sigma^2_{y_k}$, $\rho_{y_j,y_k}$, and $\tau_{y_j}$.

$$Z = \frac{(y_{ik} - \mu_k)}{\sigma_k} \tag{2.11}$$

Let $Z$ be the conditional probability that can be obtained by noting that the conditional distribution of $y^*_{ij}$ given $y_{ik}$ is normal with $\mu = \rho Z$ and variance $\sigma^2 = (1 - \rho^2)$. Therefore, the conditional distribution of $y_{ij}$ is

$$logf(y_{ij} = a | y_{ik}) = \Phi(\tau_j) - \Phi(\tau_{j-1}) \tag{2.12}$$

where

$$\tau_{j,a} = \frac{\tau_j - \rho Z}{(1 - \rho^2)^{1/2}}. \tag{2.13}$$

In case both variables (i.e., $y_{ij}, y_{ik}$) are continuous, we can calculate the Pearson correlation (e.g., Benesty et al., 2009). This correlation relies on the bivariate normal probability density function with $\log f(y_{ij}, y_{ik}; \boldsymbol{\theta}) = \log f(y_{ij}, y_{ik})$. In this two-dimensional space, the probability density function of random variables $\boldsymbol{y}_j$ and $\boldsymbol{y}_k$ equals:

$$f(y_{ij}, y_{ik}) = \frac{1}{2\pi\sigma_{y_j}\sigma_{y_k}\sqrt{1-\rho^2}} \mathrm{e}^{-\frac{1}{2(1-\rho^2)} \left[ \left( \frac{y_{ij} - \mu_{y_j}}{\sigma_{y_j}} \right)^2 - 2\rho \left( \frac{y_{ij} - \mu_{y_j}}{\sigma_{y_j}} \right) \left( \frac{y_{ik} - \mu_{y_k}}{\sigma_{y_k}} \right) + \left( \frac{y_{ik} - \mu_{y_k}}{\sigma_{y_k}} \right)^2 \right]} \tag{2.14}$$

where $\rho$ is the correlation between $\boldsymbol{y}_j$ and $\boldsymbol{y}_k$, $\sigma_{\boldsymbol{y}_j} > 0$, $\sigma_{\boldsymbol{y}_k} > 0$, and where

$$\mu = \begin{pmatrix} \mu_{\boldsymbol{y}_j} \\ \mu_{\boldsymbol{y}_k} \end{pmatrix}, \Sigma = \begin{pmatrix} \sigma^2_{\boldsymbol{y}_j} & \rho\sigma_{\boldsymbol{y}_j}\sigma_{\boldsymbol{y}_k} \\ \rho\sigma_{\boldsymbol{y}_j}\sigma_{\boldsymbol{y}_k} & \sigma^2_{\boldsymbol{y}_k} \end{pmatrix}. \tag{2.15}$$

## 2.3 Benchmark estimation method: Diagonally Weighted Least Squares

As the PML estimation method is a novel estimation method, we have used the most well-known Weighted Least Squares technique, i.e., diagonally WLS (DWLS; Muthén et al., 1997), as a benchmark. The DWLS is a well-established estimation method that is developed in the SEM framework and implemented in most SEM computer programs (e.g., MPlus; Muthén and Muthén, 2009; and the R-package *lavaan*; Rosseel, 2012). The general Weighted Least Squares method is based on a three-steps procedure. In the first stage, the thresholds are estimated using the univariate data. In the second stage, the $p\times$ polychoric correlations, polyserial correlation, and bivariate correlations are estimated (see paragraph 2.2 that describes the PML estimation method). In the third stage, the model parameters are estimated using the weighted least squares estimation method. The general discrepancy function is given by

$$F_{DWLS} = (\boldsymbol{q} - \hat{\boldsymbol{g}})'\boldsymbol{W}^{-1}(\boldsymbol{q} - \hat{\boldsymbol{g}}), \tag{2.16}$$

where $\boldsymbol{q}$ denotes a vector with non-redundant sample-based statistics, $\hat{\boldsymbol{g}}$ is a vector with the non-redundant model-based statistics (i.e., thresholds and polychoric correlations), and $\boldsymbol{W}^{-1}$ refers to the inverse of a weight matrix that estimates the asymptotic covariance matrix of $\sqrt{I}\boldsymbol{q}$ (see Muthén et al., 1997). In the least squares framework (see Browne, 1984; Muthén et al., 1997), there are different choices for $\boldsymbol{W}$ (e.g., unweighted least squares). In this study, the diagonal of the weight matrix $\boldsymbol{W}$ will be used, referred as the DWLS.

# Chapter 3

# Simulation studies

Two simulation studies were conducted to evaluate the PML estimation method for large datasets. The distinctive factor between the two simulation studies is the nature of the data. The first simulation study consists of discrete data only, and the second simulation study contains a combination of discrete and continuous data. The simulation studies allowed us to assess the accuracy and efficiency of the parameter estimates under different conditions. This chapter describes the design of the experiment, data generation process, analyses, and performance criteria.

## 3.1 Design

Several experimental conditions were created by systematically varying a number of facets. As the experimental conditions slightly differ among the two simulation studies, they will be described separately.

### 3.1.1 Simulation study 1: discrete data

Regarding the design of the first simulation study, the following facets were systematically varied: the response scales of the indicator variables (dichotomous and four-point), model size (small, medium, large, and huge), and sample size ($N \in 200, 400, 800$). This resulted in a full factorial design of 24 unique experimental conditions (see Table 3.1).

### 3.1.2 Simulation study 2: mixed data

In the second simulation study we varied model size (two, four, six, and eight factors) and sample size ($N \in 200, 400, 800$). Since the second simulation study consists of a combination between discrete and continuous data, a four-point answer scale was chosen for the discrete part of the simulation study. In total, twelve unique conditions were formed (see Table 3.2).

## 3.2 Data generation

In order to explain the set-up of the general data generation process, conditions with the small sized model (i.e., two latent variables and twelve items) will be described first (see conditions 1,

Table 3.1: Unique experimental conditions for the first simulation study with discrete data

| **cond** | **nfact** | *N* | **ncat** | **nitems** | **nmis** | **cond** | **nfact** | *N* | **ncat** | **nitems** | **nmis** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 200 | 2 | 12 | 1 | 13 | 2 | 200 | 4 | 12 | 1 |
| 2 | 4 | 200 | 2 | 24 | 2 | 14 | 4 | 200 | 4 | 24 | 2 |
| 3 | 6 | 200 | 2 | 36 | 3 | 15 | 6 | 200 | 4 | 36 | 3 |
| 4 | 8 | 200 | 2 | 48 | 4 | 16 | 8 | 200 | 4 | 48 | 4 |
| 5 | 2 | 400 | 2 | 12 | 1 | 17 | 2 | 400 | 4 | 12 | 1 |
| 6 | 4 | 400 | 2 | 24 | 2 | 18 | 4 | 400 | 4 | 24 | 2 |
| 7 | 6 | 400 | 2 | 36 | 3 | 19 | 6 | 400 | 4 | 36 | 3 |
| 8 | 8 | 400 | 2 | 48 | 4 | 20 | 8 | 400 | 4 | 48 | 4 |
| 9 | 2 | 800 | 2 | 12 | 1 | 21 | 2 | 800 | 4 | 12 | 1 |
| 10 | 4 | 800 | 2 | 24 | 2 | 22 | 4 | 800 | 4 | 24 | 2 |
| 11 | 6 | 800 | 2 | 36 | 3 | 23 | 6 | 800 | 4 | 36 | 3 |
| 12 | 8 | 800 | 2 | 48 | 4 | 24 | 8 | 800 | 4 | 48 | 4 |

*Note. cond* = conditions, *nfact* = number of latent variables, $N$ = sample size, *ncat* = number of answer categories, *nitems* = number of items, *nmis* = number of misspecifications, please note that *nitems* and *nmis* are not included as design factors in the simulation study since they gradually expand as the number of factors increases.

Table 3.2: Unique experimental conditions for the second simulation study with mixed data

| **cond** | **nfact** | *N* | **nitems** | **nmis** |
|---|---|---|---|---|
| 1 | 2 | 200 | 12 | 1 |
| 2 | 4 | 200 | 24 | 2 |
| 3 | 6 | 200 | 36 | 3 |
| 4 | 8 | 200 | 48 | 4 |
| 5 | 2 | 400 | 12 | 1 |
| 6 | 4 | 400 | 24 | 2 |
| 7 | 6 | 400 | 36 | 3 |
| 8 | 8 | 400 | 48 | 4 |
| 9 | 2 | 800 | 12 | 1 |
| 10 | 4 | 800 | 24 | 2 |
| 11 | 6 | 800 | 36 | 3 |
| 12 | 8 | 800 | 48 | 4 |

*Note. cond* = conditions, *nfact* = number of latent variables, $N$ = sample size, ncat = number of answer categories, *nitems* = number of items, *nmis* = number of misspecifications, please note that *nitems* and *nmis* are not included as design factors in the simulation study since they gradually expand as the number of factors increases.

5, 9, 13, 17, and 21 of Table 3.1). Figure 3.1 visualizes all parameters underlying the simulated data for the smallest model size. This model comprises two (correlated) latent variables, each containing six items. One multidimensional item is added to the model. This cross-loading is indicated by a dotted line in Figure 3.1.

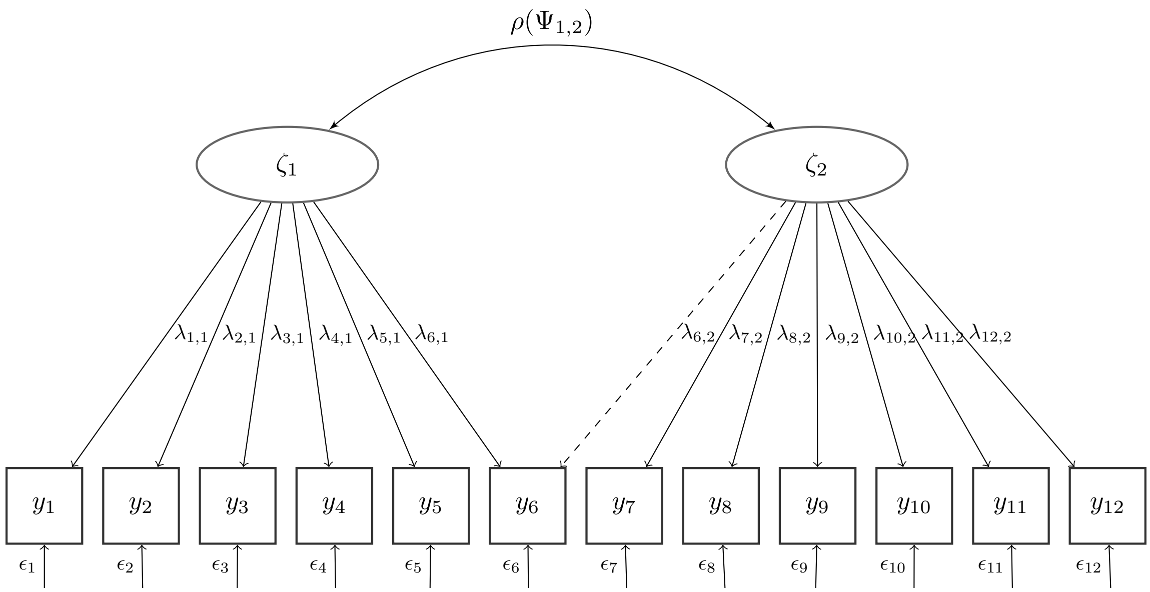


Figure 3.1: Simplest experimental design two latent variables and 12 items ($y$). Note: $\epsilon$ refers to the residual variances. Variances of the latent variables (not shown here) contain equality restrictions for model identification purposes.

Figure 3.1 visualizes all parameters underlying the simulated data for the smallest model size. This model comprises two (correlated) latent variables, each containing six items. One multidimensional item is added to the model. This cross-loading is indicated by a dotted line to item $\mathbf{y_6}$ in Figure 3.1.

Values of the variance-covariance matrix were drawn from the multivariate normal distribution with zero means and a $2 \times 2$ correlation matrix $\mathbf{\Psi}$ to obtain the $\boldsymbol{\zeta}$-values for each individual, according to Equation (2.1) and (2.2). As the mean structure was not of interest, the intercepts equal zero. The chosen true values of the parameters are shown in matrices $\mathbf{\Lambda}$, $\mathbf{\Psi}$, and $\mathbf{\Theta}$:

$$\mathbf{\Lambda} = \begin{bmatrix} 0.8 & \\ 0.8 & \\ 0.7 & \\ 0.7 & \\ 0.6 & \\ 0.6 & 0.2 \\ & 0.8 \\ & 0.8 \\ & 0.7 \\ & 0.7 \\ & 0.6 \\ & 0.6 \end{bmatrix}, \mathbf{\Psi} = \begin{bmatrix} 1 & \\ 0.3 & 1 \end{bmatrix},$$

$$\boldsymbol{\Theta} = \begin{bmatrix} 0.36 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.36 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.51 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.51 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.64 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.53 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.36 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.36 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.51 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.51 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.64 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.64 \end{bmatrix}$$

The factor loadings mentioned above range from high to medium, 0.8 and 0.6 respectively, and are representative of the values regularly observed in empirical and used simulation studies. Based on the non-centrality parameter and power (power varied from approximately .2 for the small model to .9 for the huge model), a cross-loading (indicated by a gray value) of 0.2 is chosen for the multidimensional item. The correlation coefficient between the latent factors is chosen to be 0.3, which corresponds to a medium correlation (Cohen, 1988). Error values were generated from a multivariate normal distribution with zero means and $\boldsymbol{\Theta}$ variance. As can be seen in the $\boldsymbol{\Theta}$ matrix, the values of the diagonal were calculated according to $\boldsymbol{\Theta} = I - diag(\boldsymbol{\Lambda\Psi\Lambda}')$ and referred to as $\epsilon$ in Figure 3.1.

Based on the small sized model with twelve items and two latent factors represented in Figure 3.1, other model sizes were expanded accordingly. In the medium sized models, the number of items and latent factors were doubled, implying models with 24 items and four latent variables. The data generation design of the small model was tripled for the large model (i.e., 36 items and six latent variables) and fourfold for the huge model size (i.e., 48 items and eight latent variables). In these expanded experimental designs, the data generation values in matrices $\boldsymbol{\Lambda}$, $\boldsymbol{\Psi}$, and $\boldsymbol{\Theta}$ were kept the same.

Since the nature of the data is different between the two simulation studies, the unique characteristics of the data generation will be described in the next paragraphs.

### 3.2.1 Simulation study 1: discrete data

For the first simulation study, only discrete data was analysed. To obtain the discrete data, thresholds were computed in such a way that equally sized categories were formed (see Equation (2.6)). For the dichotomous response scale, continuous scores were categorized into two categories with one threshold, i.e., 0, yielding expected proportions of .5 and .5. For the four-point response scale, continuous scores were grouped into categories with three thresholds: -1.2, 0, 1.2, yielding expected proportions of .11, .39, .39, and .11. In total, 400 random datasets were drawn for each of the 24 conditions resulting in $24 \times 400 = 9,600$ unique data sets for each estimation method separately.

### 3.2.2 Simulation study 2: mixed data

Although the design of the second simulation study is kept similar to the first simulation study, the unique part of the second simulation study entails splitting the data into two equal blocks during the data generation process. Taking Figure 3.1 into account, the first block contains the first six items (i.e., $\boldsymbol{y_1}$ to $\boldsymbol{y_6}$). This data is kept continuous. The second block represents the second six items (i.e, $\boldsymbol{y_7}$ to $\boldsymbol{y_{12}}$). For the data of the second block, the same steps were taken as for the four-point response scale of the first simulation study. Again, continuous data were discretized to obtain discrete data yielding the same three thresholds as in the first simulation study: -1.2, 0, 1.2, such that observed variables were assumed to have four response scales. Here, 250 datasets were generated*.

## 3.3 Analyses

In both studies, PML estimation method as well as DWLS estimation method were applied to the simulated data. To test the accuracy of the parameter estimates, a correctly specified model, i.e., a model including the cross-loading, was fitted to the simulated data. To test the robustness of the parameter estimates, a misspecified model, i.e., a model excluding the cross-loading, was also fitted to the data. For the first simulation study, $9,600 \times 2 \times 2 = 38,400$ analyses were conducted in total, including both methods (i.e., PML and DWLS) and model types (i.e., correctly specified and misspecified). For the second simulation study, $3,000 \times 2 \times 2 = 12,000$ analyses were conducted due to the fixed discrete part of the mixed-type data. All analyses were performed using R 4.0.2 (R Core Team, 2020). The R-package *lavaan*, suitable for latent variable modeling, was used to obtain the estimated parameters (Rosseel, 2012).

## 3.4 Performance criteria

### Parameter estimates

The performance criteria of the estimation methods are the accuracy and efficiency of the parameter estimates of the model with and without specified cross-loadings. The performance criteria aim to test the robustness of the model. The relative bias (expressed in %) of the estimated factor loadings, thresholds, and correlation(s) serve as indicator of accuracy. The relative bias of the estimated standard errors of the factor loadings, thresholds, and correlation(s) serve as indicator of efficiency. In addition, to obtain an overview of the performance of both estimation methods separately, the parameter estimate is compared to the true value among all experimental conditions. This difference is called the raw bias.

The equations of the relative bias of the estimated parameters and standard error are respectively:

$$\% \text{ bias} = \frac{100}{R} \sum_{r=1}^{R} \frac{(\hat{\theta}_r - \theta)}{\theta}, \tag{3.1}$$

$$\% \text{ bias} = \frac{100}{R} \sum_{r=1}^{R} \frac{(SE_r - SD)}{SD} \tag{3.2}$$

*Due to time limits, we chose for an unequal number of replications among the simulation studies to keep it computationally feasible. E.g., to carry out one replication for the huge model using PML took about 16 hours.

Here, R denotes the number of replications (with R $\in 400, 250$ in the current study), $\hat{\theta}_r$ denotes the estimate of the parameter at the $r^{th}$ replication and $\theta$ refers to the corresponding true value. To calculate the bias of the standard errors, the $SD$ in Equation (3.2) is used. $SD$ refers to the standard deviation of the parameter estimates across all 400 and 250 replications. The $SE_r$ denotes the estimate of the standard error at the $r^{th}$ replication. Please note that according to Equation (3.2), the $SD$ of the parameter estimates may differ among PML and DWLS. Therefore, both the parameter estimates and standard errors are computed separately for PML and DWLS.

The criteria described above were evaluated for a selected number of parameters that differs among the two simulation studies, because of the nature of the data (see Table 3.3 and Table 3.4). Since all of these values were obtained from the smallest sized model, a sanity check was performed on some other parameter estimates belonging to other blocks to detect any deviating results.

Table 3.3: Unique parameter estimates of interest to be analysed for the first simulation study with discrete data

| **Parameter** | **True value** |
|---|---|
| $\lambda_{6,1}$ | 0.60 |
| $\lambda_{6,2}$* | 0.20 |
| $\lambda_{7,2}$ | 0.80 |
| $\tau_{6,1}$ | 0.00 |
| $\tau_{7,2}$ | 0.00 |
| $\psi_{1,2}$ | 0.30 |

*Note.** Misspecified model does not include the cross-loading. To compare all experimental conditions, only parameters belonging to the small model were selected.

Table 3.4: Unique parameter estimates of interest to be analysed for the second simulation study with mixed-type data

| **Parameter** | **True value** | **Data type** |
|---|---|---|
| $\lambda_{2,1}$ | 0.80 | continuous |
| $\lambda_{6,1}$ | 0.60 | continuous |
| $\lambda_{6,2}$* | 0.20 | continuous |
| $\lambda_{8,2}$ | 0.80 | discrete (four-point) |
| $\lambda_{12,2}$ | 0.60 | discrete (four-point) |
| $\Theta_{2,1}$ | 0.36 | continuous |
| $\Theta_{6,1}$ | 0.53 | continuous |
| $\tau_{8,2}$ | 0.00 | discrete |
| $\tau_{12,2}$ | 0.00 | discrete |
| $\psi_{1,2}$ | 0.30 | continuous |

*Note.** Misspecified model does not include the cross-loading. To compare all experimental conditions, only parameters belonging to the small model were selected.

# Chapter 4

# Results

After estimating all the parameters with both the PML and the DWLS estimation method, the obtained results were inspected for possible outliers. An estimate of the parameter value was considered to be an outlier when it exceeded four standard deviations from the mean parameter estimates evaluated at each unique experimental condition separately. For the estimates of the standard errors, a more lenient approach was considered, i.e., all estimates greater or lower than eight standard deviations from the mean were removed instead. The problematic datasets were removed and replaced by new datasets to maintain the number of replicates per cell of the simulation design. Regarding the first simulation study, 30 datasets contained parameter estimates that were considered as outliers, i.e., 3.8% of all datasets. Of those 30 datasets, 22 datasets contained outliers generated by PML and DWLS as estimation method. There were five unique problematic datasets in which the DWLS was the estimation method that generated outliers and three unique datasets in which PML was the estimation method that generated outliers. For the second simulation study, only one dataset contained outliers with PML as well as DWLS as estimation method (i.e., 0.4% of all datasets).

To investigate the bias, the parameter estimates and associating raw biases will be summarized to obtain a general overview of the performance of both estimation methods. The goal of the tables that provide this overview is to compare the raw biases among different experimental conditions for both estimation methods. This allows us to detect differences between the PML and DWLS estimation method at a detailed level. The following values are reported for all unique conditions separately: the true values, raw means (i.e., parameter estimates), and raw biases (i.e., differences between the true value and parameter estimate).

To test whether the differences between estimation methods depend on the design factors, analyses of variance (ANOVA) were performed in which the estimation method was the within-subjects factor and the design factors (i.e., experimental conditions) were the between-subjects factors. Following Equations (3.1) and (3.2), the performance of the estimators and standard errors of both models are evaluated by calculating the relative bias (in %). The relative biases were entered as dependent variable in the ANOVAs. Since we are interested in differences in relative bias of the two estimation methods (i.e., PML and DWLS), we only considered significant main and interaction effects in which the estimation method was included ($p < .05$). Only effects with the highest order were interpreted following the principal of marginality. Due to the large number of datasets, the statistical power is huge, i.e., the probability that a test of significance will pick up on an effect is very high. Therefore, the generalized eta squared ($\eta^2$) serves as measure of the magnitude of the effect and is computed for all main and interaction effects.

In the following paragraphs, the results of the first simulation study and second simulation study will be described after another. For both simulation studies, the results of the parameter estimates of the correctly specified model will be discussed. After that, the results for the parameter estimates of the misspecified model will be covered. Then, the results of the standard errors of the parameters of both model types will be addressed. The subsections are divided into results based on the raw bias among experimental conditions on one hand and the ANOVA results on the other hand.

## 4.1 Simulation study 1: discrete data

After applying each of the two estimation methods to all 9,600 datasets with discrete data, the estimation methods converged.* To illustrate how both estimation methods perform independently, Appendix A contains tables that include information about estimates and standard errors of the 24 unique experimental conditions (see Table A1, A3, A4, and A5). ANOVA was employed on the relative bias of a convenient selection of parameters to detect whether the differences between estimation methods depended on the design factors. To compare the values of the relative bias based on the ANOVA results for both models, a fixed scale range of 0 to 20 for the y-axis in the plots was chosen for the parameter estimates and a fixed scale range of -5 to 5 for the y-axis was chosen for the standard errors. As indicated in Table 3.3, the parameters of both model types were analysed (i.e., $\lambda_{6,1}$, $\lambda_{6,2}$, $\lambda_{7,2}$, $\tau_{6,1}$, $\tau_{7,2}$, and $\psi_{1,2}$ for the correctly specified model and $\lambda_{6,1}$, $\lambda_{7,2}$, $\tau_{6,1}$, $\tau_{7,2}$, and $\psi_{1,2}$ for the misspecified model).

### Bias of parameter estimates

**Correctly specified model** For the correctly specified model, Table A1 shows very small biases among all experimental conditions, i.e., no estimated parameter deviates more than 0.02 from the true value. This holds for both estimation methods (PML and DWLS). Regarding the differences between the two methods, the ANOVA results revealed two significant effects. For $\lambda_{6,1}$, there was a significant main effect for method, $F(1, 9576) = 11.84, p < .001$, $\eta^2 < .001$, indicating a higher relative bias for DWLS. Although the difference between PML and DWLS in relative bias was statistically significant (i.e., $|-0.020\% - 0.290\%| \approx 0.3\%$ respectively), the difference in raw parameter estimates for PML and DWLS averaged over all experimental conditions was very small (i.e., $|0.600 - 0.602| = 0.002$ respectively). This is consistent with the small size of $\eta^2$ and the results in Table A1 in which the raw bias of PML and DWLS is below 0.01. For $\lambda_{6,2}$, no significant main or interaction effect was found, which indicates that PML and DWLS performed equally well. For $\lambda_{7,2}$, a significant two-way interaction effect between the number of answer

*Sometimes the cluster computer refused to replicate models with eight factors due to computational intensity. However, additional analyses with exactly the same number of seed showed that the analyses were doable, so it was not a structural missing.

categories and method was found, $F(1, 9576) = 4.47, p < .034$, $\eta^2 < .001$. This interaction effect is plotted in Figure 4.1. The difference in relative bias between the two estimation methods is larger in conditions with two answer categories than in the conditions with four answer categories. Based on Figure 4.1, the dotted line representing the PML estimation method is slightly closer to zero than the solid line corresponding to the DWLS estimation method in the case of two answer categories. This difference in relative bias between PML and DWLS is not visible for conditions with four answer categories. For the thresholds of item 6 ($\tau_{6,1}$) and item 7 ($\tau_{7,2}$), no effects approached significance. The same conclusion applies to the factor correlation ($\psi_{1,2}$). This means that there were no significant differences between the two estimation methods.

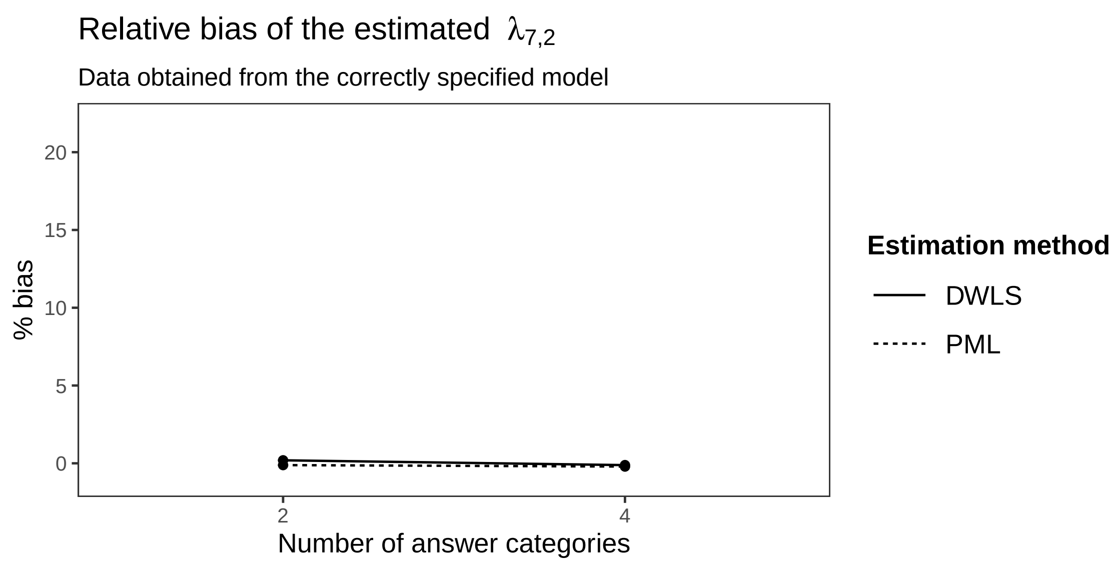


Figure 4.1: Two-way interaction effect between the number of answer categories and estimation method for $\lambda_{7,2}$.

**Misspecified model** For the misspecified model, in which the cross-loading (i.e., $\lambda_{6,2}$) is not part of the measurement model, $\lambda_{6,1}$ was expected to show a higher relative bias compared to the other parameter estimates. The estimate of $\lambda_{6,1}$ reflects the influence of the second latent factor on item 6 which is not part of the misspecified measurement model (see dotted line in Figure 3.1). Overall, as can be seen in Table 4.1, the ANOVA results revealed a number of significant interaction effects. For $\lambda_{6,1}$ and $\lambda_{7,2}$, there were significant three-way interaction effects between sample size, the number of answer categories, and method (see Figure 4.2). For $\lambda_{6,1}$, the difference in relative bias of PML and DWLS depended on both the sample size and the number of answer categories. Also for $\lambda_{7,2}$, the relative bias of the parameter estimate depended on the estimation method, the sample size, and the number of answer categories. The left plot of Figure 4.2 shows that the PML estimation method with four answer categories resulted in the lowest relative bias over all sample sizes. The DWLS estimation method yielded the highest relative bias in conditions with two answer categories when the sample size is 400 ($N$ = 400). The right plot of Figure 4.2 shows slightly larger differences in estimation method in conditions with a smaller sample size ($N$ = 200) regardless of the number of answer categories. As the sample size increases, the difference between the methods becomes smaller. Comparing the plots in Figure 4.2, it should be recognised that the relative bias of both estimation methods in the left plot is much higher than the relative bias in the right plot indicating an overestimation of the parameter estimate of $\lambda_{6,1}$. The same conclusion can be drawn based on the results of Table A3, the raw bias of $\lambda_{6,1}$ averaged over all experimental conditions is 0.11 for PML and 0.12 for DWLS. This corresponds to our expectations of $\lambda_{6,1}$ which is most affected by the misspecification in the model and can therefore be seen as the least robust parameter estimate.

Table 4.1: ANOVA results of the highest order interaction effects between design factors and estimation method for the parameter estimates of the misspecified model.

| **Par. est.** | **F-statistic** | $\eta^2$ | **Figure** | **Sign. result** |
|---|---|---|---|---|
| $\lambda_{6,1}$ | 4.49* | $< .001$ | 4.2 | $N \times$ ncat $\times$ method |
| $\lambda_{7,2}$ | 7.47* | $< .001$ | 4.2 | $N \times$ ncat $\times$ method |
| $\psi_{1,2}$ | 16.89*, 19.10*, 12.93* | $< .001$, $< .001$, $< .001$ | 4.3** | nfact $\times$ method, $N \times$ method, ncat $\times$ method |

*Note.* * indicates an associating $p$-value of <.001. ** Plot (a), (b), and (c) respectively. Par. est. = Parameter estimate, Sign. result = Significant ANOVA result, nfact = number of latent variables, $N$ = sample size, ncat = number of answer categories.

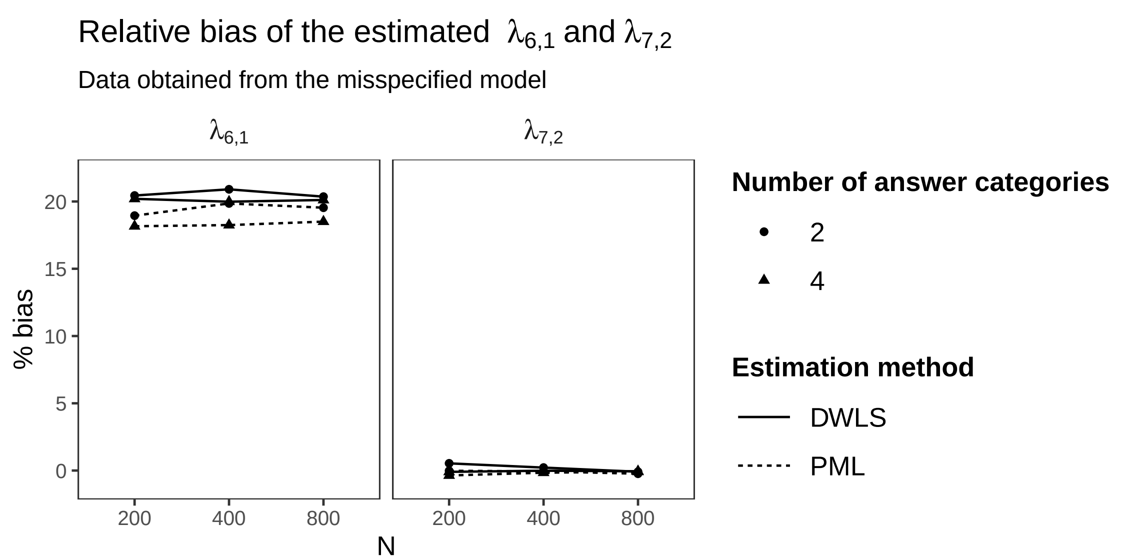


Figure 4.2: Three-way interaction effects between sample size, the number of answer categories, and method for $\lambda_{6,1}$ (left plot) and $\lambda_{7,2}$ (right plot).

No significant effects were found for the thresholds of item 6 ($\tau_{6,1}$) and item 7 ($\tau_{7,2}$), which implies that both estimation methods performed equally well. This corresponds to the results in Table A3, no estimated threshold deviated more than 0.02 from the true value. For the factor correlation ($\psi_{1,2}$), both estimation methods showed a raw bias of 0.05 indicating that the exclusion of the cross-loading in the misspecified model affects the correlation as well (see Table A3). Based on the ANOVA results, three two-way interaction effects appeared to be significant (see Figure 4.3). Regarding Figure 4.3, plot (a) displays an interaction effect between estimation method and number of factors. Plot (b) shows an interaction effect between estimation method and sample size. Plot (c) demonstrates an interaction effect between estimation method and number of answer categories. In general, each plot shows that the PML estimation method is associated with the lowest relative bias for the estimated factor correlation ($\psi_{1,2}$). Furthermore, Figure 4.3 displays a smaller difference in relative bias between the two estimation methods as the number of factors (plot (a)) or the sample size (plot (b)) increases. Plot (c) shows an opposite pattern, i.e., the difference between PML and DWLS is smaller for conditions with two answer categories (ncat = 2) than in conditions with four categories (ncat = 4). Equivalent to the results in Table A3, the relative bias for $\psi_{1,2}$ in Figure 4.3 is high compared to the other significant effects in the misspecified model.

### Bias of Standard Errors

Equivalent to the parameter estimates, tables were created for the standard errors of the parameter estimates as well to see how PML and DWLS performed separately. Furthermore, based on the relative bias of the standard errors of the estimates, the same ANOVAs were conducted on the identical selected parameters as for the parameter estimates described above.

**Correctly specified model** For the correctly specified model, Table A4 displays very small biases for the standard errors of the estimated parameters among the experimental conditions. Focusing on the differences between PML and DWLS, ANOVA revealed statistically significant effects for all the standard errors of the selected parameters. An overview of the results are displayed in Table 4.2. For the standard error of $\lambda_{6,1}$, Figure 4.4 displays subtle differences in performance between PML and DWLS. The plots show that the PML estimation method performs relatively better in terms of relative bias. More specific, if the sample size is 200 ($N = 200$) and the number of answer categories is two (ncat = 2), the four-way interaction effect for the standard error of $\lambda_{6,1}$ shows an increased difference in relative bias between the two estimation methods as the number of latent factors increases. This is indicated by a larger distance between the two methods in the plots when the number of latent factors is eight (factors = 8) compared to two (factors = 2). As the sample size increases, the difference between the two methods becomes smaller (two-way interaction). Especially for the largest sample size, the lines of the two methods coincide despite the number of latent factors. As can be deduced from the plots in Appendix A (Figures A1 - A5), the relative bias of the standard errors of the parameter estimates do not show very different patterns compared to Figure 4.4.

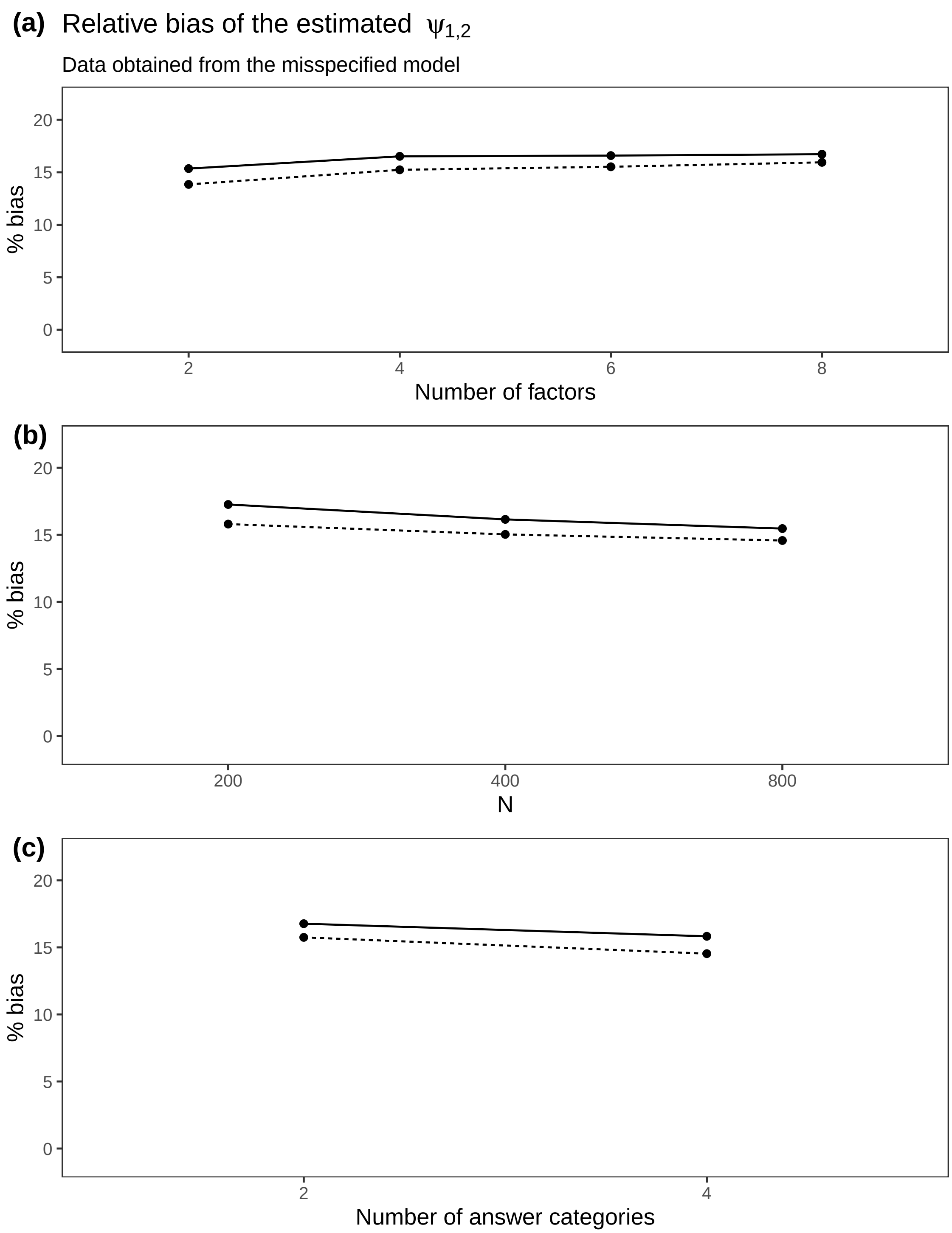


Figure 4.3: Two-way interaction effects; plot a displays interaction between the number of factors and estimation method; plot b shows interaction between sample size and estimation methods; plot c reveals interaction between number of categories and estimation methods. The dashed line and solid line respectively represent the data sets with PML and DWLS as estimation method.

Table 4.2: ANOVA results of the highest order interaction effects between design factors and estimation method for the SE of parameter estimates of the correctly specified model.

| **SE of par.** | **F-statistic** | $\eta^2$ | **Figure** | **Sign. result** |
|---|---|---|---|---|
| $\lambda_{6,1}$ | 25.80* | .001 | 4.4 | nfact × $N$ × ncat × method |
| $\lambda_{6,2}$ | 16.10* | < .001 | A1 | nfact × $N$ × ncat × method |
| $\lambda_{7,2}$ | 4.42* | < .001 | A2 | nfact × $N$ × ncat × method |
| $\tau_{6,1}$ | 80.35* | .001 | A3 | nfact × $N$ × ncat × method |
| $\tau_{7,2}$ | 103.12* | .002 | A2 | nfact × $N$ × ncat × method |
| $\psi_{1,2}$ | 22.81* | < .001 | A5 | $N$ × ncat × method |

*Note.* * indicates an associating p-value of <.001. SE of par. = Standard Error of parameter, Sign. result = Significant ANOVA result, nfact = number of latent variables, $N$ = sample size, ncat = number of answer categories.

**Misspecified model** Table A5 reveals that the standard errors of the parameter estimates are quite accurate, i.e., no raw bias exceeds 0.02. As can be deduced from Table A2, ANOVA showed statistically significant effects for all standard errors of the selected parameters for the misspecified model. Regarding the results of the standard errors of the parameter estimates for the correctly specified model, approximately the same patterns were found in plots for the misspecified model. However, some subtle inequalities can be detected, e.g., for the standard error of $\lambda_{6,1}$. According to Figure 4.5, the lines in all plots follow approximately the same pattern compared to Figure 4.4. However, for the misspecified model, the PML estimation method does not perform better in all conditions in terms of relative bias. In conditions with a sample size of 800 ($N$ = 800) and four answer categories (ncat = 4), the four-way interaction effect for the standard error of $\lambda_{6,1}$ showed an increased difference in relative bias between the two estimation methods as the number of factors increases. In general, when the sample size increases, the difference between the two methods gets smaller. Since the trends of the lines in the plots for the standard errors of the parameters follow approximately the same pattern compared to those based on the correctly specified model, the remaining plots are provided in Appendix A (see Figures A6 - A9).

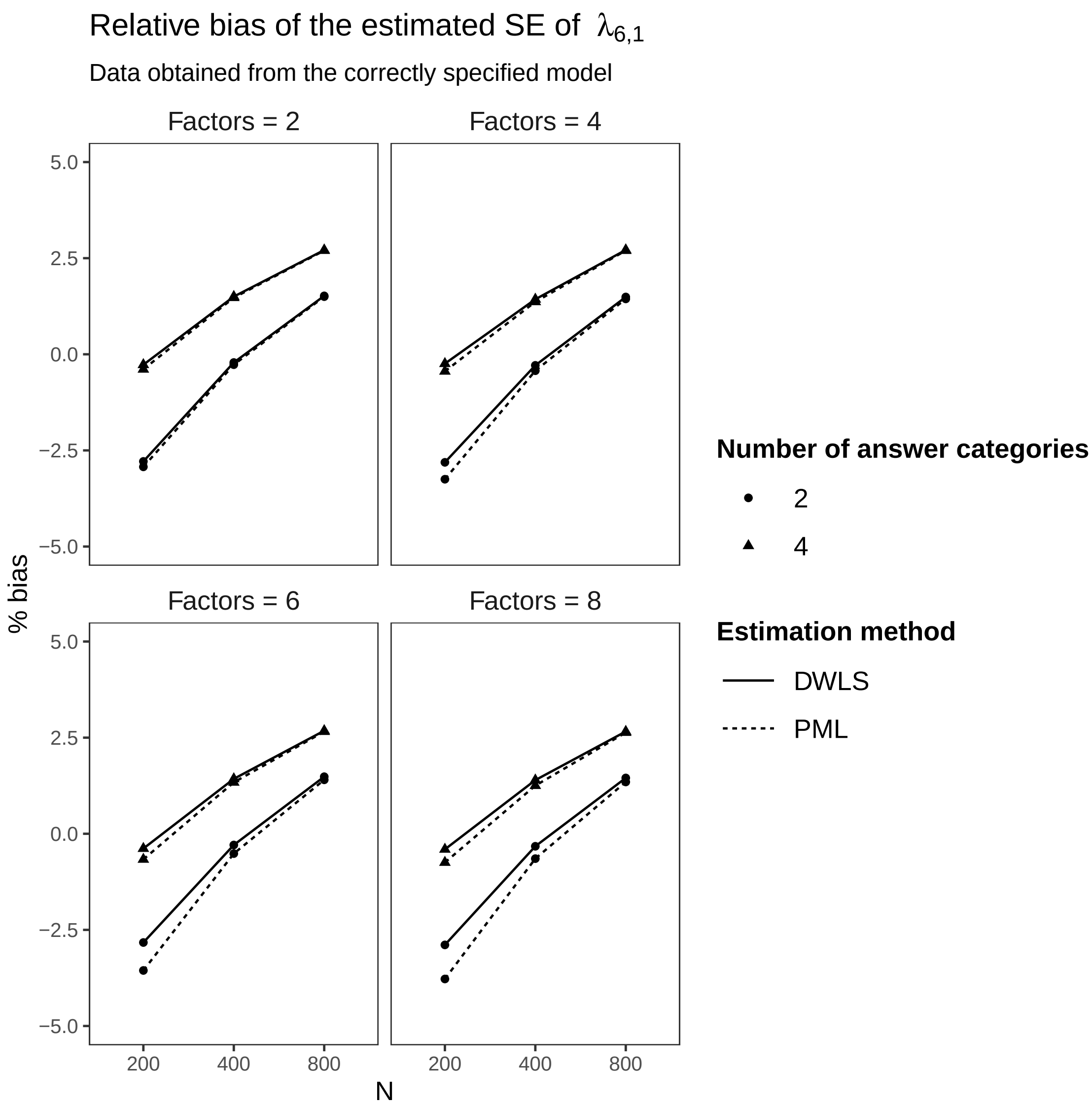


Figure 4.4: Four-way interaction effect between number of latent factors, sample size, number of answer categories and estimation method for $\lambda_{6,1}$

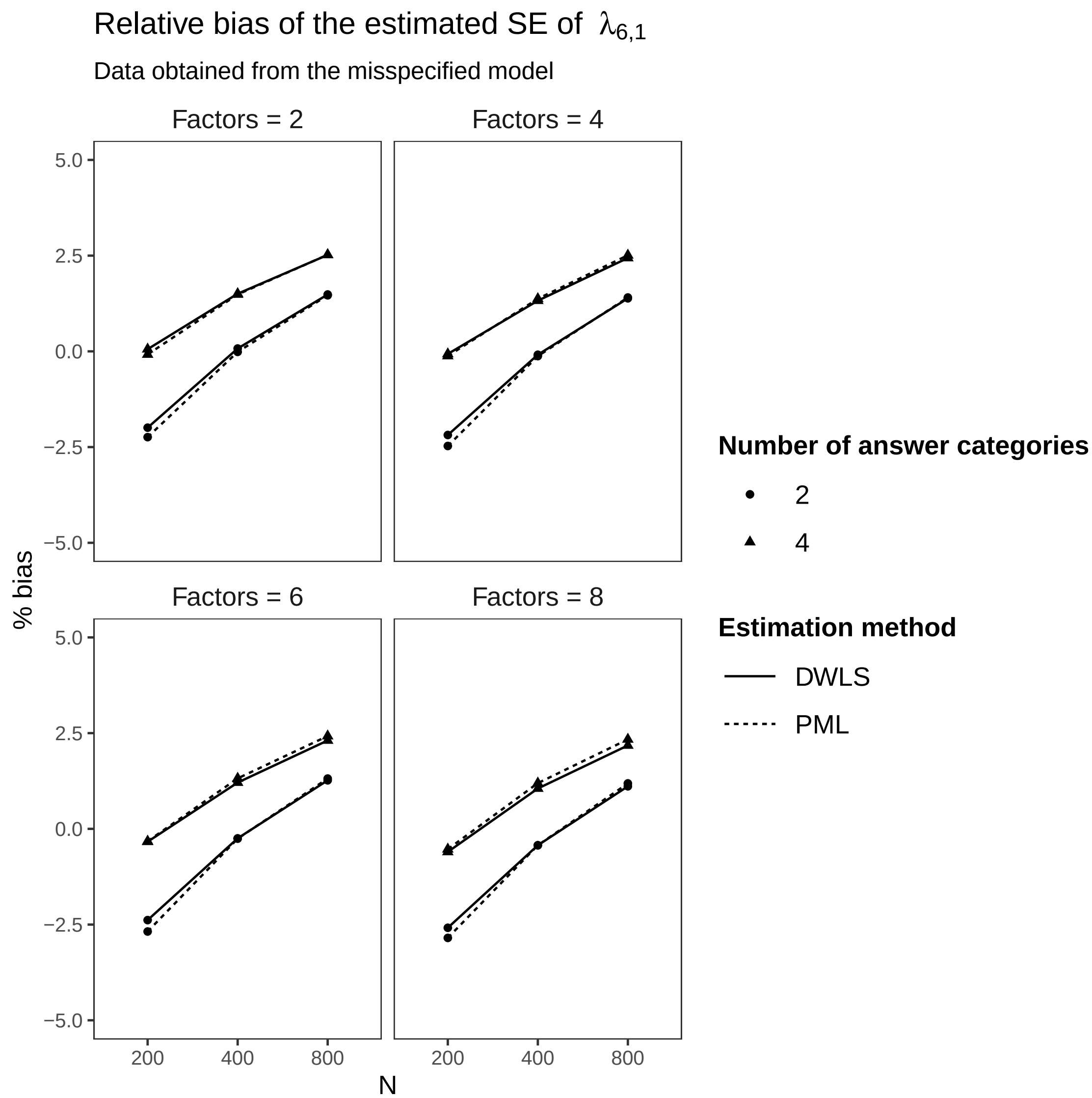


Figure 4.5: Four-way interaction effect between the number of latent factors, sample size, number of answer categories and estimation method for $\lambda_{6,1}$

## 4.2 Simulation study 2: mixed data

After applying each of the two estimation methods to all 6,000 datasets with mixed-type data, the estimation methods converged. Similar to the first simulation study, the performance of the estimation methods separately were shown in Tables B1, B3, B4, and B6 in Appendix B. These tables provide information about the estimates and standard errors of all twelve experimental conditions. Equally, ANOVAs were conducted to discover whether the differences between estimation methods depended on the design factors. However, the design of the second simulation study is somewhat different since one option in the number of answer categories was excluded resulting in fewer experimental conditions. Given the nature of the data of the second simulation study, another series of parameters were selected to evaluate the relative bias. As outlined in Table 3.4, continuous as well as discrete parameter estimates are included in the correctly specified model (i.e., $\lambda_{2,1}$, $\lambda_{6,1}$, $\lambda_{6,2}$, $\lambda_{8,2}$, $\lambda_{12,2}$, $\theta_{2,1}$, $\theta_{6,1}$, $\tau_{8,2}$, $\tau_{12,2}$, and $\psi_{1,2}$). The same parameter estimates were selected for the misspecified model with the exception of the cross-loading, i.e., $\lambda_{6,2}$. The selection of error values, thresholds, and correlations are identical for both the correctly specified and misspecified model. It is worthwhile to note that the y-axis of the plots displaying the relative bias of parameter estimates and the standard errors are fixed in a different way, i.e., ranging from -40 to 20 and ranging from -5 to 5 respectively, to improve comparability.

### Bias of parameter estimates

**Correctly specified model** For the correctly specified model, very small biases were found for both estimation methods. The parameter estimates suggest very accurate results for PML and DWLS, i.e., no raw bias exceeded 0.02. Regarding the differences between PML and DWLS, the results are presented in Table 4.3. No significant effect was found for $\lambda_{2,1}$. This indicates that the performance of the estimation methods is approximately the same. For $\lambda_{6,1}$ and $\lambda_{6,2}$, main effects for estimation method appeared to be significant in which PML was associated with a lower relative bias. Based on the result of $\lambda_{6,1}$, the difference in relative bias between PML and DWLS is small, (i.e., $|-0.19\% - -0.28\%| \approx 0.1\%$ respectively). Regarding $\lambda_{6,2}$, approximately the same conclusion can be drawn about the difference in relative bias between PML and DWLS ($|-0.33\% - -0.65\%| \approx 0.3\%$ respectively). These results were consistent with Table B1 in Appendix B in which raw biases across all conditions were rather small, i.e., no raw biases of PML or DWLS were above 0.02 or below -0.02 for $\lambda_{6,1}$ and $\lambda_{6,2}$. Furthermore, a three-way interaction effect between the estimation method, number of latent factors, and sample size for $\lambda_{8,2}$ was found. Partially due to the wide range of the fixed scale of the y-axis, Figure B1 shows very small differences in relative bias. Even though differences between the two estimation methods can be barely detected with the unaided eye, after manually adjusting the range of y-axis such that the result is amplified, it appears that the the difference in relative bias between the two estimation methods decreases as the number of factors and the sample size ($N$) increase. Based on Table B1 in Appendix B, very low raw biases appeared for PML and DWLS. Furthermore, two-way interaction effects for $\lambda_{12,2}$ were found. The number of latent factors and method had a significant effect on the relative bias of $\lambda_{12,2}$ (see plot a in Figure B2). The estimation method and sample size also showed a significant effect on the relative bias of $\lambda_{12,2}$ (see plot b in Figure B2).

Regarding the selected error values (i.e., $\theta_{2,1}$ and $\theta_{6,1}$), only $\theta_{6,1}$ was showing a statistically significant effect. A two-way interaction effect between estimation method and sample size was found. Figure 4.6 illustrates a larger difference between PML and DWLS in relative bias for the error value for the smallest sample size ($N = 200$). As the sample size increases, the difference between the two estimation methods seems to disappear. Again, because of the scale of the y-axis, differences are hard to detect. Surprisingly, the relative bias of $\theta_{6,1}$ is considerably lower (around -20%, indicating an underestimation of the parameter estimate) than the relative bias of the factor loadings (around 0%).

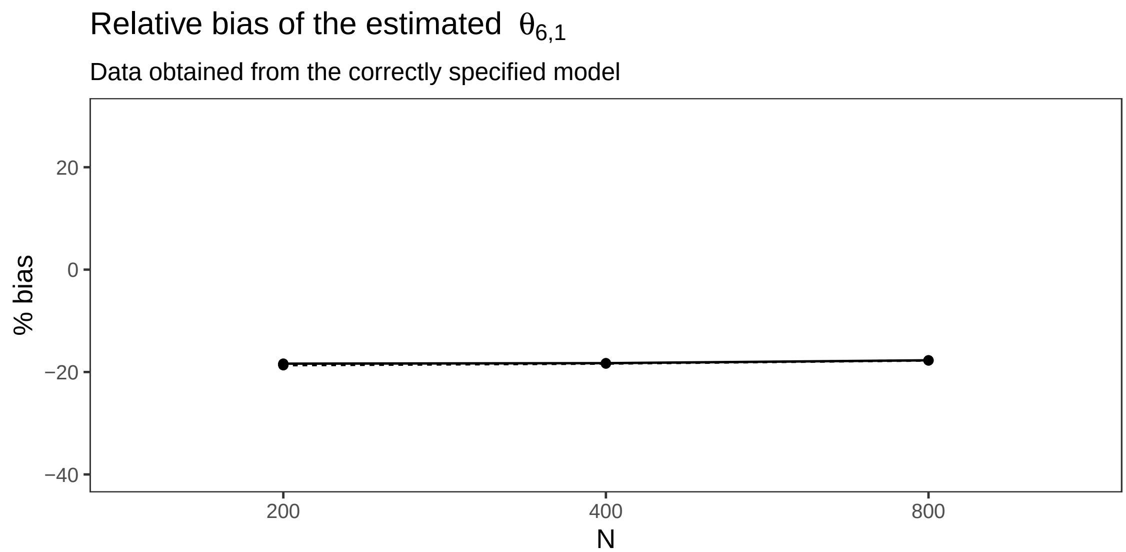


Figure 4.6: Two-way interaction effect between sample size and estimation method for $\theta_{6,1}$. The dashed line and solid line respectively represent the data sets with PML and DWLS as estimation method.

No significant effects were found for the selected thresholds (i.e., $\tau_{8,2}$ and $\tau_{12,2}$) indicating a similar performance of both PML and DWLS. For the factor correlation ($\psi_{1,2}$), a three-way interaction effect between the estimation method, number of latent factors, and sample size was statistically significant. As can be seen in Figure B3, if the number of latent factors is 2 (factors = 2), both estimation methods showed a somewhat larger relative bias for the highest sample size ($N$ = 800). This difference was not visible for the other number of latent factors.

Table 4.3: ANOVA results of the highest order interaction effects between design factors and estimation method for the parameter estimates of the correctly specified model.

| **Par. est.** | **F-statistic** | ***p*-value** | $\eta^2$ | **Figure** | **Sign. result** |
|---|---|---|---|---|---|
| $\lambda_{6,1}$ | 5.14 | .023 | $< .001$ | - | method |
| $\lambda_{6,2}$ | 25.62 | $< .001$ | $< .001$ | - | method |
| $\lambda_{8,2}$ | 2.31 | .031 | $< .001$ | B1 | nfact × $N$ × method |
| $\lambda_{12,2}$ | 7.18, 155.82 | $< .001$, $<.001$ | $< .001$, $< .001$ | B2* | nfact × method, $N$ × method |
| $\theta_{6,1}$ | 5.50 | .004 | $< .001$ | 4.6 | nfact × method, $N$ × method |
| $\psi_{1,2}$ | 2.32 | .031 | $< .001$ | B3 | nfact × $N$ × method |

*Note.* * Plot (a) and (b) respectively. Par. est = parameter estimate, Sign. result = Significant ANOVA result, nfact = number of latent variables, $N$ = sample size.

**Misspecified model** For the misspecified model, in which the cross-loading is not part of the model, some parameter estimates showed higher raw biases. As can be seen in Table B3, especially the parameters estimated with DWLS as estimation method revealed higher biases. ANOVAs were performed to see whether the differences between PML and DWLS were significant. Table 4.4 provides an overview of all significant results. Below, only remarkable deviating results will be discussed. According to Table B3, the raw bias of $\lambda_{2,1}$ for PML and DWLS was -0.01 and -0.03, respectively. Based on the ANOVA results, this difference in performance between the estimation methods was illustrated by a significant main effect for $\lambda_{2,1}$. The relative bias of PML was -0.192 and for DWLS -0.280 (i.e., difference of $|-0.19 - -0.28| = 0.09\%$). In addition, the effect size was higher for this parameter estimate compared other parameter estimates. Furthermore, Table B3 illustrates high raw biases for $\lambda_{6,1}$. This is the factor loading that is most affected by the exclusion of the cross-loading. Raw biases of 0.10 and 0.14 were found for PML and DWLS, respectively. Based on the ANOVA results, a two-way interaction effect appeared between estimation method and the number of factors. The difference between PML and DWLS in terms of relative bias decreases as the number of factors increases. Furthermore, Figure 4.7 displays a clear distinction between the PML and DWLS estimation method which is consistent with the results of the raw biases in Table B3. Although the difference between PML and DWLS decreases as the number of factors increases, the relative bias associated with PML remains lower every model size (i.e., number of latent factors).

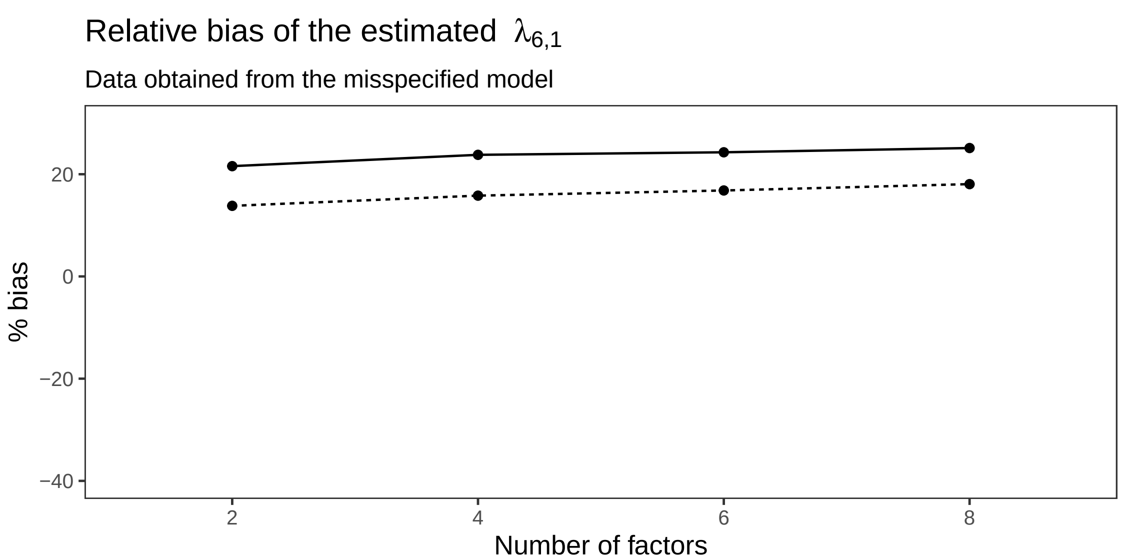


Figure 4.7: Two-way interaction effect between sample size and estimation method for $\lambda_{6,1}$. The dashed line and solid line respectively represent the data sets with PML and DWLS as estimation method.

As indicated in Table B3, a significant main effect appeared for method for $\theta_{2,1}$. The relative bias was 2.957 and 9.000 for PML and DWLS respectively (i.e., difference is $|2.957 - 9.000| = 6.043\%$). This significant result was associated with a relatively high effect size ($\eta^2 = .048$). Based on the results in Table B3, the difference in estimation method was also exposed (raw bias of 0.01 and 0.03 for PML and DWLS respectively). For $\theta_{6,1}$, a two-way interaction effect between estimation method and sample size showed a distinctive difference in relative bias for the estimation methods. Figure B6 in Appendix B illustrates a smaller bias for PML compared to DWLS over all sample sizes. Again, this points towards a better performance of PML for estimating the error value ($\theta_{6,1}$) for the misspecified model. This corresponds to the results of Table B3 in which the raw biases were -0.02 and -0.08 for PML and DWLS respectively. As can be seen in Table B5, no significant effects appeared for the thresholds. For $\psi_{1,2}$, a three-way interaction effect between the estimation method, number of latent factors, and sample size was statistically significant. Based on Figure B7 in Appendix B, the degree of relative bias is similar to the factor correlation in the misspecified model of the first simulation study (see Figure 4.3). Higher relative biases were found across all sample sizes and number of latent factors. As before, the PML indicates a better performance compared to DWLS, i.e., the dashed line continues to be below the solid line indicating a lower relative bias. This is consistent with the results of the raw bias in Table B3 in which PML was associated with a lower raw bias than DWLS.

Table 4.4: ANOVA results of the highest order interaction effects between design factors and estimation method for the parameter estimates of the misspecified model.

| **Par. est** | **F-statistic** | ***p*-value** | $\boldsymbol{\eta^2}$ | **Figure** | **Sign. result** |
|---|---|---|---|---|---|
| $\lambda_{2,1}$ | 2432.33 | $< .001$ | .020 | - | method |
| $\lambda_{6,1}$ | 12.44 | $< .001$ | $< .001$ | 4.7 | nfact $\times$ method |
| $\lambda_{8,2}$ | 2.65 | .014 | $< .001$ | B4 | nfact $\times$ $N$ $\times$ method |
| $\lambda_{12,2}$ | 6.61, 153.91 | $< .001$ | $< .001$, $< .001$ | B5* | nfact $\times$ method, $N$ $\times$ method |
| $\theta_{2,1}$ | 2369.86 | $< .001$ | .048 | - | method |
| $\theta_{6,1}$ | 5.56 | $< .001$ | $< .001$ | B6 | $N$ $\times$ method |
| $\psi_{1,2}$ | 2.62 | .015 | $< .001$ | B7 | nfact $\times$ $N$ $\times$ method |

*Note.* * Plot (a) and (b) respectively. Par. est = parameter estimate, Sign. result = Significant ANOVA result, nfact = number of latent variables, $N$ = sample size.

### Bias of Standard Errors

Since the results of the relative bias of the standard error of the parameter estimates did not exceed values below -5 and above 5, the scale of the y-axis was adjust to a range of [-5,5]. Although ANOVAs indicated many significant results, it implies that the relative bias does not deviate a lot from zero.

**Correctly specified model** As can be derived from Table B4, very small biases were found for both estimation methods indicating accurate results for PML and DWLS, i.e., no raw bias exceeded 0.02. Regarding the ANOVAs, Table B2 depicts the results for the standard errors of the parameter estimates for the correctly specified model. Figures B8 to B17 display the associating plots which do not show very deviating results. Although the figures indicate small differences between the two estimation methods, it is remarkable that the effect size is relatively high for $\lambda_{2,1}$ and $\psi_{1,2}$, indicating that PML and DWLS differ in relative bias. For $\lambda_{2,1}$, two significant two-way interactions were found (see Table B2). As can be deduced from Figure B8, plot (a) indicates that difference in relative bias between the estimation methods increases as the number of factors increases. Plot (b) displays a smaller difference in relative bias between the estimation methods as the sample size increases. For $\psi_{1,2}$, two two-way interaction effects appeared to be statistically significant. Based on Figure B17, plot (a) shows an interaction effect between the estimation method and the number of factors. Plot (b) displays an interaction effect between the estimation method and sample size. Although the effect size of the second two-way interaction effect was relatively high ($\eta^2 = .022$), the difference in relative bias between the two estimation methods is barely noticeable in plot (b) in Figure B17. Only a small decrease in difference in relative bias between PML and DWLS is visible as the sample size increases.

**Misspecified model** Based on the results of the standard error of the parameter estimates of the misspecified model, Table B6 reports relatively small raw biases for both estimation methods (i.e., no raw bias exceeded 0.03).

As can be deduced from Table B5, ANOVA showed statistically significant effects for all standard errors of the selected parameters for the misspecified model. Figures B18 to B26 provide the associating plots. Based on the size of $\eta^2$, the two two-way interaction effects of the standard error of $\lambda_{1,2}$ stand out compared to the other standard errors of the parameter estimates. Regarding Figure B18, plot (a) indicates an interaction effect between the estimation method and the number of factors. As the number of factors increases, the difference between the estimation method also increases. Plot (b) shows an interaction effect between the estimation method and sample size. Here, the difference between the two estimation methods becomes smaller as the sample size increases.

# Chapter 5

# Discussion

Discrete data is often analysed with limited information methods within SEM. A well-known limited information method is the Diagonally Weighted Least Squares estimation method (DWLS). DWLS is a multiple-step procedure in which parameter estimates are obtained using three sequential stages. In this study, we examined a rather unknown limited information method, called Pairwise Maximum Likelihood (PML). The PML estimation method is able to estimate all model parameters in one step.

Two simulation studies were conducted to investigate the performance of PML in large datasets in terms of the accuracy and efficiency. In these simulation studies PML was compared to DWLS, where DWLS served as benchmark to judge the performance of the PML estimation method. Discrete and mixed-type data were used in the first and second simulation study respectively. By systematically varying design factors, several experimental conditions were created.

Overall, PML achieved accurate results in parameter estimation among almost all experimental conditions, i.e., the mean raw biases for PML across all conditions for the first and second simulation study were $|0.003|$ and $|0.006|$ respectively. Regarding the accuracy of the parameter estimates of both simulation studies, PML and DWLS showed similar results with respect to relative bias. The resemblance in parameter estimates is not surprising since both estimation methods rely on bivariate data. The ANOVA results, which display whether the differences between the estimation methods depended on the design factors, indicated that sample size and the number of answer categories occurred most often in the first simulation study. A trend seems to appear in which the smallest sample size ($N = 200$) and fewer number of factors (nfact = 2) are associated with a larger difference in relative bias between the estimation methods. In the second simulation study, sample size was also the most prevalent design factor. Although various effects appeared to be statistically significant, the difference between the PML and DWLS estimation method remained small with a subtle advantage of PML over DWLS. Due to the large number of the datasets, the huge power caused subtle differences to be statistically significant. One main effect worth mentioning is the error value ($\theta_{2,1}$) in the second simulation study for the misspecified model ($\eta^2 = 0.048$) in which PML shows better results compared to DWLS. In general, the relative bias is very small for the correctly specified model (ranging from approximately 0% to 20% for the first simulation study and ranging from -20% to 5% for the second simulation study) and slightly higher in conditions with a misspecification (ranging from approximately -30% to 20% for the first simulation study and ranging from -30% to 30% for the second simulation study). Equivalent to our prior expectations, the most sensitive parameter estimate $\lambda_{6,1}$ in the misspecified model was associated with a higher relative bias for both estimation methods. Due to the exclusion of the cross-loading in the misspecified model, the relative bias of the estimated factor correlation ($\psi_{1,2}$) was also higher for the misspecified model compared to the correctly specified model. For these parameter estimates, PML revealed reduced relative bias compared to DWLS regardless of the design factors. This indicates a higher robustness of PML as estimation method. Regarding the efficiency of the parameter estimates, the standard errors of the parameter estimates are associated with small (relative) biases for both PML and DWLS, i.e., approximately zero bias.

In short, the main result of both simulation studies is that PML performed at least as good or even better than DWLS in terms of (relative) bias. For the majority of the selected parameters, PML showed slightly lower (relative) biases compared to DWLS. The overall conclusion is that PML is comparable to DWLS in terms of accuracy and efficiency since the estimation methods are associated with very close parameter estimates and standard errors. Katsikatsou et al. (2012) investigated the performance of the PML estimator, and its standard error during a preliminary simulation study where they only included six experimental conditions. We expanded the experimental design to see how PML would perform on more elaborate models. Therefore, under the simulated conditions, this study provides evidence to use PML as estimation method in the case of discrete and mixed-type data. We tried to comprise common experimental conditions to keep the design manageable (see for example Barendse and Rosseel, 2020; Xi, 2011; Nestler, 2013). However, it is unfeasible to include all experimental conditions in only two simulation studies. A recommendation would be to include the number of items as separate experimental design factor to be able to investigate its influence.

Overall, analysing data with PML as estimation method comes with several advantages. As Katsikatsou et al. (2012) pointed out earlier, one-step procedures such as the PML estimation method ought to be more efficient than multiple step approaches. In our simulation studies, this could be an explanation of a better performance of PML, since DWLS depends on the diagonal of a huge weight matrix before model parameter estimates can be calculated (as explained in paragraph 2.3). Due to small sample sizes, this weight matrix becomes quite unstable (Li, 2016). PML, by contrast, is able to estimate model parameters without such a matrix. In this one step procedure, final parameter estimates include all sampling variability (Katsikatsou et al., 2012). As a result, PML has various advanced options including multilevel modeling extensions (e.g., Barendse and Rosseel, 2020). Besides, PML estimates are associated with low computational complexity (De Leon, 2005; Katsikatsou et al., 2012). Obtaining the estimates is relatively easy since it only involves evaluation of up to two-dimensional integrals irrespective of the number of observed variables (Katsikatsou et al., 2012).

However, PML is also accompanied by some limitations. Although PML is associated with less computational complexity from a theoretical point of view, the computational time necessary to perform all analyses was much higher for PML compared to DWLS. For one replication, it took several hours to estimate parameters of the most elaborate model with PML as estimation method (see condition 24 in Table 3.1). Hence, the bivariate marginal likelihood is maximized for every distinct pair in each iteration, i.e., for the condition with 48 items, $\binom{48}{2} = 1128$ pairs has to be evaluated. Therefore, improving the PML algorithm to gain computational efficiency could be done in future research. One suggestion could be to evaluate every bivariate item pair after each iteration and assess to what extent the pair adds additional value to the log likelihood. If pairs do not add to the likelihood, they can be deleted. To decrease the computational time of the algorithm, another option would be to obtain bivariate correlations of all items. Before the estimation procedure starts, item pairs that are not related to each other will then be deleted. Besides, although PML is freely available in the *lavaan* package in R (Rosseel, 2012), it should be accessible in other SEM software programs as well, e.g., MPlus, Lisrel, and EQS (Muthén and Muthén, 2009; Jöreskog and Sörbom, 1996; Byrne, 1994).

The results of the current study offer a promising perspective to further expand the knowledge regarding the applicability of the PML estimation method. Further research is needed to apply PML on real mixed-type data to compare estimates and standard errors of PML and DWLS in an illustrative example. Furthermore, additional research should provide overall measures to evaluate the goodness-of-fit (e.g., fit measures indicated by Barendse et al., 2016; Katsikatsou et al., 2012) associated with PML in large datasets (especially in the case of mixed-type data). Such information would allow researchers to test the model fit of PML. Lastly, future research might examine the potential role of missing values in the parameter estimation process. These values can be encountered in the simulation study to further test the robustness of the estimation methods and mimic real-life research settings. Based on promising results of Katsikatsou et al. (2021), PML is expected to perform better compared to DWLS in case of incomplete data with large datasets.

# Chapter 6

# Appendix

## 6.1 Appendix A

**Simulation study 1: discrete data**

Table A1: Table with true values, raw means, and raw bias for the parameter estimates for each of the 24 conditions separately. Values are obtained from the correctly specified model.

| | | | Experimental condition | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Value** | **Parameter** | **Method** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** | **Mean*** |
| True | $\lambda_{6,1}$ | Both | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | - |
| Estimate | $\lambda_{6,1}$ | PML | 0.60 | 0.60 | 0.59 | 0.60 | 0.60 | 0.60 | 0.60 | 0.61 | 0.60 | 0.60 | 0.60 | 0.60 | 0.59 | 0.61 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 |
| Raw bias | $\lambda_{6,1}$ | PML | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Estimate | $\lambda_{6,1}$ | DWLS | 0.60 | 0.60 | 0.59 | 0.60 | 0.60 | 0.60 | 0.60 | 0.61 | 0.60 | 0.60 | 0.60 | 0.60 | 0.59 | 0.61 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 |
| Raw bias | $\lambda_{6,1}$ | DWLS | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| True | $\lambda_{6,2}$ | Both | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | - |
| Estimate | $\lambda_{6,2}$ | PML | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.21 | 0.20 | 0.19 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.19 | 0.20 | 0.19 | 0.20 | 0.20 | 0.20 | 0.19 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 |
| Raw bias | $\lambda_{6,2}$ | PML | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Estimate | $\lambda_{6,2}$ | DWLS | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.21 | 0.20 | 0.19 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.19 | 0.20 | 0.19 | 0.20 | 0.20 | 0.20 | 0.19 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 |
| Raw bias | $\lambda_{6,2}$ | DWLS | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| True | $\lambda_{7,2}$ | Both | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | - |
| Estimate | $\lambda_{7,2}$ | PML | 0.80 | 0.80 | 0.80 | 0.80 | 0.81 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.79 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 |
| Raw bias | $\lambda_{7,2}$ | PML | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Estimate | $\lambda_{7,2}$ | DWLS | 0.80 | 0.80 | 0.80 | 0.80 | 0.81 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.79 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 |
| Raw bias | $\lambda_{7,2}$ | DWLS | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| True | $\tau_{6,1}$ | Both | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | - |
| Estimate | $\tau_{6,1}$ | PML | -0.01 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Raw bias | $\tau_{6,1}$ | PML | -0.01 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Estimate | $\tau_{6,1}$ | DWLS | -0.01 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Raw bias | $\tau_{6,1}$ | DWLS | -0.01 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| True | $\tau_{7,2}$ | Both | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | - |
| Estimate | $\tau_{7,2}$ | PML | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Raw bias | $\tau_{7,2}$ | PML | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Estimate | $\tau_{7,2}$ | DWLS | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Raw bias | $\tau_{7,2}$ | DWLS | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| True | $\psi_{1,2}$ | Both | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | - |
| Estimate | $\psi_{1,2}$ | PML | 0.31 | 0.30 | 0.30 | 0.31 | 0.30 | 0.31 | 0.30 | 0.31 | 0.30 | 0.30 | 0.30 | 0.30 | 0.29 | 0.30 | 0.31 | 0.31 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 |
| Raw bias | $\psi_{1,2}$ | PML | 0.01 | 0.00 | 0.00 | 0.01 | 0.00 | 0.01 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Estimate | $\psi_{1,2}$ | DWLS | 0.31 | 0.30 | 0.30 | 0.31 | 0.30 | 0.31 | 0.30 | 0.31 | 0.30 | 0.30 | 0.30 | 0.30 | 0.29 | 0.30 | 0.31 | 0.31 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 |
| Raw bias | $\psi_{1,2}$ | DWLS | 0.01 | 0.00 | 0.00 | 0.01 | 0.00 | 0.01 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

*Note.* Values < 0.01 are rounded to 0.00. The experimental conditions correspond to those specified in Table 3.1. * calculated across all conditions.

Table A2: ANOVA results of the highest order interaction effects between design factors and estimation method for the SE of parameter estimates of the misspecified model.

| **SE of par.** | **F-statistic** | $\eta^2$ | **Figure** | **Sign. result** |
|---|---|---|---|---|
| $\lambda_{6,1}$ | 7.03* | $<.001$ | 4.5 | nfact $\times$ $N$ $\times$ ncat $\times$ method |
| $\lambda_{7,2}$ | 3.84* | $<.001$ | A6 | nfact $\times$ $N$ $\times$ ncat $\times$ method |
| $\tau_{6,1}$ | 51.98* | $<.001$ | A7 | nfact $\times$ $N$ $\times$ ncat $\times$ method |
| $\tau_{7,2}$ | 99.47* | .002 | A6 | nfact $\times$ $N$ $\times$ ncat $\times$ method |
| $\psi_{1,2}$ | 19.90* | $<.001$ | A9 | $N$ $\times$ ncat $\times$ method |

*Note.* * indicates an associating p-value of <.001. SE of par. = Standard Error of parameter, nfact = number of latent variables, $N$ = sample size, ncat = number of answer categories

Table A3: Table with true values, raw means, and raw bias for the parameter estimates for each of the 24 conditions separately. Values are obtained from the misspecified model.

| Value | Parameter | Method | Experimental condition | | | | | | | | | | | | | | | | | | | | | | | | Mean* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | |
| True | $\lambda_{6,1}$ | Both | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | - |
| Estimate | $\lambda_{6,1}$ | PML | 0.70 | 0.71 | 0.72 | 0.73 | 0.71 | 0.72 | 0.72 | 0.73 | 0.70 | 0.71 | 0.72 | 0.73 | 0.69 | 0.71 | 0.71 | 0.72 | 0.70 | 0.70 | 0.72 | 0.72 | 0.70 | 0.71 | 0.72 | 0.72 | 0.71 |
| Raw bias | $\lambda_{6,1}$ | PML | 0.10 | 0.11 | 0.12 | 0.13 | 0.11 | 0.12 | 0.12 | 0.13 | 0.10 | 0.11 | 0.12 | 0.13 | 0.09 | 0.11 | 0.11 | 0.12 | 0.10 | 0.10 | 0.12 | 0.12 | 0.10 | 0.11 | 0.12 | 0.12 | 0.11 |
| Estimate | $\lambda_{6,1}$ | DWLS | 0.71 | 0.72 | 0.73 | 0.74 | 0.71 | 0.72 | 0.73 | 0.73 | 0.71 | 0.72 | 0.73 | 0.73 | 0.71 | 0.72 | 0.72 | 0.73 | 0.71 | 0.72 | 0.73 | 0.73 | 0.71 | 0.72 | 0.73 | 0.73 | 0.72 |
| Raw bias | $\lambda_{6,1}$ | DWLS | 0.11 | 0.12 | 0.13 | 0.14 | 0.11 | 0.12 | 0.13 | 0.13 | 0.11 | 0.12 | 0.13 | 0.13 | 0.11 | 0.12 | 0.12 | 0.13 | 0.11 | 0.12 | 0.13 | 0.13 | 0.11 | 0.12 | 0.13 | 0.13 | 0.12 |
| True | $\lambda_{7,2}$ | Both | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | - |
| Estimate | $\lambda_{7,2}$ | PML | 0.80 | 0.80 | 0.80 | 0.80 | 0.81 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.79 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 |
| Raw bias | $\lambda_{7,2}$ | PML | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Estimate | $\lambda_{7,2}$ | DWLS | 0.81 | 0.80 | 0.81 | 0.80 | 0.81 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 |
| Raw bias | $\lambda_{7,2}$ | DWLS | 0.01 | 0.00 | 0.01 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| True | $\tau_{6,1}$ | Both | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | - |
| Estimate | $\tau_{6,1}$ | PML | -0.01 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Raw bias | $\tau_{6,1}$ | PML | -0.01 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Estimate | $\tau_{6,1}$ | DWLS | -0.01 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Raw bias | $\tau_{6,1}$ | DWLS | -0.01 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| True | $\tau_{7,2}$ | Both | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | - |
| Estimate | $\tau_{7,2}$ | PML | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Raw bias | $\tau_{7,2}$ | PML | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Estimate | $\tau_{7,2}$ | DWLS | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Raw bias | $\tau_{7,2}$ | DWLS | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| True | $\psi_{1,2}$ | Both | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | - |
| Estimate | $\psi_{1,2}$ | PML | 0.35 | 0.35 | 0.35 | 0.35 | 0.34 | 0.35 | 0.35 | 0.35 | 0.34 | 0.35 | 0.35 | 0.34 | 0.34 | 0.34 | 0.35 | 0.35 | 0.34 | 0.34 | 0.35 | 0.35 | 0.34 | 0.34 | 0.34 | 0.34 | 0.35 |
| Raw bias | $\psi_{1,2}$ | PML | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.05 | 0.05 | 0.05 | 0.04 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.05 | 0.05 | 0.04 | 0.04 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 | 0.05 |
| Estimate | $\psi_{1,2}$ | DWLS | 0.35 | 0.35 | 0.35 | 0.36 | 0.35 | 0.35 | 0.35 | 0.35 | 0.34 | 0.35 | 0.35 | 0.35 | 0.34 | 0.35 | 0.35 | 0.35 | 0.34 | 0.34 | 0.35 | 0.35 | 0.35 | 0.35 | 0.35 | 0.35 | 0.35 |
| Raw bias | $\psi_{1,2}$ | DWLS | 0.05 | 0.05 | 0.05 | 0.06 | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.05 | 0.05 | 0.05 | 0.04 | 0.05 | 0.05 | 0.05 | 0.04 | 0.04 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |

*Note.* Values $< 0.01$ are rounded to 0.00. The experimental conditions correspond to those specified in Table 3.1. * calculated across all conditions.

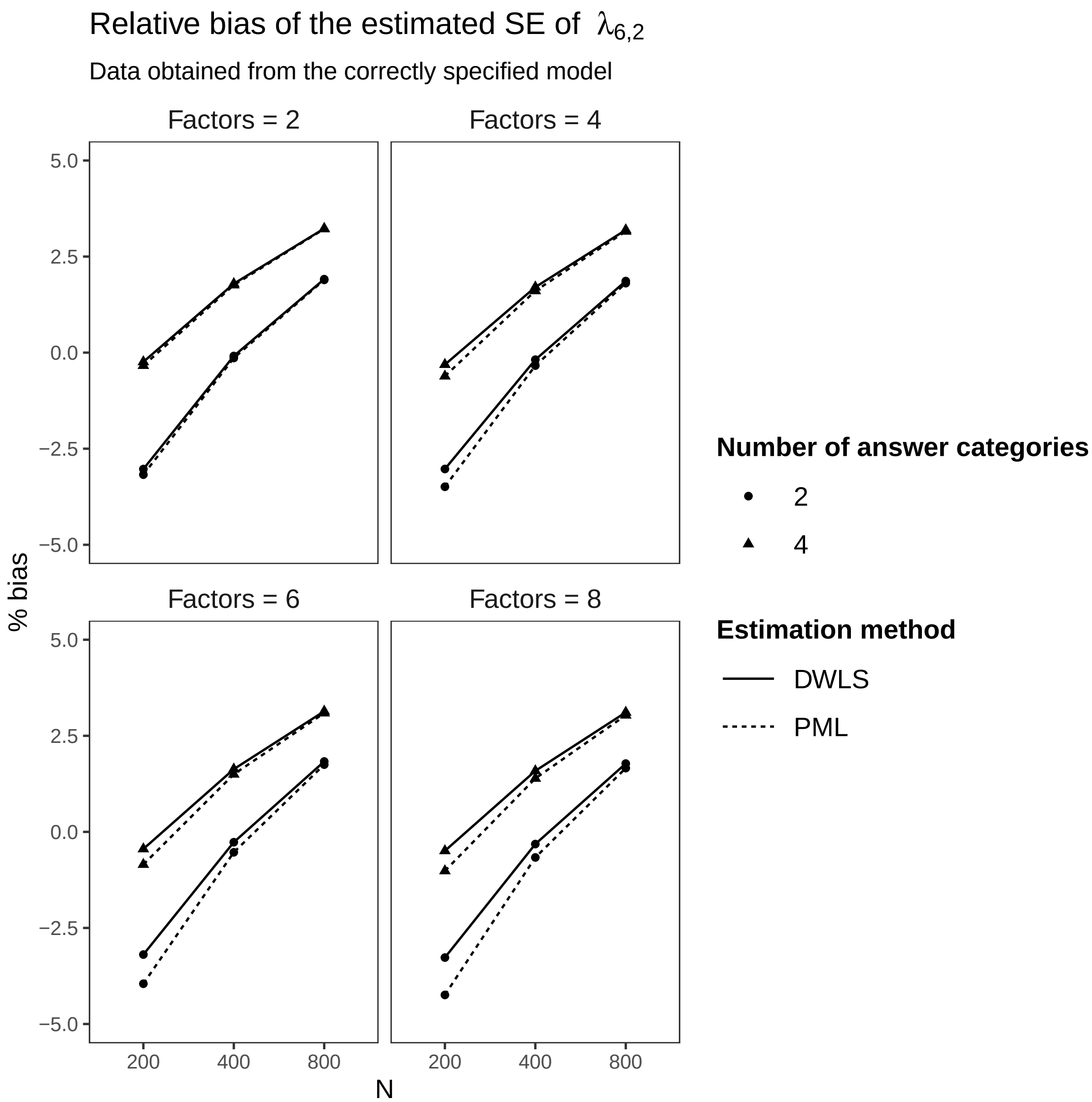


Figure A1: Four-way interaction effect between the number of latent variables, sample size, number of answer categories, and estimation method for the SE of $\lambda_{6,2}$

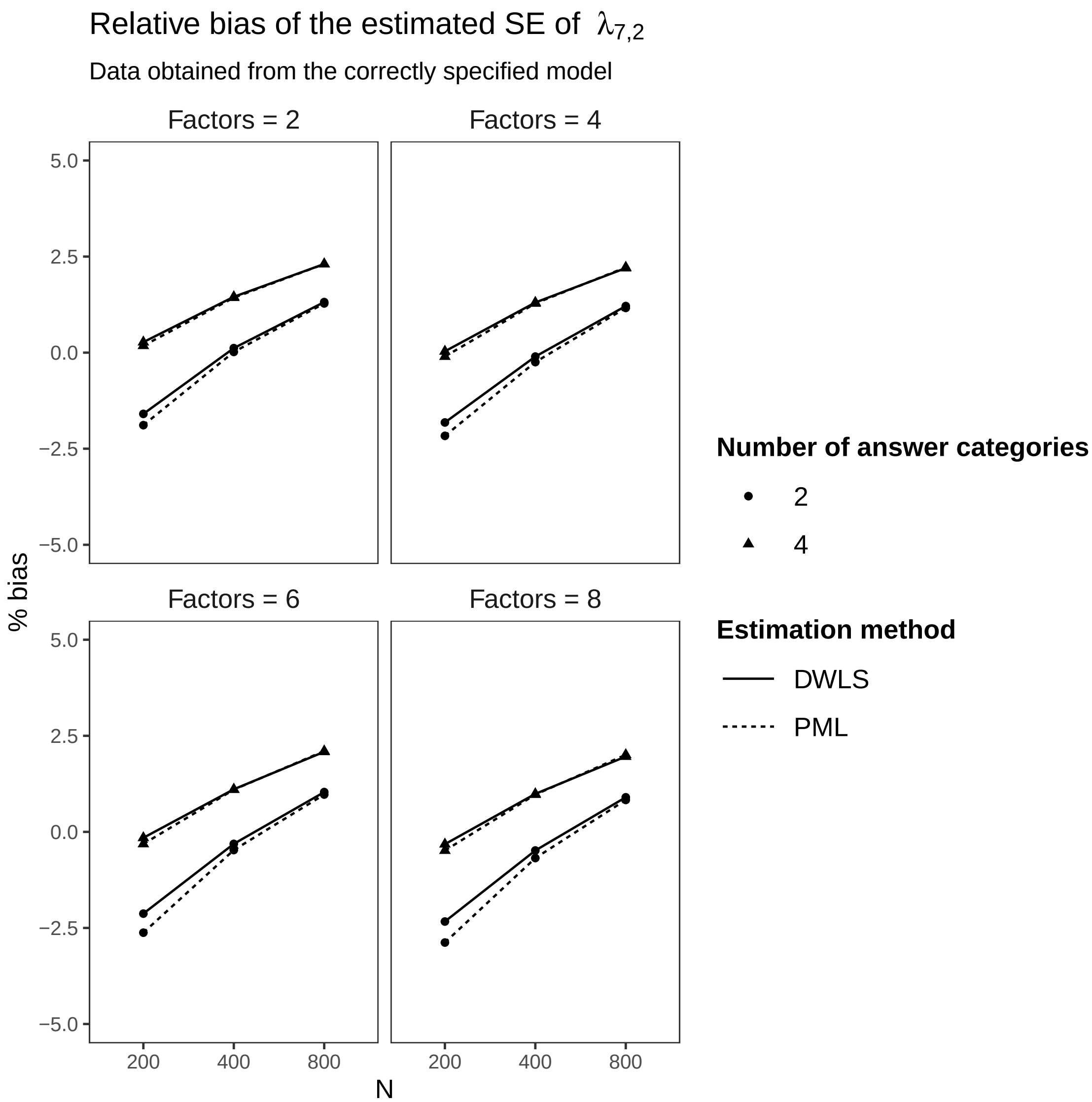


Figure A2: Four-way interaction effect between the number of latent variables, sample size, number of answer categories, and estimation method for the SE of $\lambda_{7,2}$

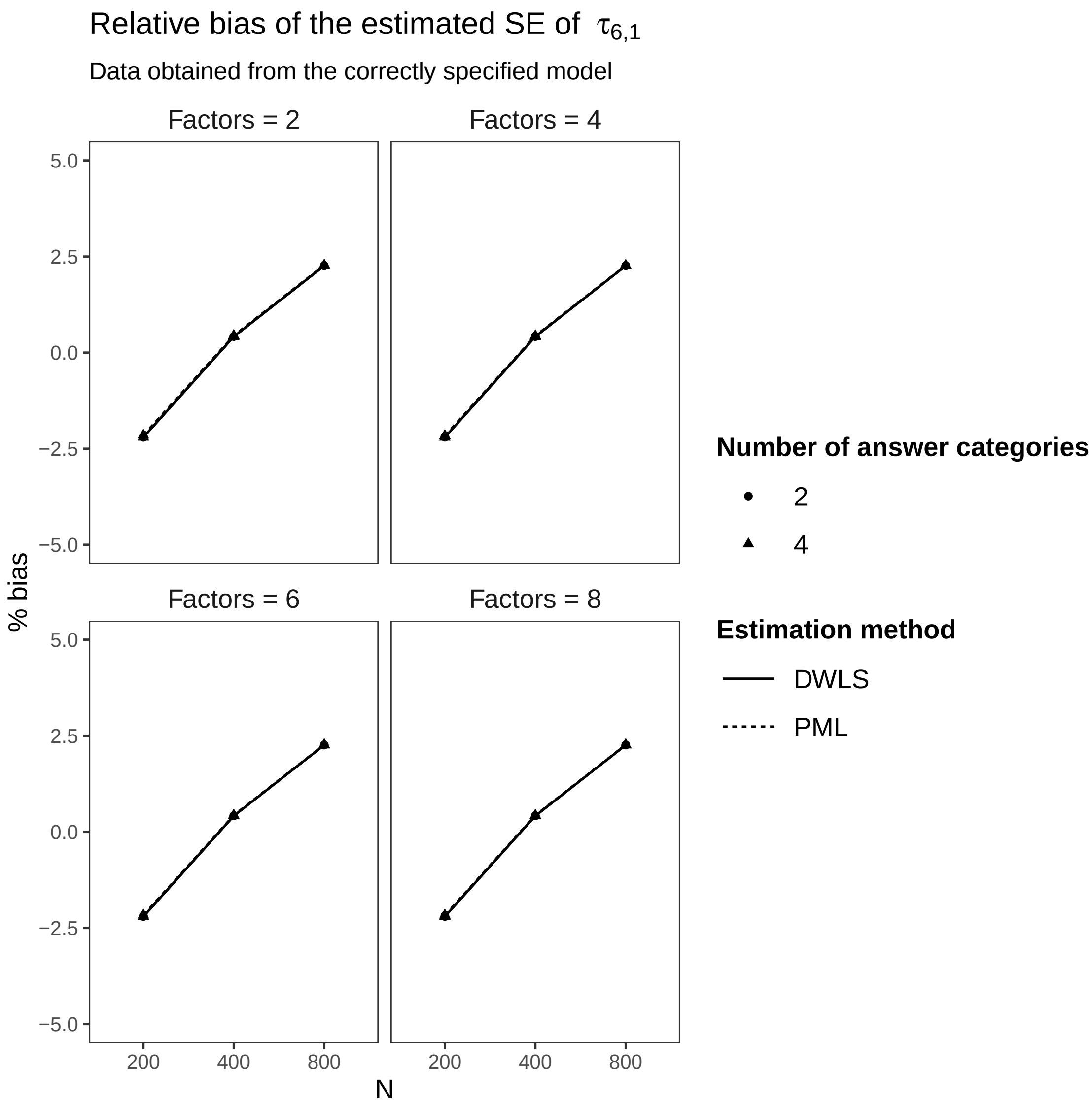


Figure A3: Four-way interaction effect between the number of latent variables, sample size, number of answer categories, and estimation method for the SE of $\tau_{6,1}$

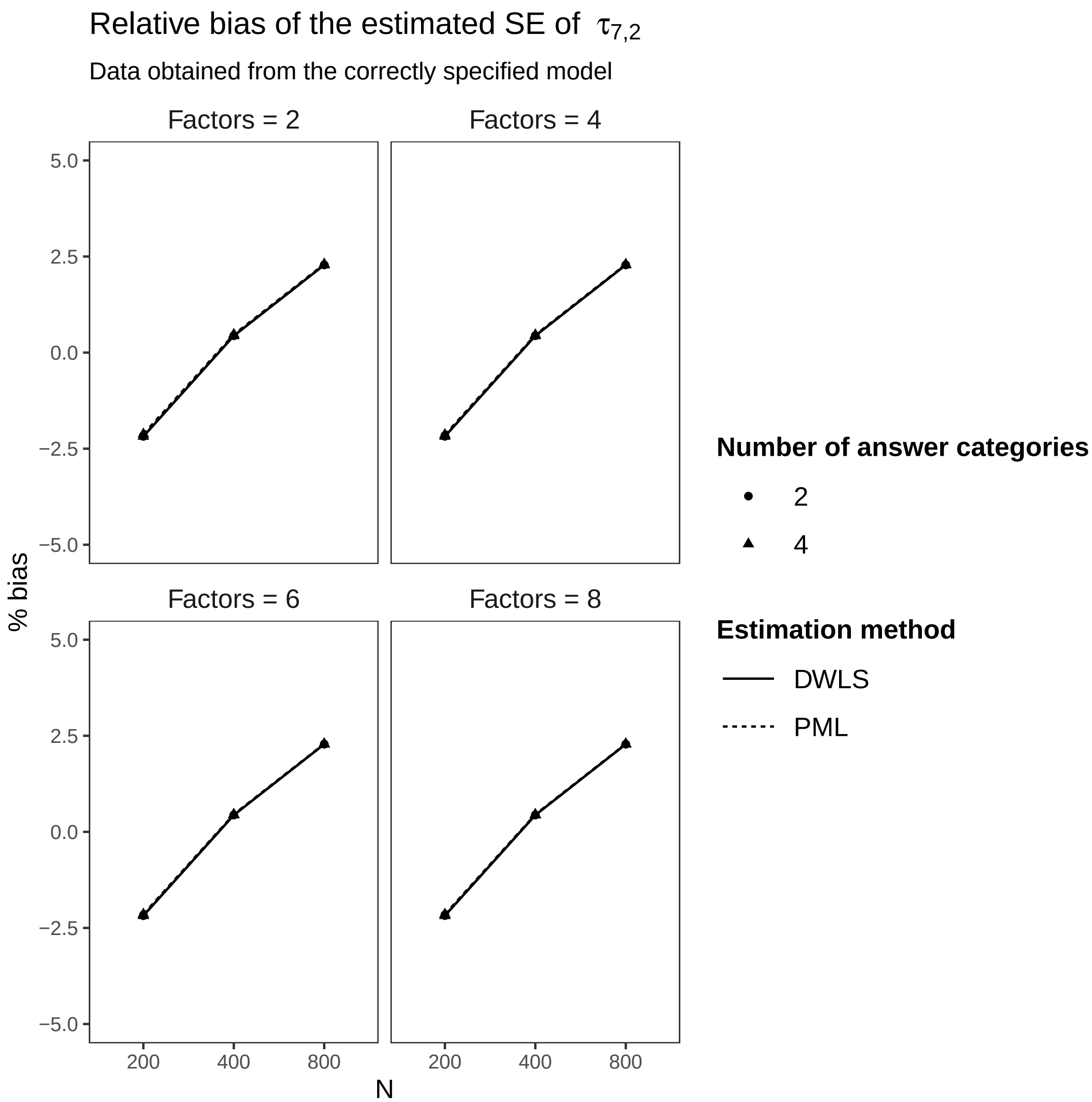


Figure A4: Four-way interaction effect between the number of latent variables, sample size, number of answer categories, and estimation method for the SE of $\tau_{7,2}$

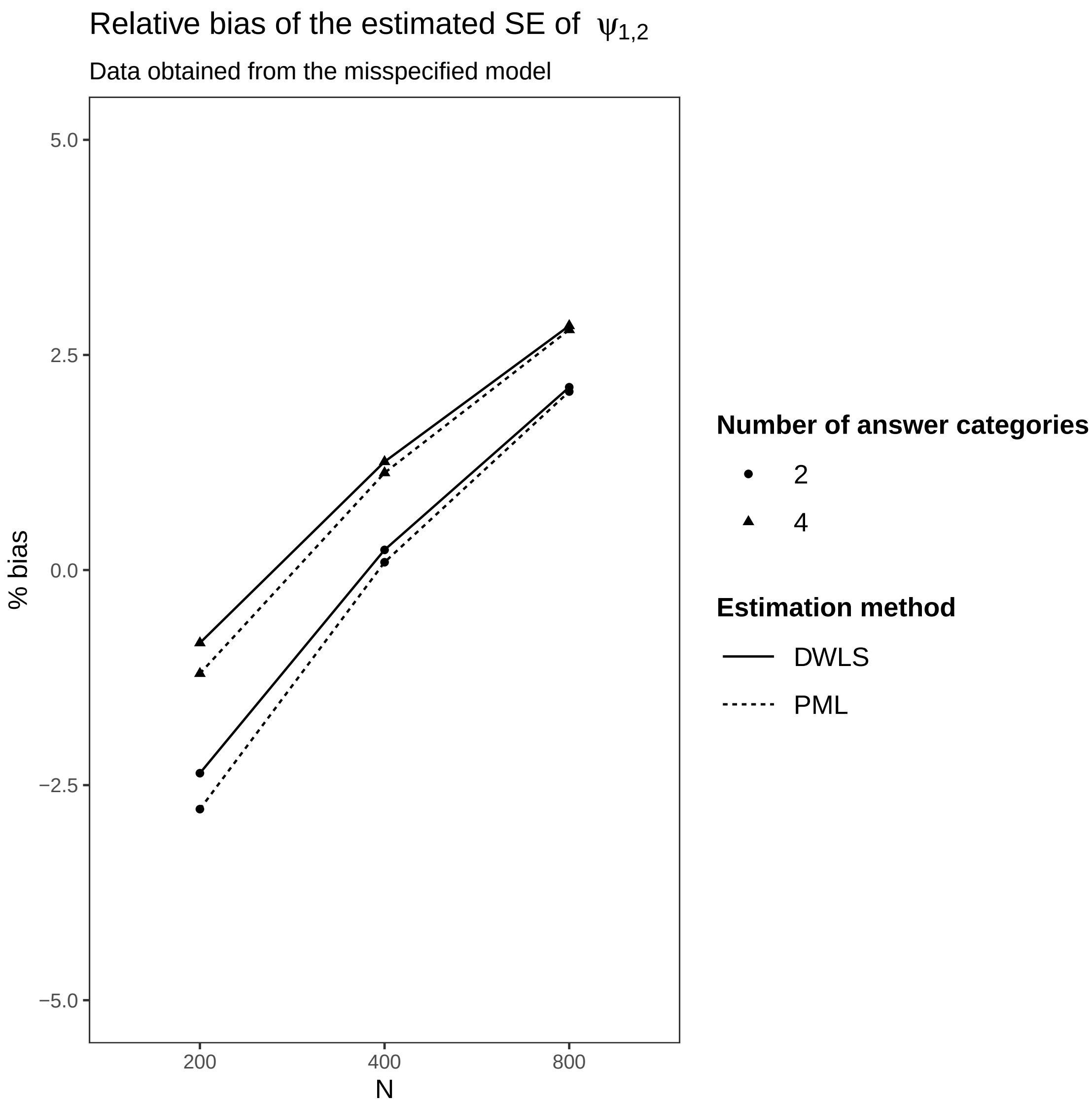


Figure A5: Four-way interaction effect between the number of latent variables, sample size, number of answer categories, and estimation method for the SE of $\psi_{1,2}$

Table A4: Table with true values, raw means, and raw bias for the SE of the parameter estimates for each of the 24 conditions separately. Values are obtained from the correctly specified model.

| | | | Experimental condition | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Value** | **Parameter** | **Method** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** | **Mean*** |
| SD of parameter estimate | $\lambda_{6,1}$ | PML | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | - |
| Estimate | SE of $\lambda_{6,1}$ | PML | 0.09 | 0.09 | 0.09 | 0.10 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.06 | 0.06 | 0.06 | 0.07 | 0.04 | 0.04 | 0.04 | 0.05 | 0.03 | 0.03 | 0.03 | 0.03 | 0.05 |
| Raw bias | SE of $\lambda_{6,1}$ | PML | 0.03 | 0.03 | 0.04 | 0.04 | 0.00 | 0.00 | 0.01 | 0.01 | -0.01 | -0.01 | -0.01 | -0.01 | 0.00 | 0.00 | 0.01 | 0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.03 | -0.03 | -0.03 | -0.03 | 0.00 |
| SD of parameter estimate | $\lambda_{6,1}$ | DWLS | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | - |
| Estimate | SE of $\lambda_{6,1}$ | DWLS | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.03 | 0.03 | 0.03 | 0.03 | 0.05 |
| Raw bias | SE of $\lambda_{6,1}$ | DWLS | 0.03 | 0.03 | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.01 | -0.01 | -0.01 | -0.03 | -0.03 | -0.03 | -0.03 | 0.00 |
| SD of parameter estimate | $\lambda_{6,2}$ | PML | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | - |
| Estimate | SE of $\lambda_{6,2}$ | PML | 0.10 | 0.10 | 0.11 | 0.11 | 0.07 | 0.07 | 0.07 | 0.07 | 0.05 | 0.05 | 0.05 | 0.05 | 0.07 | 0.07 | 0.08 | 0.08 | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 | 0.07 |
| Raw bias | SE of $\lambda_{6,2}$ | PML | 0.03 | 0.04 | 0.04 | 0.04 | 0.00 | 0.00 | 0.01 | 0.01 | -0.02 | -0.02 | -0.02 | -0.02 | 0.00 | 0.01 | 0.01 | 0.01 | -0.02 | -0.02 | -0.01 | -0.01 | -0.03 | -0.03 | -0.03 | -0.03 | 0.00 |
| SD of parameter estimate | $\lambda_{6,2}$ | DWLS | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | - |
| Estimate | SE of $\lambda_{6,2}$ | DWLS | 0.10 | 0.10 | 0.10 | 0.10 | 0.07 | 0.07 | 0.07 | 0.07 | 0.05 | 0.05 | 0.05 | 0.05 | 0.07 | 0.07 | 0.07 | 0.07 | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 | 0.06 |
| Raw bias | SE of $\lambda_{6,2}$ | DWLS | 0.03 | 0.03 | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | -0.02 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | -0.02 | -0.03 | -0.03 | -0.03 | -0.03 | -0.01 |
| SD of parameter estimate | $\lambda_{7,2}$ | PML | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | - |
| Estimate | SE of $\lambda_{7,2}$ | PML | 0.06 | 0.07 | 0.07 | 0.07 | 0.04 | 0.05 | 0.05 | 0.05 | 0.03 | 0.03 | 0.03 | 0.04 | 0.04 | 0.05 | 0.05 | 0.05 | 0.03 | 0.03 | 0.03 | 0.03 | 0.02 | 0.02 | 0.02 | 0.02 | 0.04 |
| Raw bias | SE of $\lambda_{7,2}$ | PML | 0.02 | 0.02 | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.01 | -0.01 | -0.01 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | -0.01 | -0.01 | -0.01 | -0.02 | -0.02 | -0.02 | -0.02 | 0.00 |
| SD of parameter estimate | $\lambda_{7,2}$ | DWLS | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | - |
| Estimate | SE of $\lambda_{7,2}$ | DWLS | 0.06 | 0.06 | 0.07 | 0.07 | 0.04 | 0.05 | 0.05 | 0.05 | 0.03 | 0.03 | 0.03 | 0.04 | 0.04 | 0.04 | 0.05 | 0.05 | 0.03 | 0.03 | 0.03 | 0.03 | 0.02 | 0.02 | 0.02 | 0.02 | 0.04 |
| Raw bias | SE of $\lambda_{7,2}$ | DWLS | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | -0.01 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | -0.01 | -0.01 | -0.01 | -0.02 | -0.02 | -0.02 | -0.02 | 0.00 |
| SD of parameter estimate | $\tau_{6,1}$ | PML | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | - |
| Estimate | SE of $\tau_{6,1}$ | PML | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.06 |
| Raw bias | SE of $\tau_{6,1}$ | PML | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | -0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | -0.02 | 0.00 |
| SD of parameter estimate | $\tau_{6,1}$ | DWLS | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | - |
| Estimate | SE of $\tau_{6,1}$ | DWLS | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.06 |
| Raw bias | SE of $\tau_{6,1}$ | DWLS | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | -0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | -0.02 | 0.00 |
| SD of parameter estimate | $\tau_{7,2}$ | PML | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | - |
| Estimate | SE of $\tau_{7,2}$ | PML | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.06 |
| Raw bias | SE of $\tau_{7,2}$ | PML | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | -0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | -0.02 | 0.00 |
| SD of parameter estimate | $\tau_{7,2}$ | DWLS | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | - |
| Estimate | SE of $\tau_{7,2}$ | DWLS | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.06 |
| Raw bias | SE of $\tau_{7,2}$ | DWLS | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | -0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | -0.02 | 0.00 |
| SD of parameter estimate | $\psi_{1,2}$ | PML | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | - |
| Estimate | SE of $\psi_{1,2}$ | PML | 0.10 | 0.10 | 0.10 | 0.10 | 0.07 | 0.07 | 0.07 | 0.07 | 0.05 | 0.05 | 0.05 | 0.05 | 0.08 | 0.08 | 0.08 | 0.08 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.07 |
| Raw bias | SE of $\psi_{1,2}$ | PML | 0.03 | 0.03 | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | -0.02 | 0.01 | 0.01 | 0.01 | 0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.03 | -0.03 | -0.03 | -0.03 | 0.00 |
| SD of parameter estimate | $\psi_{1,2}$ | DWLS | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | - |
| Estimate | SE of $\psi_{1,2}$ | DWLS | 0.09 | 0.09 | 0.09 | 0.09 | 0.07 | 0.07 | 0.07 | 0.07 | 0.05 | 0.05 | 0.05 | 0.05 | 0.08 | 0.08 | 0.08 | 0.08 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.07 |
| Raw bias | SE of $\psi_{1,2}$ | DWLS | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | -0.02 | 0.01 | 0.01 | 0.01 | 0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.03 | -0.03 | -0.03 | -0.03 | -0.01 |

*Note.* Values < 0.01 are rounded to 0.00. The experimental conditions correspond to those specified in Table 3.1. * calculated across all conditions.

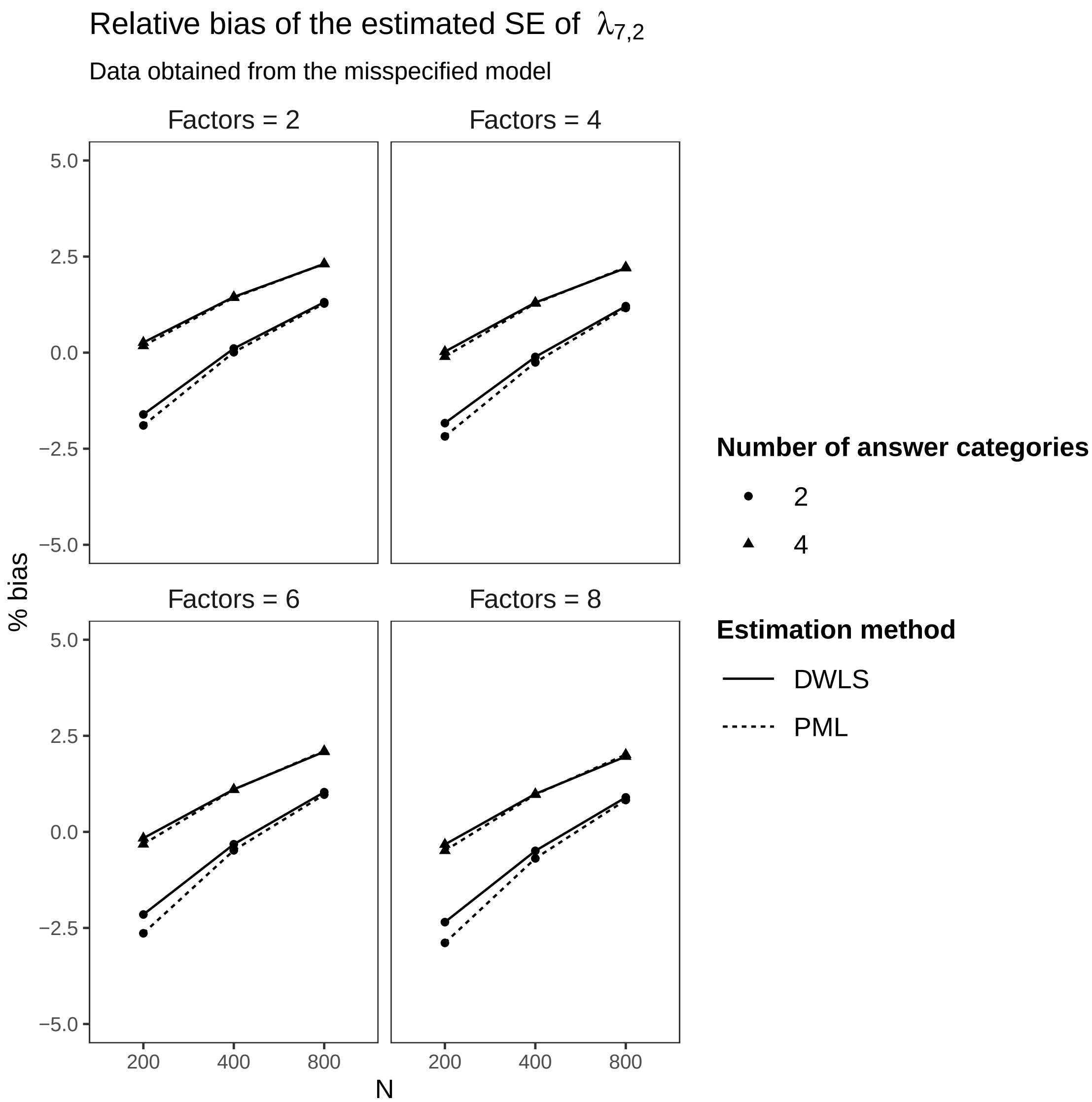


Figure A6: Four-way interaction effect between the number of latent variables, sample size, number of answer categories, and estimation method for the SE of $\lambda_{7,2}$

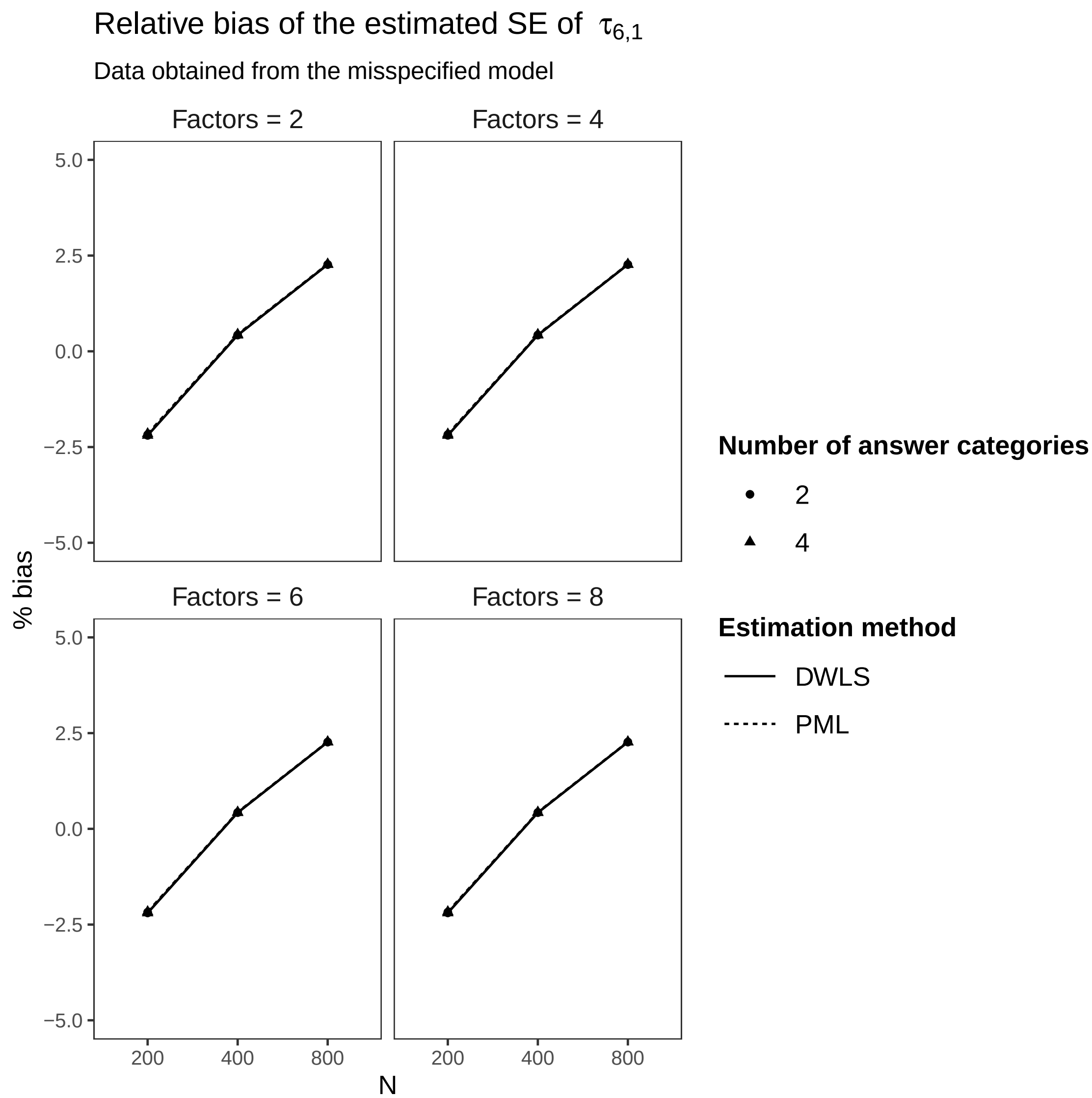


Figure A7: Four-way interaction effect between the number of latent variables, sample size, number of answer categories, and estimation method for the SE of $\tau_{6,1}$

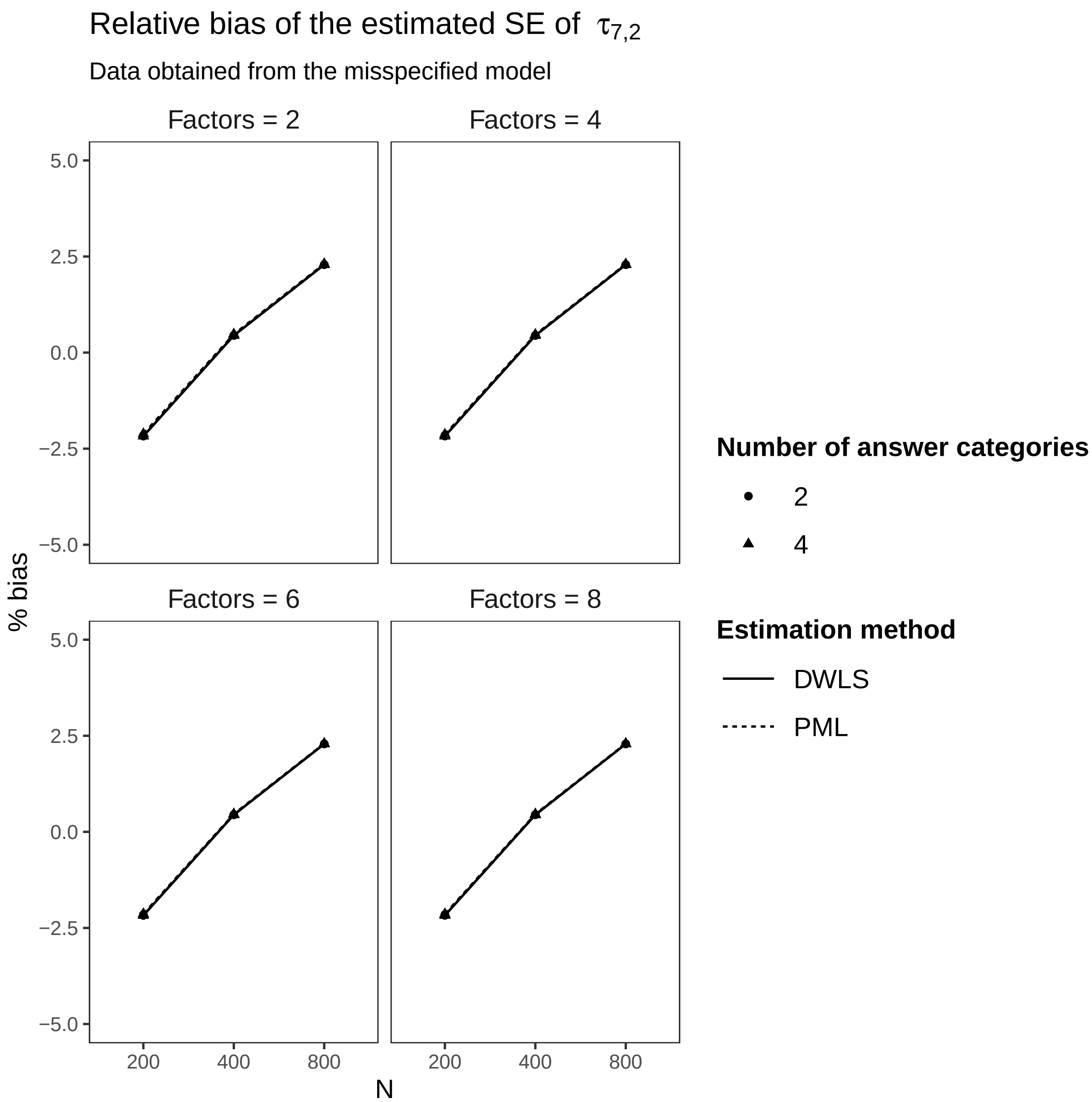


Figure A8: Four-way interaction effect between the number of latent variables, sample size, number of answer categories, and estimation method for the SE of $\tau_{7,2}$

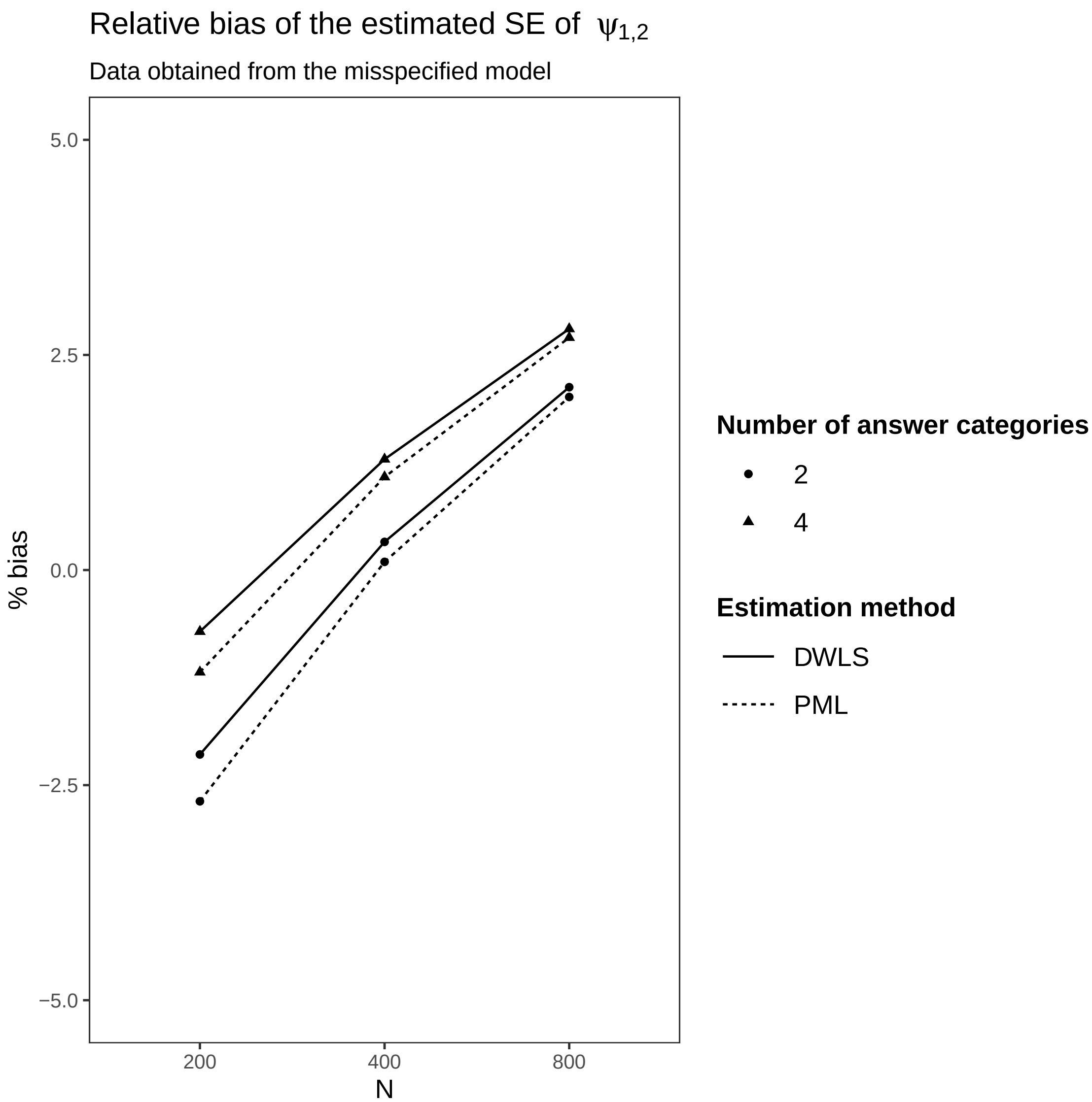


Figure A9: Four-way interaction effect between the number of latent variables, sample size, number of answer categories, and estimation method for the SE of $\psi_{1,2}$

Table A5: Table with true values, raw means, and raw bias for the SE of the parameter estimates for each of the 24 conditions separately. Values are obtained from the misspecified model.

| Value | Parameter | Method | Experimental condition 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | Mean* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SD of parameter estimate | $\lambda_{6,1}$ | PML | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | - |
| Estimate | SE of $\lambda_{6,1}$ | PML | 0.07 | 0.07 | 0.07 | 0.08 | 0.05 | 0.05 | 0.05 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.05 | 0.05 | 0.05 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.03 | 0.03 | 0.03 | 0.03 | 0.05 |
| Raw bias | SE of $\lambda_{6,1}$ | PML | 0.02 | 0.02 | 0.02 | 0.03 | 0.00 | 0.00 | 0.00 | 0.01 | -0.01 | -0.01 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.02 | -0.02 | -0.02 | -0.02 | 0.00 |
| SD of parameter estimate | $\lambda_{6,1}$ | DWLS | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | - |
| Estimate | SE of $\lambda_{6,1}$ | DWLS | 0.07 | 0.08 | 0.08 | 0.08 | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 | 0.05 | 0.05 | 0.05 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.03 | 0.03 | 0.03 | 0.03 | 0.05 |
| Raw bias | SE of $\lambda_{6,1}$ | DWLS | 0.02 | 0.02 | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.01 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.01 | -0.01 | -0.01 | -0.03 | -0.03 | -0.02 | -0.02 | 0.00 |
| SD of parameter estimate | $\lambda_{7,2}$ | PML | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | - |
| Estimate | SE of $\lambda_{7,2}$ | PML | 0.06 | 0.06 | 0.07 | 0.07 | 0.04 | 0.05 | 0.05 | 0.05 | 0.03 | 0.03 | 0.03 | 0.04 | 0.04 | 0.04 | 0.05 | 0.05 | 0.03 | 0.03 | 0.03 | 0.03 | 0.02 | 0.02 | 0.02 | 0.02 | 0.04 |
| Raw bias | SE of $\lambda_{7,2}$ | PML | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | -0.01 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | -0.01 | -0.01 | -0.01 | -0.02 | -0.02 | -0.02 | -0.02 | 0.00 |
| SD of parameter estimate | $\lambda_{7,2}$ | DWLS | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | - |
| Estimate | SE of $\lambda_{7,2}$ | DWLS | 0.06 | 0.07 | 0.07 | 0.07 | 0.04 | 0.05 | 0.05 | 0.05 | 0.03 | 0.03 | 0.03 | 0.04 | 0.04 | 0.05 | 0.05 | 0.05 | 0.03 | 0.03 | 0.03 | 0.03 | 0.02 | 0.02 | 0.02 | 0.02 | 0.04 |
| Raw bias | SE of $\lambda_{7,2}$ | DWLS | 0.02 | 0.02 | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.01 | -0.01 | -0.01 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | -0.01 | -0.01 | -0.01 | -0.02 | -0.02 | -0.02 | -0.02 | 0.00 |
| SD of parameter estimate | $\tau_{6,1}$ | PML | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | - |
| Estimate | SE of $\tau_{6,1}$ | PML | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.06 |
| Raw bias | SE of $\tau_{6,1}$ | PML | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | -0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | -0.02 | 0.00 |
| SD of parameter estimate | $\tau_{6,1}$ | DWLS | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | - |
| Estimate | SE of $\tau_{6,1}$ | DWLS | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.06 |
| Raw bias | SE of $\tau_{6,1}$ | DWLS | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | -0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | -0.02 | 0.00 |
| SD of parameter estimate | $\tau_{7,2}$ | PML | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | - |
| Estimate | SE of $\tau_{7,2}$ | PML | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.06 |
| Raw bias | SE of $\tau_{7,2}$ | PML | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | -0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | -0.02 | 0.00 |
| SD of parameter estimate | $\tau_{7,2}$ | DWLS | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | - |
| Estimate | SE of $\tau_{7,2}$ | DWLS | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.06 |
| Raw bias | SE of $\tau_{7,2}$ | DWLS | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | -0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | -0.02 | 0.00 |
| SD of parameter estimate | $\psi_{1,2}$ | PML | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | - |
| Estimate | SE of $\psi_{1,2}$ | PML | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.07 | 0.07 | 0.07 | 0.07 | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 | 0.06 |
| Raw bias | SE of $\psi_{1,2}$ | PML | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | -0.02 | 0.01 | 0.01 | 0.01 | 0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.03 | -0.03 | -0.03 | -0.03 | -0.01 |
| SD of parameter estimate | $\psi_{1,2}$ | DWLS | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | - |
| Estimate | SE of $\psi_{1,2}$ | DWLS | 0.09 | 0.09 | 0.09 | 0.09 | 0.07 | 0.06 | 0.06 | 0.06 | 0.05 | 0.05 | 0.05 | 0.05 | 0.08 | 0.08 | 0.08 | 0.08 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.06 |
| Raw bias | SE of $\psi_{1,2}$ | DWLS | 0.03 | 0.03 | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | -0.02 | 0.01 | 0.01 | 0.01 | 0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.03 | -0.03 | -0.03 | -0.03 | 0.00 |

*Note.* Values $< 0.01$ are rounded to 0.00. The experimental conditions correspond to those specified in Table 3.1. * calculated across all conditions.

## 6.2 Appendix B

### Simulation study 2: mixed data

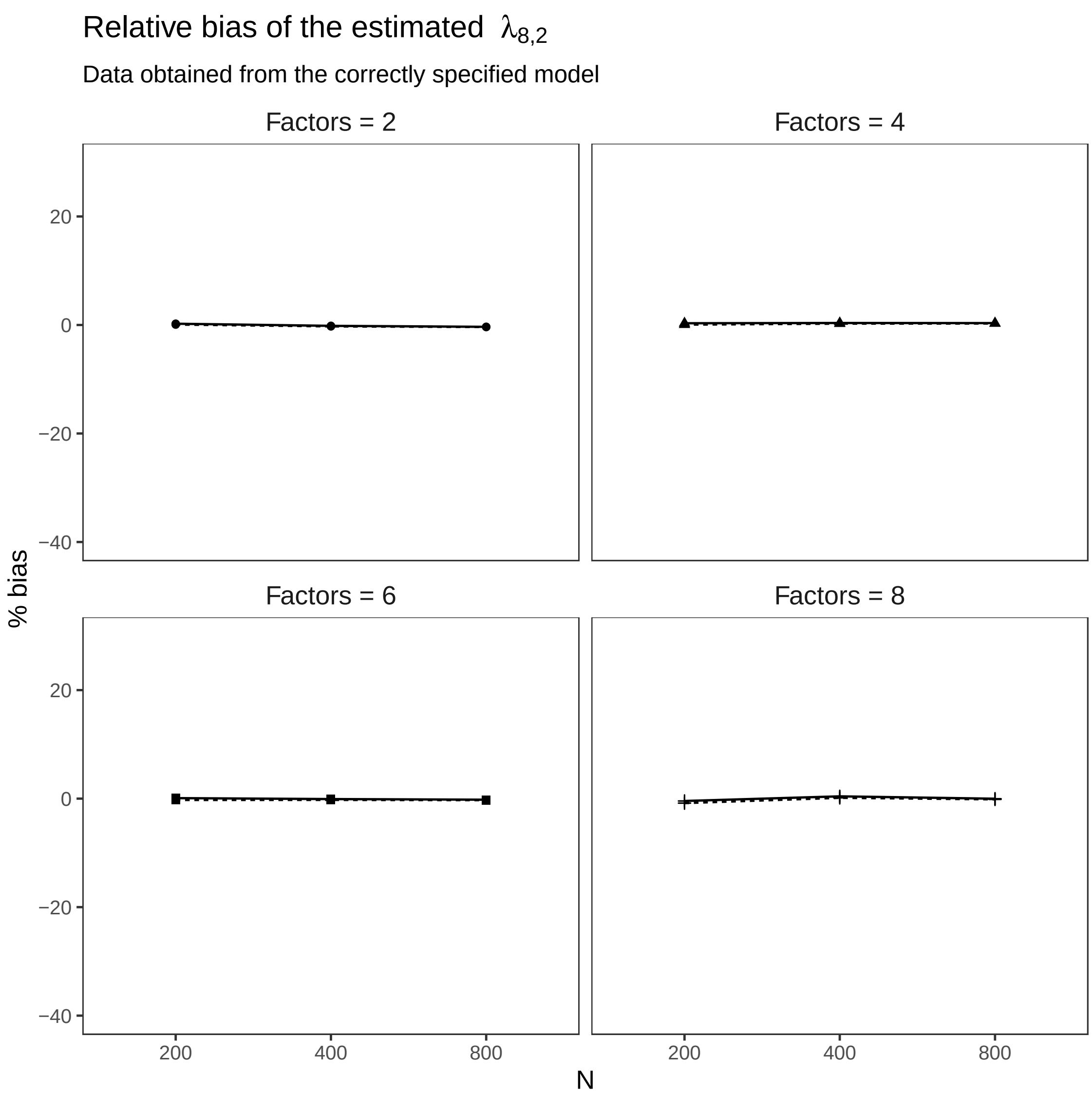


Figure B1: Three-way interaction effect between the number of latent factors, sample size, and estimation method for $\lambda_{8,2}$. The dashed line and solid line respectively represent the data sets with PML and DWLS as estimation method.

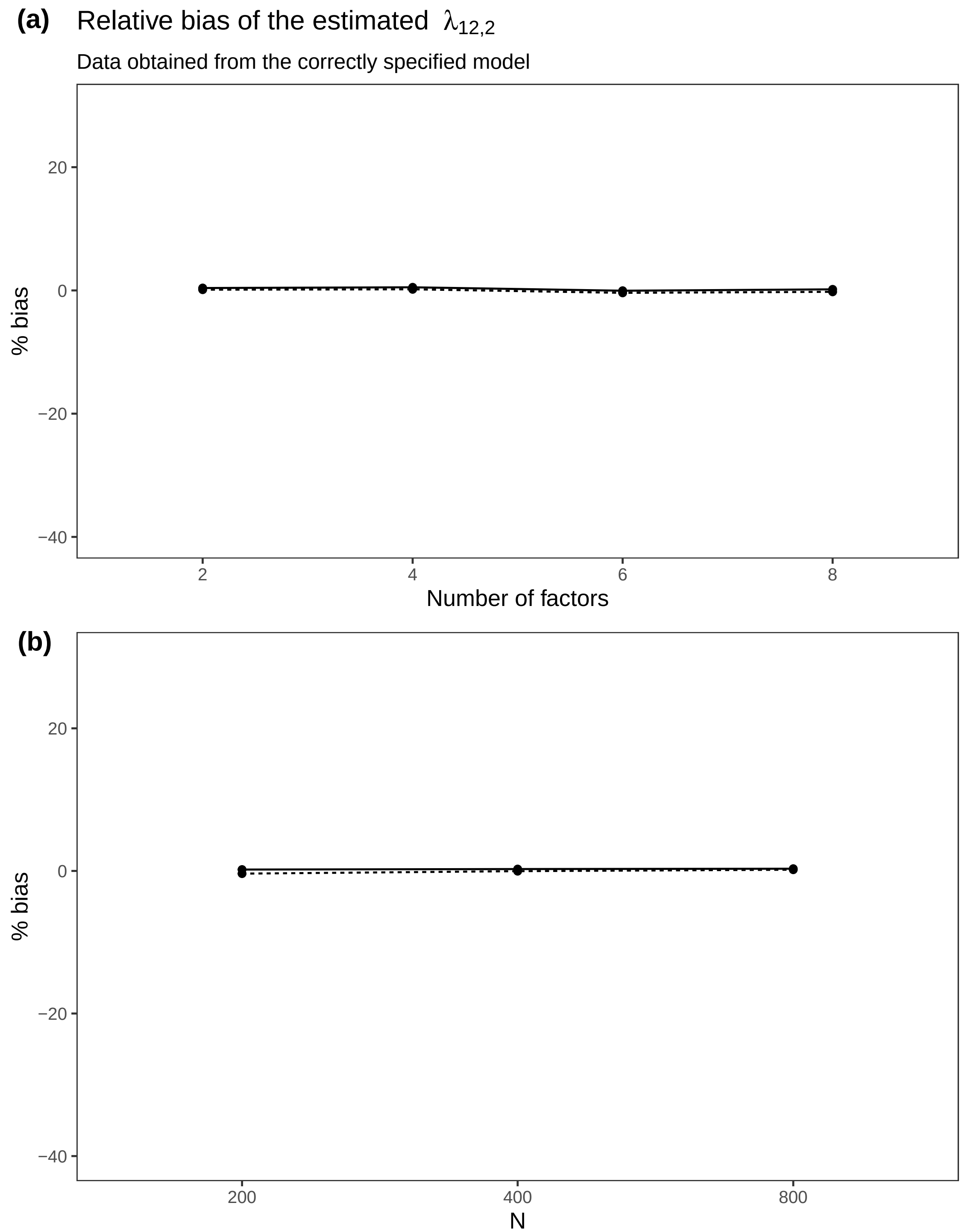


Figure B2: Two two-way interaction effects; plot a displays interaction effect between the number of latent factors and the estimation method for $\lambda_{12,2}$; plot b shows the interaction effect between sample size and estimation method for $\lambda_{12,2}$. The dashed line and solid line respectively represent the data sets with PML and DWLS as estimation method.

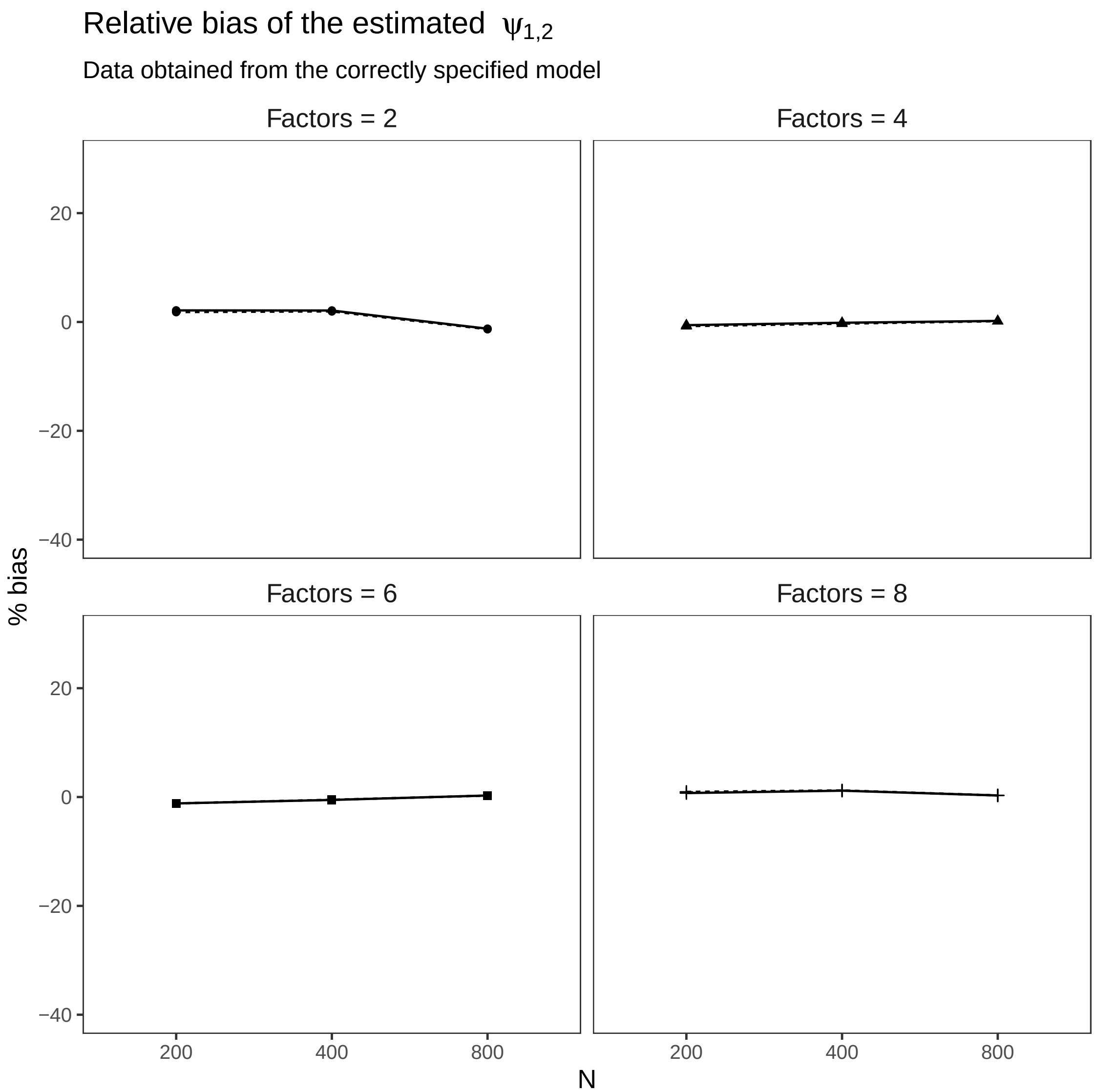


Figure B3: Three-way interaction effect between number of latent factors, sample size, and estimation method for $\psi_{1,2}$. The dashed line and solid line respectively represent the data sets with PML and DWLS as estimation method.

Table B1: Table with true values, raw means, and raw bias for the parameter estimates for each of the 12 conditions separately. Values are obtained from the correctly specified model.

| | | | Experimental condition | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Value** | **Parameter** | **Method** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **Mean*** |
| True | $\lambda_{2,1}$ | Both | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | - |
| Estimate | $\lambda_{2,1}$ | PML | 0.80 | 0.79 | 0.80 | 0.79 | 0.80 | 0.79 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 |
| Raw bias | $\lambda_{2,1}$ | PML | 0.00 | -0.01 | 0.00 | -0.01 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Estimate | $\lambda_{2,1}$ | DWLS | 0.80 | 0.79 | 0.79 | 0.79 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 |
| Raw bias | $\lambda_{2,1}$ | DWLS | 0.00 | -0.01 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| True | $\lambda_{6,1}$ | Both | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | - |
| Estimate | $\lambda_{6,1}$ | PML | 0.60 | 0.60 | 0.59 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.59 | 0.60 | 0.60 | 0.60 | 0.60 |
| Raw bias | $\lambda_{6,1}$ | PML | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| Estimate | $\lambda_{6,1}$ | DWLS | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.59 | 0.60 | 0.60 | 0.60 | 0.60 |
| Raw bias | $\lambda_{6,1}$ | DWLS | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| True | $\lambda_{6,2}$ | Both | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | - |
| Estimate | $\lambda_{6,2}$ | PML | 0.19 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.19 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 |
| Raw bias | $\lambda_{6,2}$ | PML | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Estimate | $\lambda_{6,2}$ | DWLS | 0.19 | 0.20 | 0.20 | 0.20 | 0.20 | 0.21 | 0.20 | 0.19 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 |
| Raw bias | $\lambda_{6,2}$ | DWLS | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| True | $\lambda_{8,2}$ | Both | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | - |
| Estimate | $\lambda_{8,2}$ | PML | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 |
| Raw bias | $\lambda_{8,2}$ | PML | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Estimate | $\lambda_{8,2}$ | DWLS | 0.80 | 0.80 | 0.80 | 0.79 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 |
| Raw bias | $\lambda_{8,2}$ | DWLS | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| True | $\lambda_{12,2}$ | Both | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | - |
| Estimate | $\lambda_{12,2}$ | PML | 0.61 | 0.61 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 |
| Raw bias | $\lambda_{12,2}$ | PML | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Estimate | $\lambda_{12,2}$ | DWLS | 0.60 | 0.60 | 0.59 | 0.59 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 |
| Raw bias | $\lambda_{12,2}$ | DWLS | 0.00 | 0.00 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| True | $\theta_{2,1}$ | Both | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | - |
| Estimate | $\theta_{2,1}$ | PML | 0.35 | 0.36 | 0.35 | 0.37 | 0.35 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 |
| Raw bias | $\theta_{2,1}$ | PML | -0.01 | 0.00 | -0.01 | 0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Estimate | $\theta_{2,1}$ | DWLS | 0.35 | 0.36 | 0.35 | 0.36 | 0.35 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 |
| Raw bias | $\theta_{2,1}$ | DWLS | -0.01 | 0.00 | -0.01 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| True | $\theta_{6,1}$ | Both | 0.53 | 0.53 | 0.53 | 0.53 | 0.53 | 0.53 | 0.53 | 0.53 | 0.53 | 0.53 | 0.53 | 0.53 | - |
| Estimate | $\theta_{6,1}$ | PML | 0.52 | 0.53 | 0.53 | 0.52 | 0.52 | 0.52 | 0.52 | 0.53 | 0.53 | 0.52 | 0.53 | 0.53 | 0.53 |
| Raw bias | $\theta_{6,1}$ | PML | -0.01 | 0.00 | 0.00 | -0.01 | -0.01 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Estimate | $\theta_{6,1}$ | DWLS | 0.52 | 0.52 | 0.52 | 0.52 | 0.52 | 0.52 | 0.52 | 0.53 | 0.53 | 0.52 | 0.53 | 0.53 | 0.52 |
| Raw bias | $\theta_{6,1}$ | DWLS | -0.01 | 0.00 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| True | $\tau_{8,2}$ | Both | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | - |
| Estimate | $\tau_{8,2}$ | PML | 0.00 | -0.01 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Raw bias | $\tau_{8,2}$ | PML | 0.00 | -0.01 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Estimate | $\tau_{8,2}$ | DWLS | 0.00 | -0.01 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Raw bias | $\tau_{8,2}$ | DWLS | 0.00 | -0.01 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| True | $\tau_{12,2}$ | Both | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | - |
| Estimate | $\tau_{12,2}$ | PML | -0.01 | -0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Raw bias | $\tau_{12,2}$ | PML | -0.01 | -0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Estimate | $\tau_{12,2}$ | DWLS | -0.01 | -0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Raw bias | $\tau_{12,2}$ | DWLS | -0.01 | -0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| True | $\psi_{1,2}$ | Both | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | - |
| Estimate | $\psi_{1,2}$ | PML | 0.31 | 0.30 | 0.30 | 0.30 | 0.31 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 |
| Raw bias | $\psi_{1,2}$ | PML | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Estimate | $\psi_{1,2}$ | DWLS | 0.31 | 0.30 | 0.30 | 0.30 | 0.31 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 |
| Raw bias | $\psi_{1,2}$ | DWLS | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

*Note.* Values $< 0.01$ are rounded to 0.00. The experimental conditions correspond to those specified in Table 3.2. * calculated across all conditions.

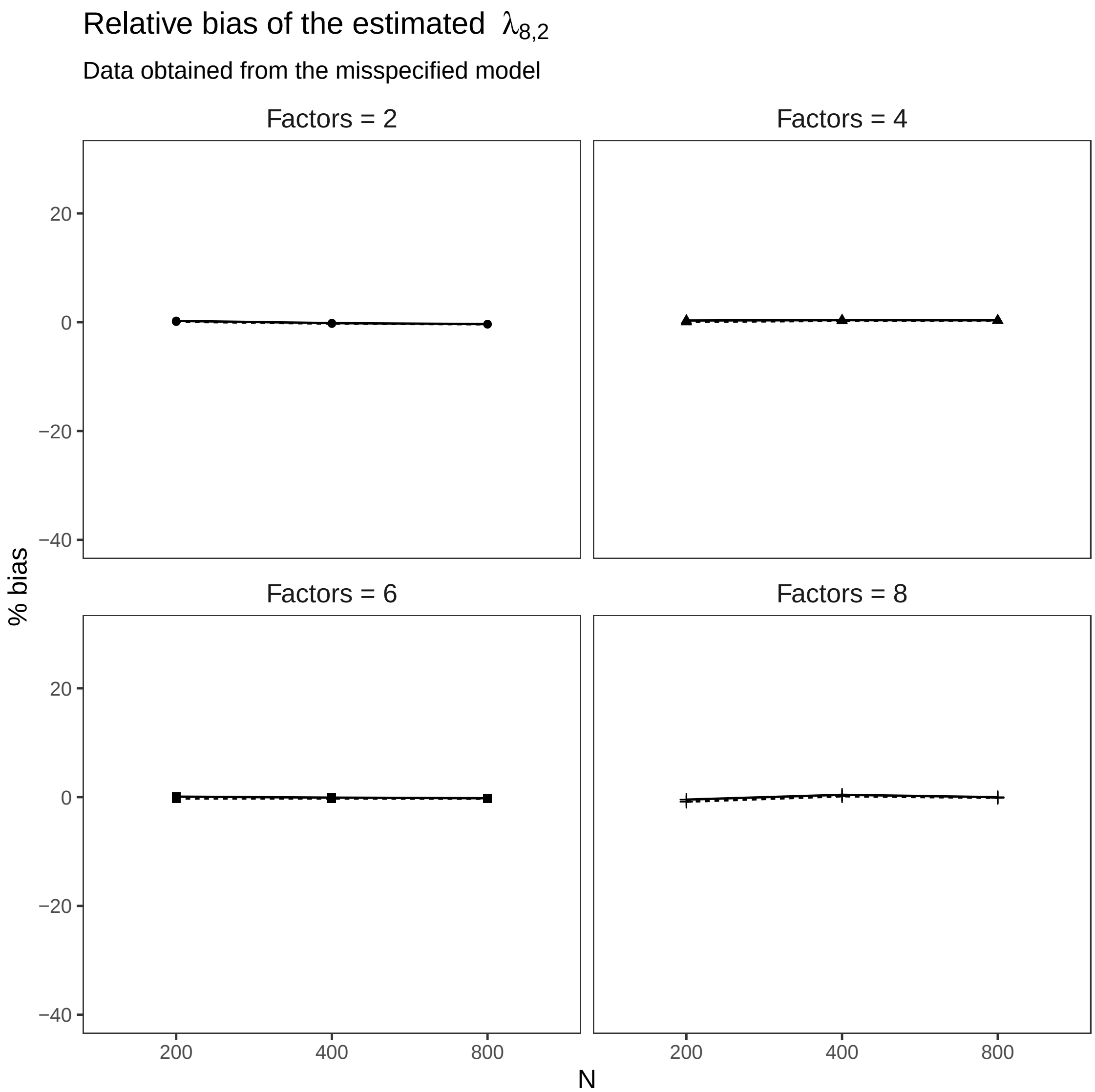


Figure B4: Three-way interaction effect between number of latent factors, sample size, and estimation method for $\lambda_{8,2}$. The dashed line and solid line respectively represent the data sets with PML and DWLS as estimation method.

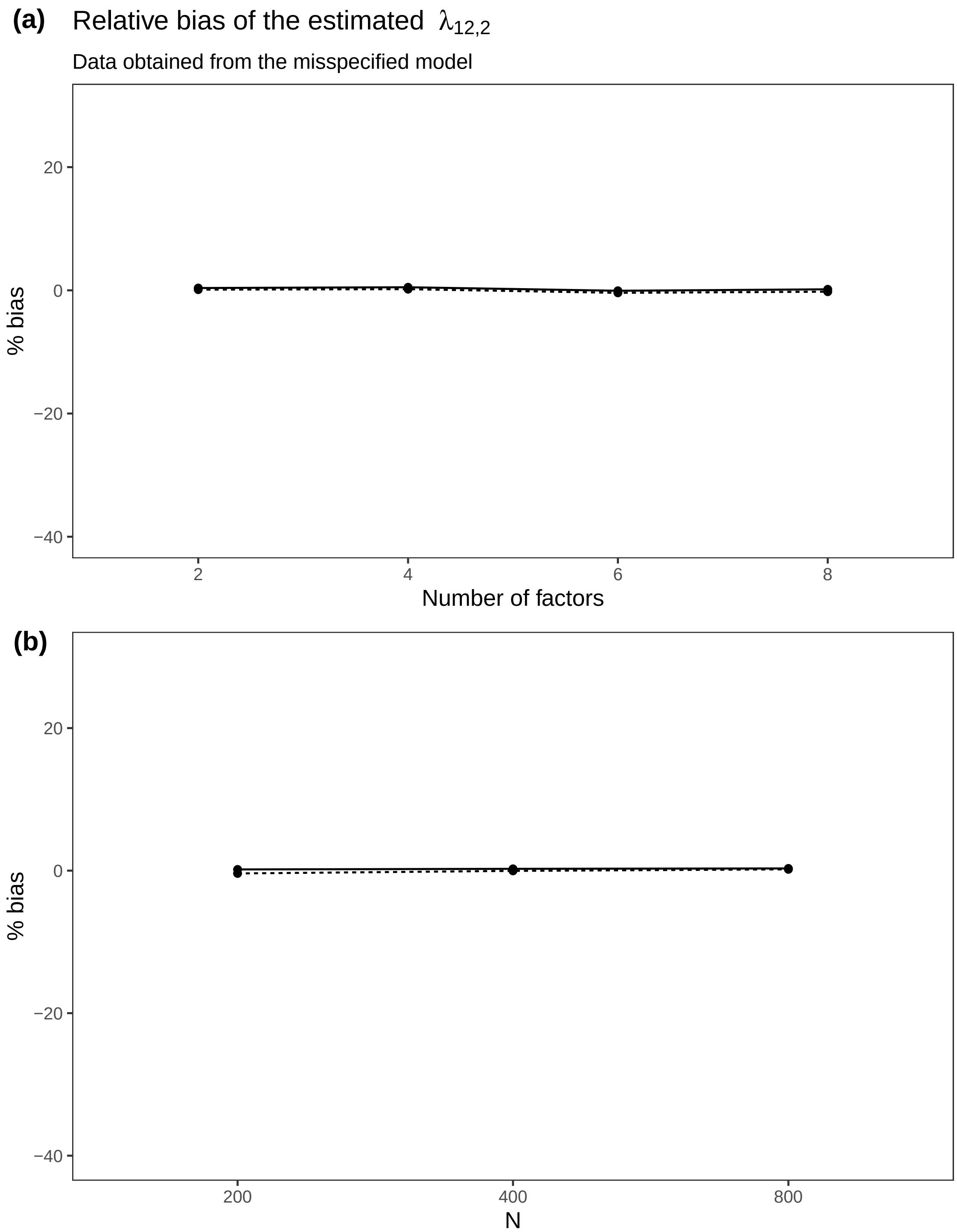


Figure B5: Two two-way interaction effects; plot a displays interaction effect between the number of latent factors and the estimation method for $\lambda_{12,2}$; plot b shows the interaction effect between sample size and estimation method for $\lambda_{12,2}$. The dashed line and solid line respectively represent the data sets with PML and DWLS as estimation method.

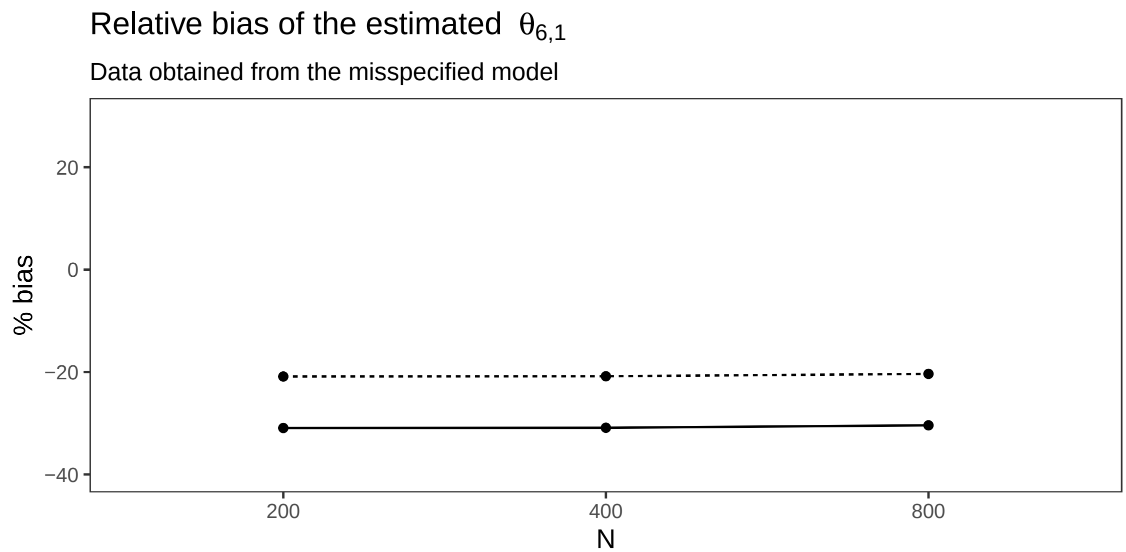


Figure B6: Two-way interaction effect between sample size and estimation method for $\theta_{6,1}$. The dashed line and solid line respectively represent the data sets with PML and DWLS as estimation method.

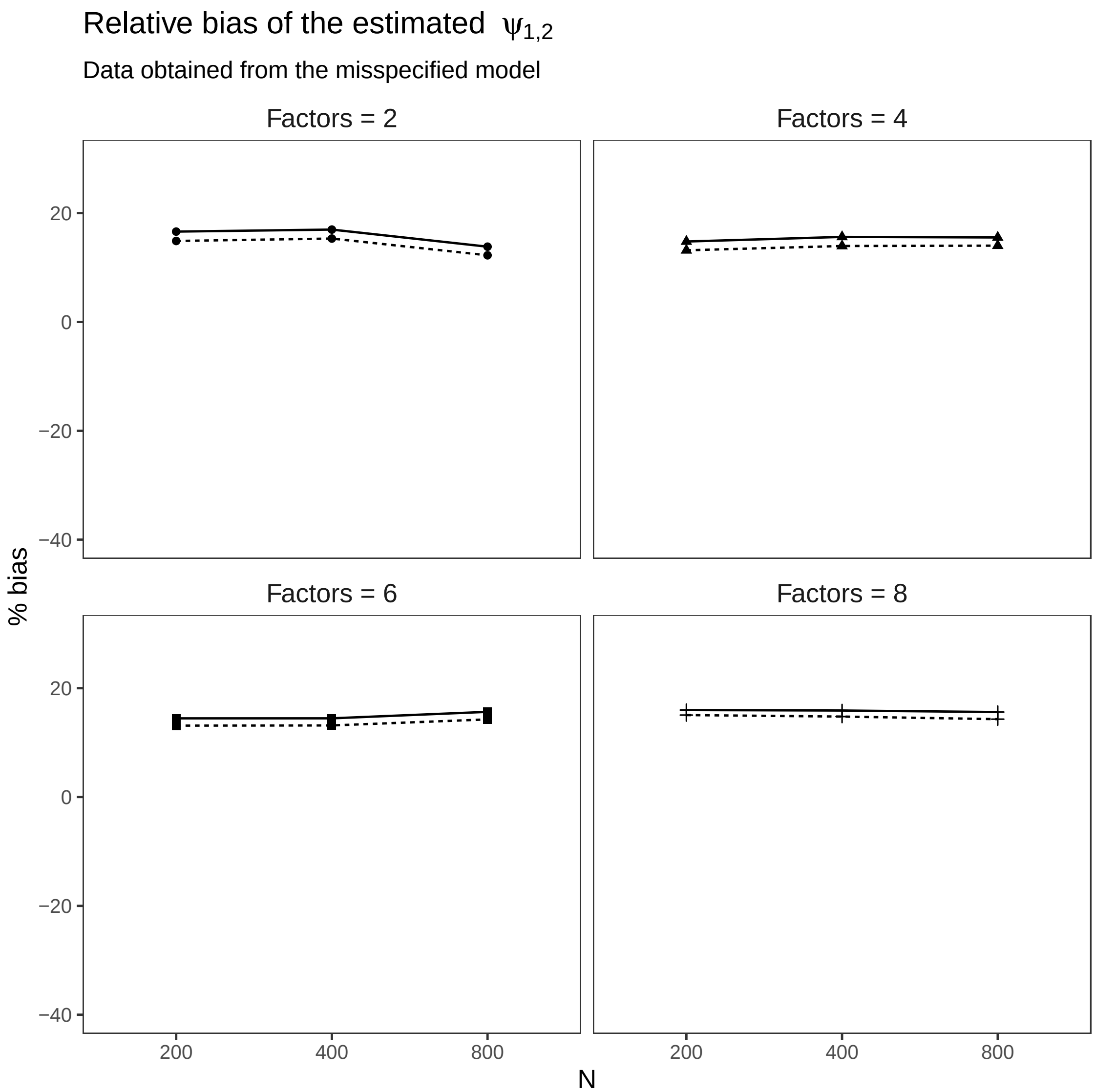


Figure B7: Three-way interaction effect between number of latent factors, sample size, and estimation method for $\psi_{1,2}$. The dashed line and solid line respectively represent the data sets with PML and DWLS as estimation method.

Table B2: ANOVA results of the highest order interaction effects between design factors and estimation method for the parameter estimates of the correctly specified model.

| **SE of par.** | **F-statistic** | ***p*-value** | $\eta^2$ | **Figure** | **Sign. result** |
|---|---|---|---|---|---|
| $\lambda_{2,1}$ | 68.29, 108.40 | <.001, <.001 | .042, .045 | B8* | nfact × method, $N$ × method |
| $\lambda_{6,1}$ | 47.11, 18.30 | <.001, <.001 | .016, .004 | B9* | nfact × method, $N$ × method |
| $\lambda_{6,2}$ | 20.94 | <.001 | .005 | B10 | nfact × $N$ × method |
| $\lambda_{8,2}$ | 6.62 | <.001 | <.001 | B11 | nfact × $N$ × method |
| $\lambda_{12,2}$ | 2.29 | .033 | <.001 | B12 | nfact × $N$ × method |
| $\theta_{2,1}$ | 24.68 | <.001 | .009 | B13 | nfact × $N$ × method |
| $\theta_{6,1}$ | 16.91 | <.001 | .005 | B14 | nfact × $N$ × method |
| $\tau_{8,2}$ | 79.92 | <.001 | .007 | B15 | nfact × $N$ × method |
| $\tau_{12,2}$ | 52.54 | <.001 | .003 | B16 | nfact × $N$ × method |
| $\psi_{1,2}$ | 4.15, 261.20 | .006, <.001 | <.001, .022 | B17* | nfact × method, $N$ × method |

*Note* * Plot (a) and (b) respectively. SE of par. = Standard Error of parameter, Sign. result = Significant ANOVA result, nfact = number of latent variables, $N$ = sample size

Table B3: Table with true values, raw means, and raw bias for the parameter estimates for each of the 12 conditions separately. Values are obtained from the misspecified model.

| | | | **Experimental condition** | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Value** | **Parameter** | **Method** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **Mean*** |
| True | $\lambda_{2,1}$ | Both | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | - |
| Estimate | $\lambda_{2,1}$ | PML | 0.79 | 0.79 | 0.79 | 0.78 | 0.80 | 0.79 | 0.79 | 0.79 | 0.79 | 0.79 | 0.79 | 0.79 | 0.79 |
| Raw bias | $\lambda_{2,1}$ | PML | -0.01 | -0.01 | -0.01 | -0.02 | 0.00 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 |
| Estimate | $\lambda_{2,1}$ | DWLS | 0.78 | 0.77 | 0.77 | 0.77 | 0.78 | 0.77 | 0.78 | 0.78 | 0.78 | 0.78 | 0.77 | 0.77 | 0.78 |
| Raw bias | $\lambda_{2,1}$ | DWLS | -0.02 | -0.03 | -0.03 | -0.03 | -0.02 | -0.03 | -0.02 | -0.02 | -0.02 | -0.02 | -0.03 | -0.03 | -0.03 |
| True | $\lambda_{6,1}$ | Both | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | - |
| Estimate | $\lambda_{6,1}$ | PML | 0.68 | 0.69 | 0.70 | 0.71 | 0.68 | 0.69 | 0.70 | 0.71 | 0.68 | 0.70 | 0.70 | 0.71 | 0.70 |
| Raw bias | $\lambda_{6,1}$ | PML | 0.08 | 0.09 | 0.10 | 0.11 | 0.08 | 0.09 | 0.10 | 0.11 | 0.08 | 0.10 | 0.10 | 0.11 | 0.10 |
| Estimate | $\lambda_{6,1}$ | DWLS | 0.73 | 0.74 | 0.74 | 0.75 | 0.73 | 0.74 | 0.74 | 0.75 | 0.73 | 0.74 | 0.75 | 0.75 | 0.74 |
| Raw bias | $\lambda_{6,1}$ | DWLS | 0.13 | 0.14 | 0.14 | 0.15 | 0.13 | 0.14 | 0.14 | 0.15 | 0.13 | 0.14 | 0.15 | 0.15 | 0.14 |
| True | $\lambda_{8,2}$ | Both | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | - |
| Estimate | $\lambda_{8,2}$ | PML | 0.80 | 0.80 | 0.80 | 0.79 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 |
| Raw bias | $\lambda_{8,2}$ | PML | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Estimate | $\lambda_{8,2}$ | DWLS | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 |
| Raw bias | $\lambda_{8,2}$ | DWLS | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| True | $\lambda_{12,2}$ | Both | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | - |
| Estimate | $\lambda_{12,2}$ | PML | 0.60 | 0.60 | 0.59 | 0.59 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 |
| Raw bias | $\lambda_{12,2}$ | PML | 0.00 | 0.00 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Estimate | $\lambda_{12,2}$ | DWLS | 0.61 | 0.61 | 0.59 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 |
| Raw bias | $\lambda_{12,2}$ | DWLS | 0.01 | 0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| True | $\theta_{2,1}$ | Both | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | - |
| Estimate | $\theta_{2,1}$ | PML | 0.36 | 0.37 | 0.37 | 0.38 | 0.36 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.38 | 0.38 | 0.37 |
| Raw bias | $\theta_{2,1}$ | PML | 0.00 | 0.01 | 0.01 | 0.02 | 0.00 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.02 | 0.02 | 0.01 |
| Estimate | $\theta_{2,1}$ | DWLS | 0.38 | 0.39 | 0.38 | 0.40 | 0.39 | 0.40 | 0.39 | 0.39 | 0.39 | 0.39 | 0.40 | 0.40 | 0.39 |
| Raw bias | $\theta_{2,1}$ | DWLS | 0.02 | 0.03 | 0.02 | 0.04 | 0.03 | 0.04 | 0.03 | 0.03 | 0.03 | 0.03 | 0.04 | 0.04 | 0.03 |
| True | $\theta_{6,1}$ | Both | 0.53 | 0.53 | 0.53 | 0.53 | 0.53 | 0.53 | 0.53 | 0.53 | 0.53 | 0.53 | 0.53 | 0.53 | - |
| Estimate | $\theta_{6,1}$ | PML | 0.52 | 0.51 | 0.50 | 0.49 | 0.52 | 0.51 | 0.50 | 0.50 | 0.53 | 0.51 | 0.50 | 0.50 | 0.51 |
| Raw bias | $\theta_{6,1}$ | PML | -0.01 | -0.01 | -0.02 | -0.04 | -0.01 | -0.02 | -0.03 | -0.03 | 0.00 | -0.02 | -0.03 | -0.03 | -0.02 |
| Estimate | $\theta_{6,1}$ | DWLS | 0.45 | 0.45 | 0.44 | 0.43 | 0.46 | 0.44 | 0.44 | 0.44 | 0.46 | 0.44 | 0.44 | 0.44 | 0.44 |
| Raw bias | $\theta_{6,1}$ | DWLS | -0.07 | -0.08 | -0.09 | -0.10 | -0.07 | -0.09 | -0.09 | -0.09 | -0.06 | -0.08 | -0.09 | -0.09 | -0.08 |
| True | $\tau_{8,2}$ | Both | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | - |
| Estimate | $\tau_{8,2}$ | PML | 0.00 | -0.01 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Raw bias | $\tau_{8,2}$ | PML | 0.00 | -0.01 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Estimate | $\tau_{8,2}$ | DWLS | 0.00 | -0.01 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Raw bias | $\tau_{8,2}$ | DWLS | 0.00 | -0.01 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| True | $\tau_{12,2}$ | Both | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | - |
| Estimate | $\tau_{12,2}$ | PML | -0.01 | -0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Raw bias | $\tau_{12,2}$ | PML | -0.01 | -0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Estimate | $\tau_{12,2}$ | DWLS | -0.01 | -0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Raw bias | $\tau_{12,2}$ | DWLS | -0.01 | -0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| True | $\psi_{1,2}$ | Both | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | - |
| Estimate | $\psi_{1,2}$ | PML | 0.34 | 0.34 | 0.34 | 0.35 | 0.35 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 |
| Raw bias | $\psi_{1,2}$ | PML | 0.04 | 0.04 | 0.04 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 |
| Estimate | $\psi_{1,2}$ | DWLS | 0.35 | 0.34 | 0.34 | 0.35 | 0.35 | 0.35 | 0.34 | 0.35 | 0.34 | 0.35 | 0.35 | 0.35 | 0.35 |
| Raw bias | $\psi_{1,2}$ | DWLS | 0.05 | 0.04 | 0.04 | 0.05 | 0.05 | 0.05 | 0.04 | 0.05 | 0.04 | 0.05 | 0.05 | 0.05 | 0.05 |

*Note.* Values < 0.01 are rounded to 0.00. * calculated across all conditions.

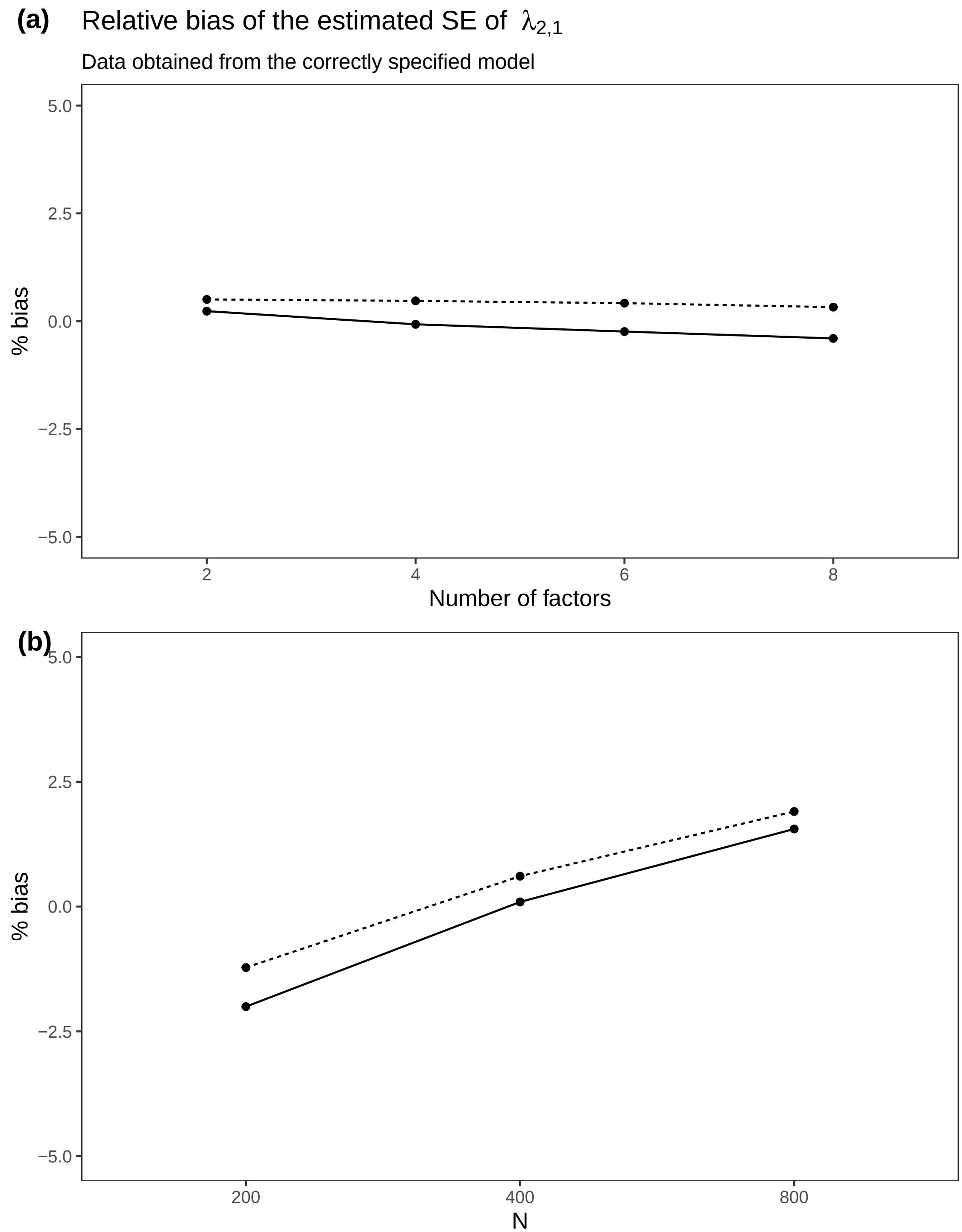


Figure B8: Two two-way interaction effects; plot a displays interaction effect between the number of latent factors and the estimation method for $\lambda_{2,2}$; plot b shows the interaction effect between sample size and estimation method for the SE of $\lambda_{2,1}$. The dashed line and solid line respectively represent the data sets with PML and DWLS as estimation method.

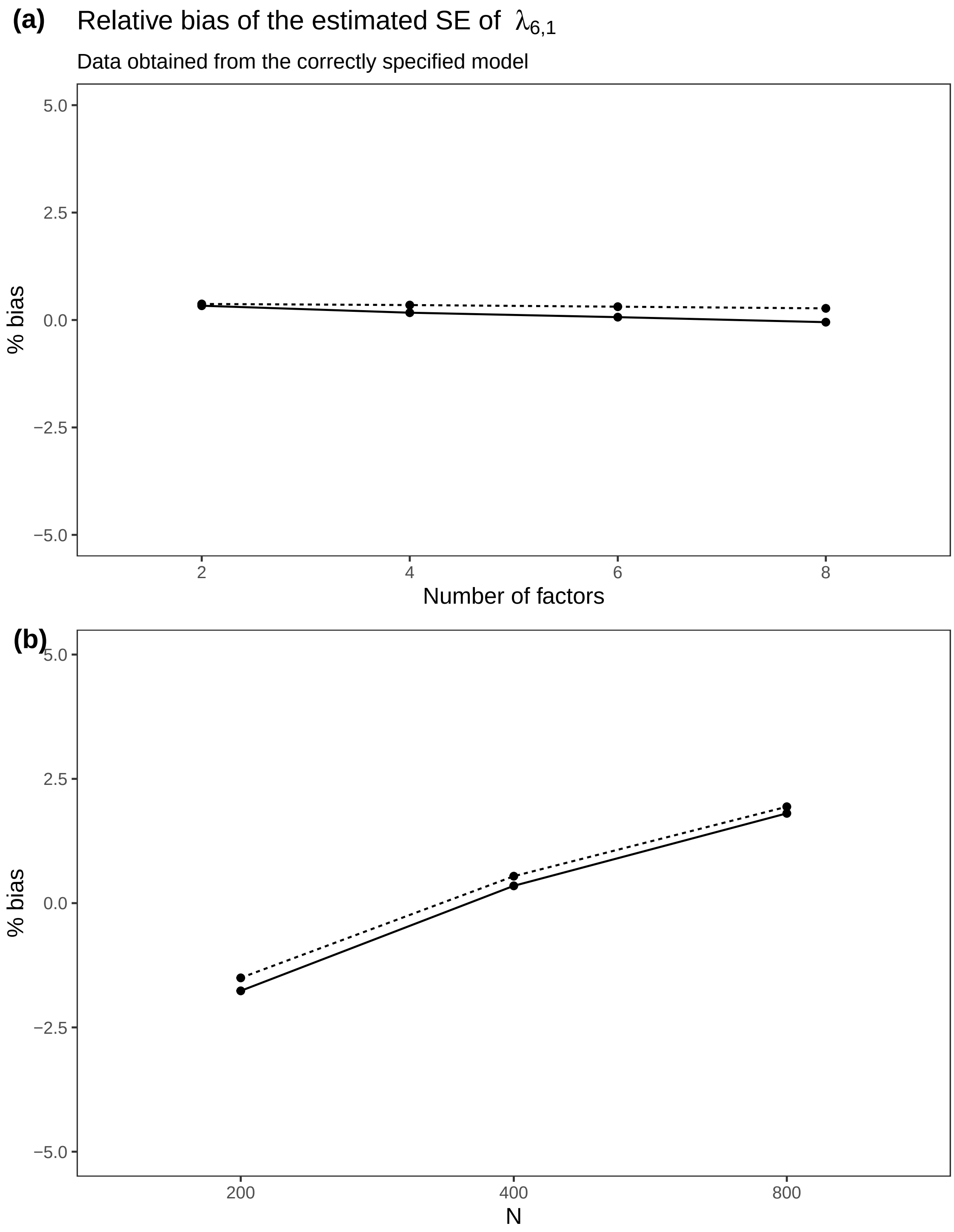


Figure B9: Two two-way interaction effects; plot (a) displays the interaction effect between the number of latent factors and the estimation method for $\lambda_{12,2}$; plot (b) shows the interaction effect between sample size and estimation method for the SE of $\lambda_{6,1}$. The solid line and dashed line respectively represent the datasets with DWLS and PML as estimation method.

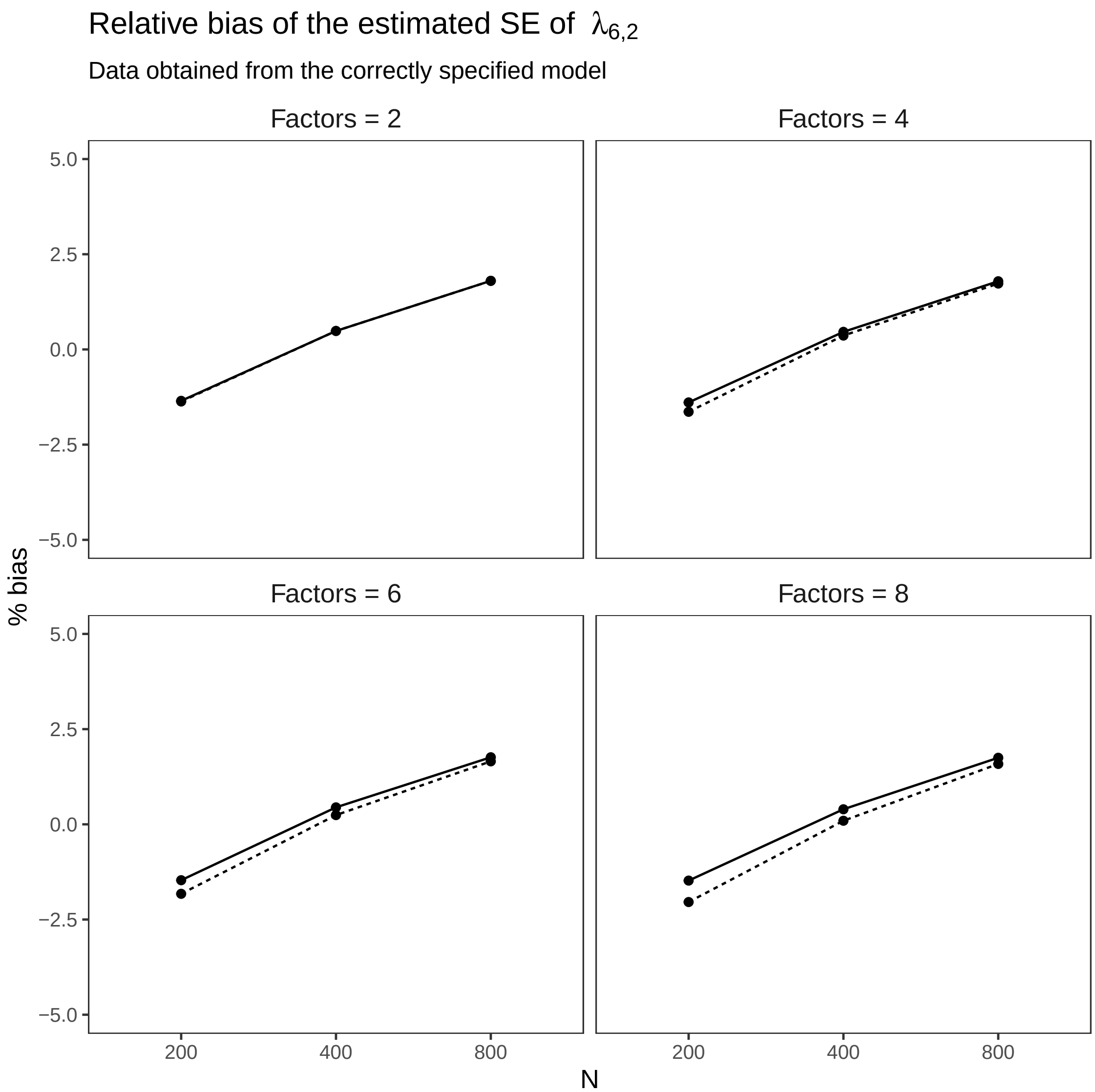


Figure B10: Three-way interaction effect between number of latent factors, sample size, and estimation method for the SE of $\lambda_{6,2}$. The solid line and dashed line respectively represent the datasets with DWLS and PML as estimation method.

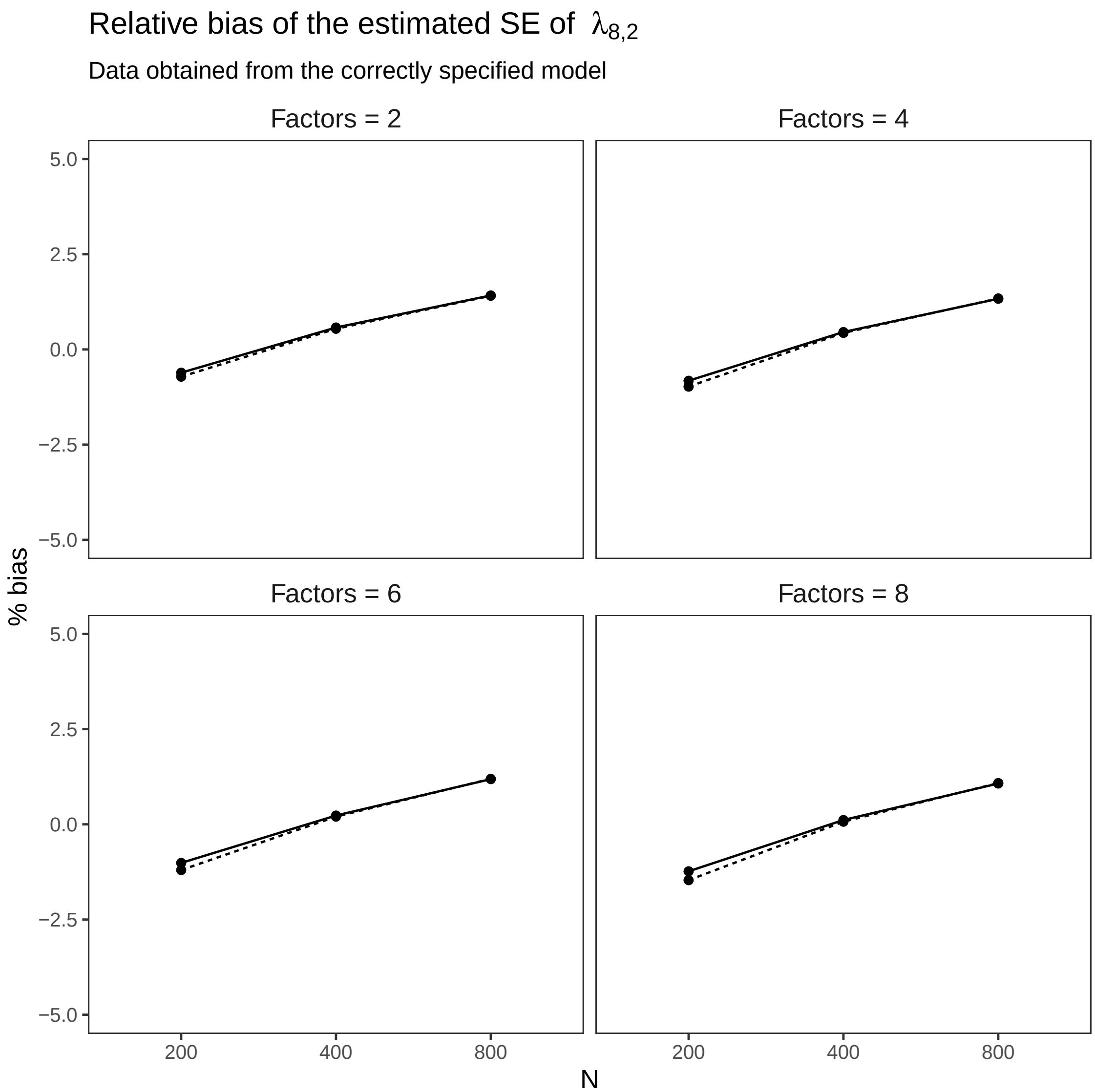


Figure B11: Three-way interaction effect between number of latent factors, sample size, and estimation method for the SE of $\lambda_{8,2}$. The solid line and dashed line respectively represent the datasets with DWLS and PML as estimation method.

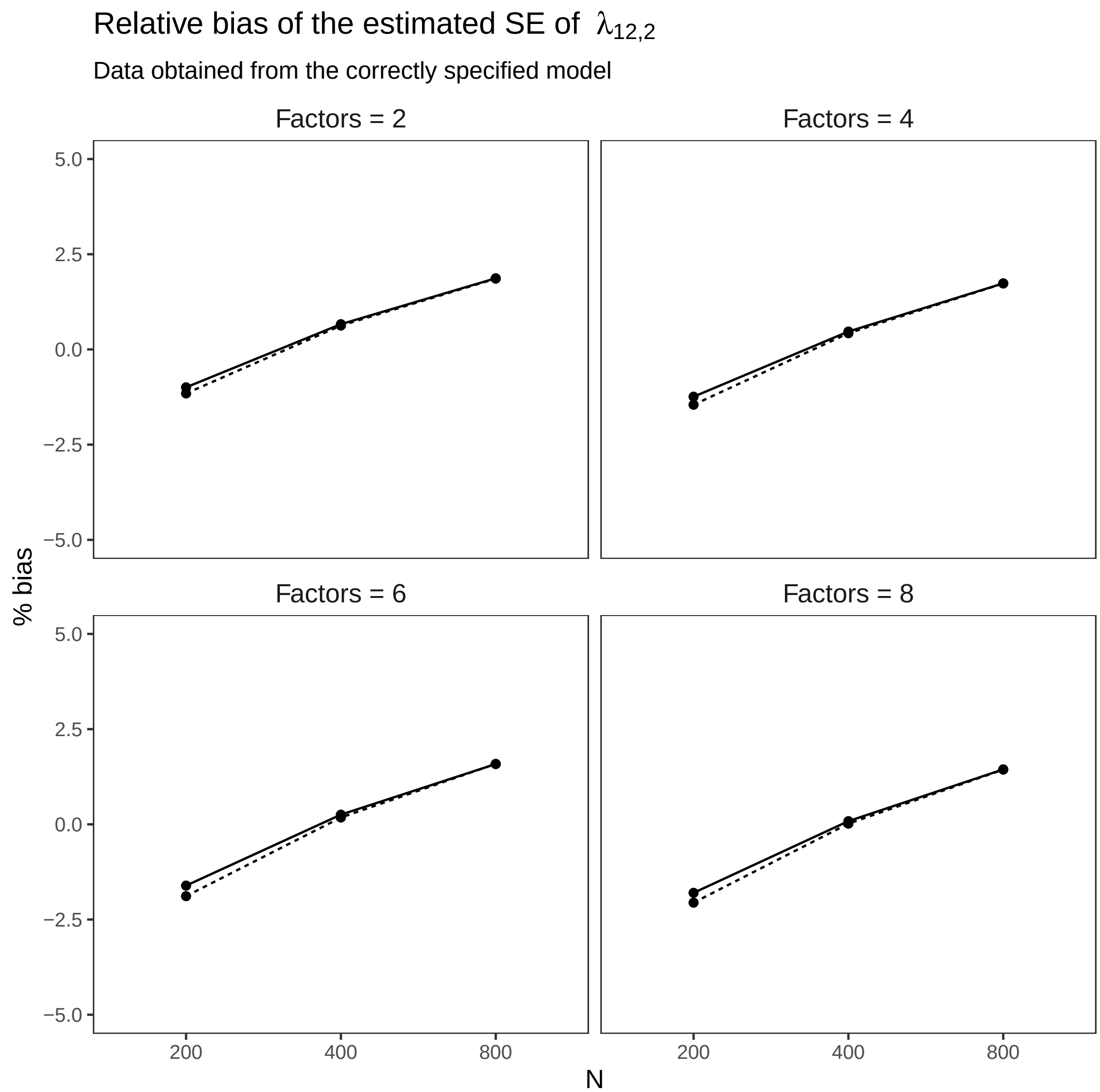


Figure B12: Three-way interaction effect between number of latent factors, sample size, and estimation method for the SE of $\lambda_{12,2}$. The solid line and dashed line respectively represent the datasets with DWLS and PML as estimation method.

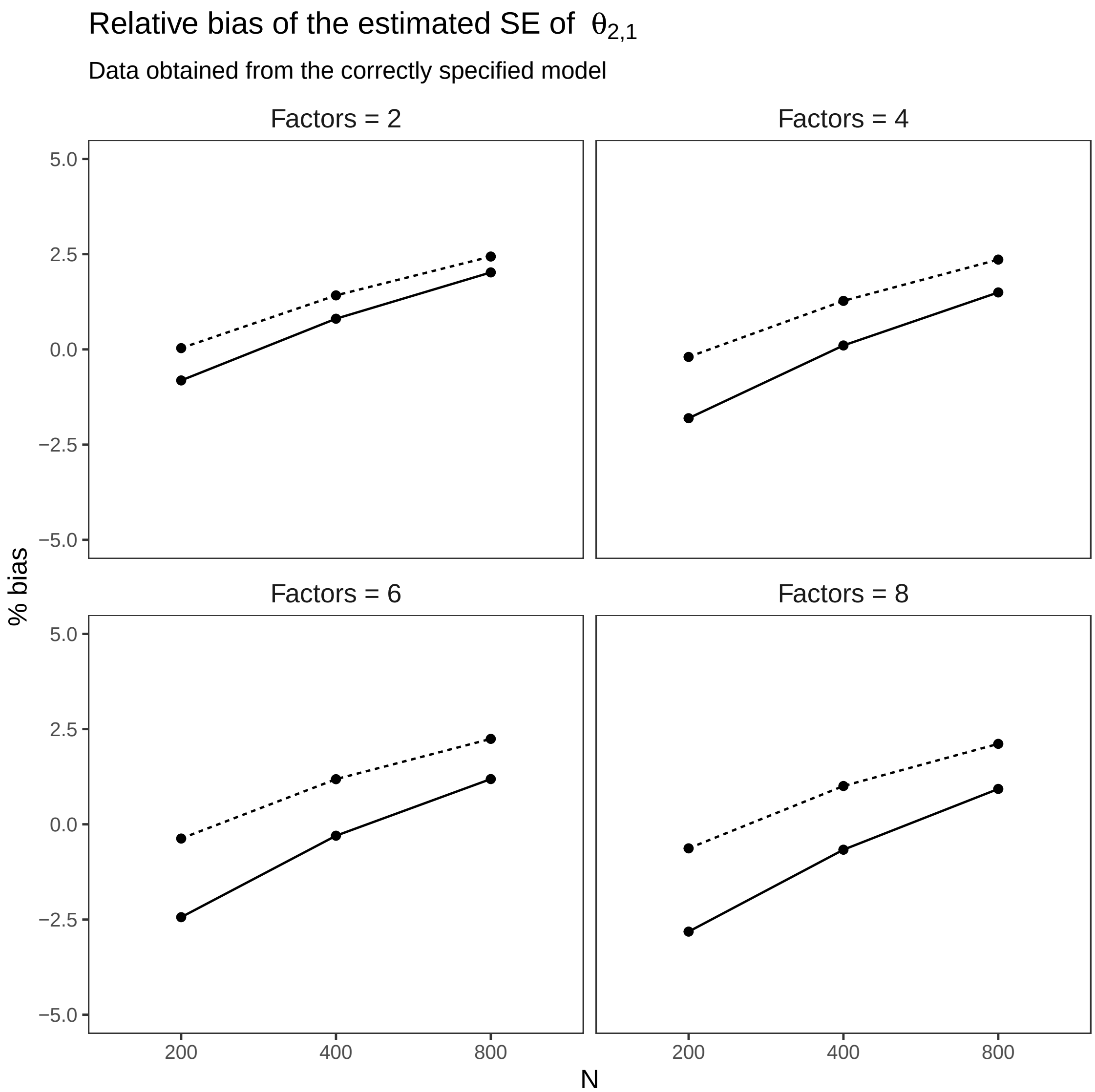


Figure B13: Three-way interaction effect between number of latent factors, sample size, and estimation method for the SE of $\theta_{2,1}$. The solid line and dashed line respectively represent the datasets with DWLS and PML as estimation method.

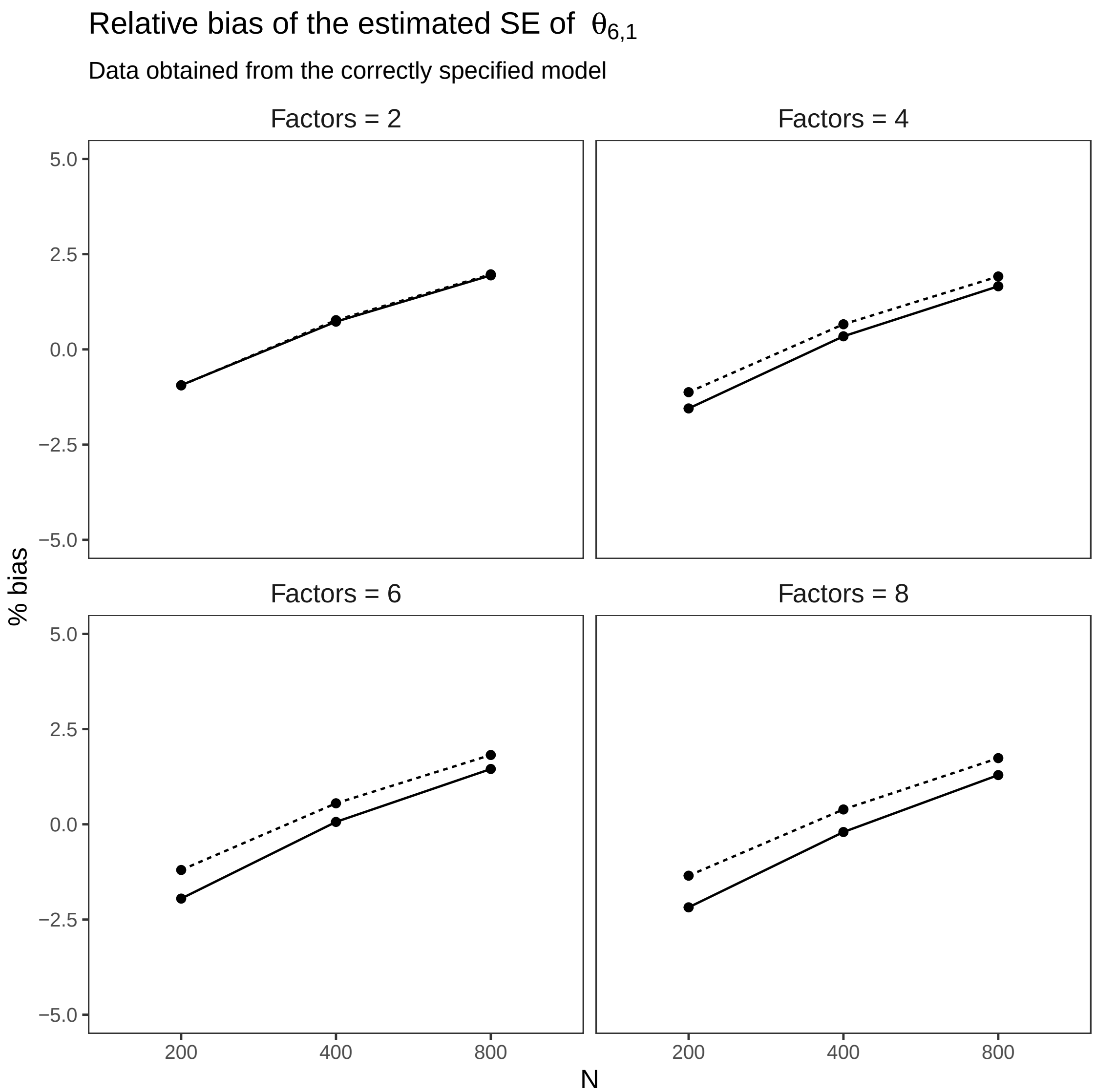


Figure B14: Three-way interaction effect between number of latent factors, sample size, and estimation method for the SE of $\theta_{6,1}$. The solid line and dashed line respectively represent the datasets with DWLS and PML as estimation method.

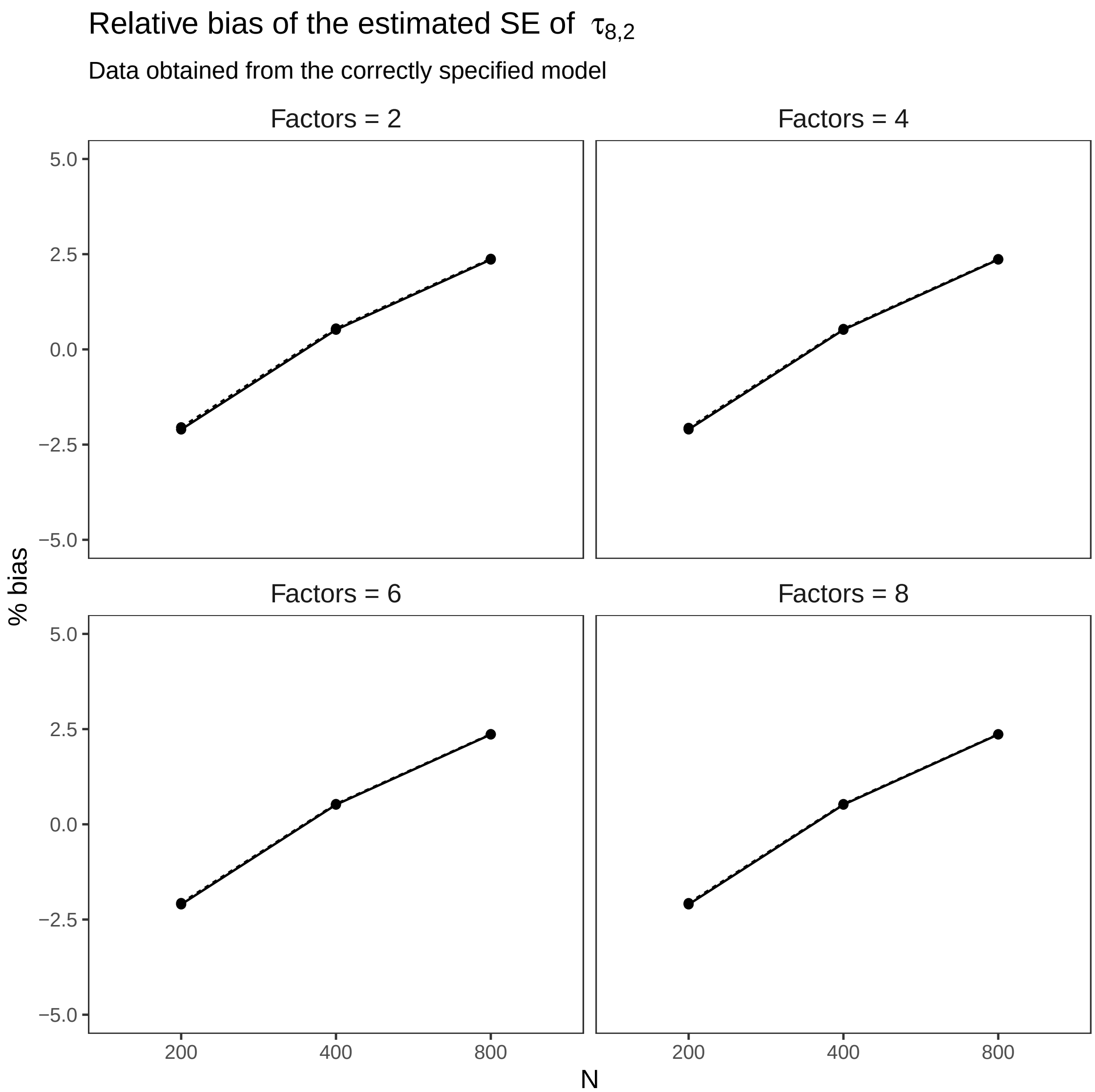


Figure B15: Three-way interaction effect between number of latent factors, sample size, and estimation method for the SE of $\tau_{8,2}$. The solid line and dashed line respectively represent the datasets with DWLS and PML as estimation method.

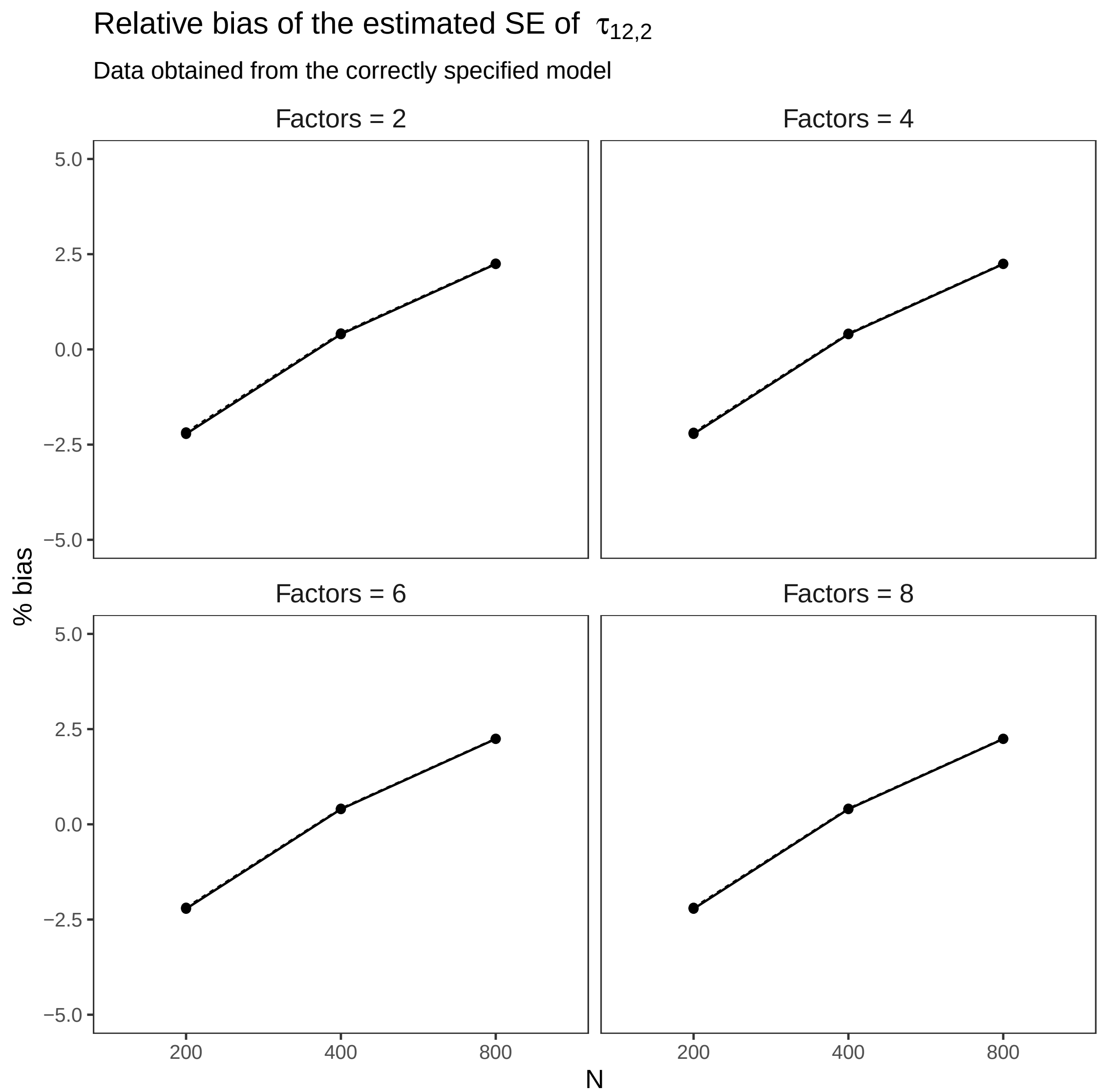


Figure B16: Three-way interaction effect between number of latent factors, sample size, and estimation method for the SE of $\tau_{12,2}$. The solid line and dashed line respectively represent the datasets with DWLS and PML as estimation method.

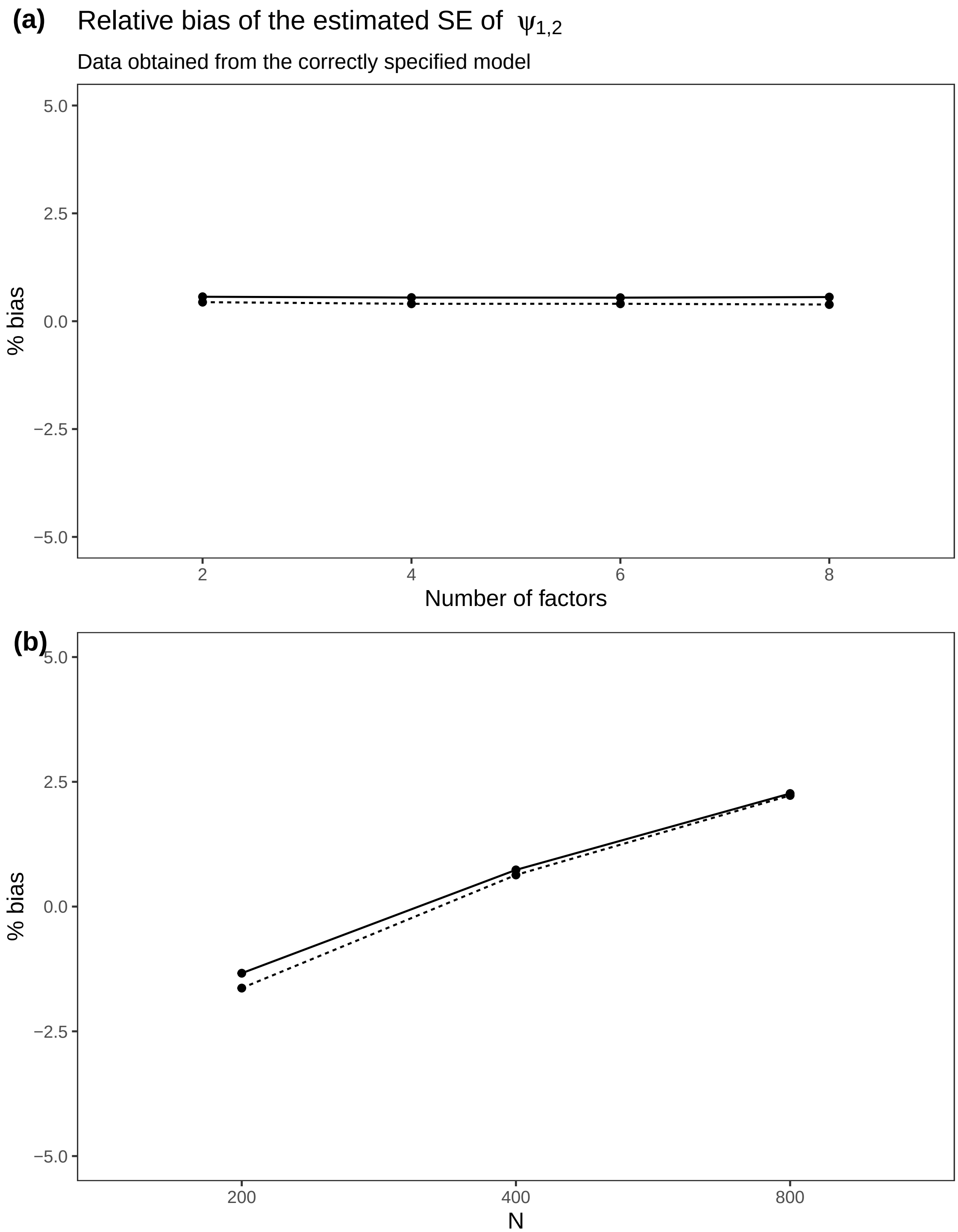


Figure B17: Two two-way interaction effects; plot (a) displays the interaction effect between the number of latent factors and the estimation method for the SE of $\psi_{1,2}$; plot (b) shows the interaction effect between sample size and estimation method for the SE of $\psi_{1,2}$. The solid line and dashed line respectively represent the datasets with DWLS and PML as estimation method.

Table B4: Table with true values, raw means, and raw bias for the SE of the parameter estimates for each of the 12 conditions separately. Values are obtained from the correctly specified model.

| | | | Experimental condition | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Value** | **Parameter** | **Method** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **Mean*** |
| SD of parameter estimate | $\lambda_{2,1}$ | PML | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | - |
| Estimate | SE of $\lambda_{2,1}$ | PML | 0.07 | 0.07 | 0.07 | 0.07 | 0.05 | 0.05 | 0.05 | 0.05 | 0.03 | 0.03 | 0.04 | 0.04 | 0.05 |
| Raw bias | SE of $\lambda_{2,1}$ | PML | -0.01 | -0.02 | -0.02 | -0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 |
| SD of parameter estimate | $\lambda_{2,1}$ | DWLS | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | - |
| Estimate | SE of $\lambda_{2,1}$ | DWLS | 0.07 | 0.07 | 0.07 | 0.07 | 0.05 | 0.05 | 0.05 | 0.05 | 0.03 | 0.03 | 0.04 | 0.04 | 0.05 |
| Raw bias | SE of $\lambda_{2,1}$ | DWLS | -0.02 | -0.02 | -0.02 | -0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.01 | 0.01 | 0.01 | 0.00 |
| SD of parameter estimate | $\lambda_{6,1}$ | PML | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | - |
| Estimate | SE of $\lambda_{6,1}$ | PML | 0.07 | 0.07 | 0.07 | 0.07 | 0.05 | 0.05 | 0.05 | 0.05 | 0.03 | 0.03 | 0.04 | 0.04 | 0.05 |
| Raw bias | SE of $\lambda_{6,1}$ | PML | -0.01 | -0.02 | -0.02 | -0.02 | 0.01 | 0.01 | 0.00 | 0.00 | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 |
| SD of parameter estimate | $\lambda_{6,1}$ | DWLS | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | - |
| Estimate | SE of $\lambda_{6,1}$ | DWLS | 0.07 | 0.07 | 0.07 | 0.07 | 0.05 | 0.05 | 0.05 | 0.05 | 0.03 | 0.03 | 0.04 | 0.04 | 0.05 |
| Raw bias | SE of $\lambda_{6,1}$ | DWLS | -0.02 | -0.02 | -0.02 | -0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 |
| SD of parameter estimate | $\lambda_{6,2}$ | PML | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | - |
| Estimate | SE of $\lambda_{6,2}$ | PML | 0.06 | 0.06 | 0.06 | 0.06 | 0.05 | 0.05 | 0.05 | 0.05 | 0.03 | 0.03 | 0.03 | 0.03 | 0.05 |
| Raw bias | SE of $\lambda_{6,2}$ | PML | -0.01 | -0.01 | -0.01 | -0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 |
| SD of parameter estimate | $\lambda_{6,2}$ | DWLS | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | - |
| Estimate | SE of $\lambda_{6,2}$ | DWLS | 0.06 | 0.06 | 0.06 | 0.06 | 0.05 | 0.05 | 0.05 | 0.05 | 0.03 | 0.03 | 0.03 | 0.03 | 0.05 |
| Raw bias | SE of $\lambda_{6,2}$ | DWLS | -0.01 | -0.01 | -0.01 | -0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 |
| SD of parameter estimate | $\lambda_{8,2}$ | PML | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | - |
| Estimate | SE of $\lambda_{8,2}$ | PML | 0.04 | 0.04 | 0.05 | 0.05 | 0.03 | 0.03 | 0.03 | 0.03 | 0.02 | 0.02 | 0.02 | 0.02 | 0.03 |
| Raw bias | SE of $\lambda_{8,2}$ | PML | -0.01 | -0.01 | -0.01 | -0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.01 | 0.01 | 0.00 |
| SD of parameter estimate | $\lambda_{8,2}$ | DWLS | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | - |
| Estimate | SE of $\lambda_{8,2}$ | DWLS | 0.04 | 0.04 | 0.05 | 0.05 | 0.03 | 0.03 | 0.03 | 0.03 | 0.02 | 0.02 | 0.02 | 0.02 | 0.03 |
| Raw bias | SE of $\lambda_{8,2}$ | DWLS | -0.01 | -0.01 | -0.01 | -0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.01 | 0.01 | 0.00 |
| SD of parameter estimate | $\lambda_{12,2}$ | PML | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | - |
| Estimate | SE of $\lambda_{12,2}$ | PML | 0.06 | 0.06 | 0.06 | 0.07 | 0.04 | 0.04 | 0.05 | 0.05 | 0.03 | 0.03 | 0.03 | 0.03 | 0.05 |
| Raw bias | SE of $\lambda_{12,2}$ | PML | -0.01 | -0.01 | -0.02 | -0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.02 | 0.02 | 0.02 | 0.01 | 0.00 |
| SD of parameter estimate | $\lambda_{12,2}$ | DWLS | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | - |
| Estimate | SE of $\lambda_{12,2}$ | DWLS | 0.06 | 0.06 | 0.06 | 0.07 | 0.04 | 0.04 | 0.05 | 0.05 | 0.03 | 0.03 | 0.03 | 0.03 | 0.05 |
| Raw bias | SE of $\lambda_{12,2}$ | DWLS | -0.01 | -0.01 | -0.02 | -0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.02 | 0.02 | 0.02 | 0.01 | 0.00 |
| SD of parameter estimate | $\theta_{2,1}$ | PML | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | - |
| Estimate | SE of $\theta_{2,1}$ | PML | 0.06 | 0.07 | 0.07 | 0.08 | 0.04 | 0.05 | 0.05 | 0.06 | 0.03 | 0.03 | 0.04 | 0.04 | 0.05 |
| Raw bias | SE of $\theta_{2,1}$ | PML | -0.01 | -0.02 | -0.02 | -0.03 | 0.01 | 0.00 | 0.00 | 0.00 | 0.02 | 0.02 | 0.01 | 0.01 | 0.00 |
| SD of parameter estimate | $\theta_{2,1}$ | DWLS | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | - |
| Estimate | SE of $\theta_{2,1}$ | DWLS | 0.06 | 0.07 | 0.07 | 0.08 | 0.04 | 0.05 | 0.05 | 0.06 | 0.03 | 0.03 | 0.04 | 0.04 | 0.05 |
| Raw bias | SE of $\theta_{2,1}$ | DWLS | -0.01 | -0.02 | -0.03 | -0.03 | 0.01 | 0.00 | -0.01 | -0.01 | 0.02 | 0.01 | 0.01 | 0.01 | 0.00 |
| SD of parameter estimate | $\theta_{6,1}$ | PML | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | - |
| Estimate | SE of $\theta_{6,1}$ | PML | 0.06 | 0.06 | 0.07 | 0.07 | 0.04 | 0.05 | 0.05 | 0.05 | 0.03 | 0.03 | 0.03 | 0.04 | 0.05 |
| Raw bias | SE of $\theta_{6,1}$ | PML | 0.00 | 0.00 | -0.01 | -0.01 | 0.02 | 0.02 | 0.01 | 0.01 | 0.03 | 0.03 | 0.03 | 0.02 | 0.01 |
| SD of parameter estimate | $\theta_{6,1}$ | DWLS | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | - |
| Estimate | SE of $\theta_{6,1}$ | DWLS | 0.06 | 0.06 | 0.07 | 0.07 | 0.04 | 0.05 | 0.05 | 0.05 | 0.03 | 0.03 | 0.03 | 0.04 | 0.05 |
| Raw bias | SE of $\theta_{6,1}$ | DWLS | 0.00 | 0.00 | -0.01 | -0.01 | 0.02 | 0.02 | 0.01 | 0.01 | 0.03 | 0.03 | 0.03 | 0.02 | 0.01 |
| SD of parameter estimate | $\tau_{8,2}$ | PML | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | - |
| Estimate | SE of $\tau_{8,2}$ | PML | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.06 |
| Raw bias | SE of $\tau_{8,2}$ | PML | -0.02 | -0.02 | -0.02 | -0.02 | 0.01 | 0.01 | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 |
| SD of parameter estimate | $\tau_{8,2}$ | DWLS | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | - |
| Estimate | SE of $\tau_{8,2}$ | DWLS | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.06 |
| Raw bias | SE of $\tau_{8,2}$ | DWLS | -0.02 | -0.02 | -0.02 | -0.02 | 0.01 | 0.01 | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 |
| SD of parameter estimate | $\tau_{12,2}$ | PML | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | - |
| Estimate | SE of $\tau_{12,2}$ | PML | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.06 |
| Raw bias | SE of $\tau_{12,2}$ | PML | -0.02 | -0.02 | -0.02 | -0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 |
| SD of parameter estimate | $\tau_{12,2}$ | DWLS | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | - |
| Estimate | SE of $\tau_{12,2}$ | DWLS | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.06 |
| Raw bias | SE of $\tau_{12,2}$ | DWLS | -0.02 | -0.02 | -0.02 | -0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 |
| SD of parameter estimate | $\psi_{1,2}$ | PML | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | - |
| Estimate | SE of $\psi_{1,2}$ | PML | 0.07 | 0.07 | 0.07 | 0.07 | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 | 0.05 |
| Raw bias | SE of $\psi_{1,2}$ | PML | -0.02 | -0.02 | -0.02 | -0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 |
| SD of parameter estimate | $\psi_{1,2}$ | DWLS | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | - |
| Estimate | SE of $\psi_{1,2}$ | DWLS | 0.07 | 0.07 | 0.07 | 0.07 | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 | 0.05 |
| Raw bias | SE of $\psi_{1,2}$ | DWLS | -0.03 | -0.03 | -0.03 | -0.03 | -0.01 | -0.01 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |

*Note*. Values $< 0.01$ are rounded to 0.00. The experimental conditions correspond to those specified in Table 3.2. * calculated across all conditions.

Table B5: ANOVA results of the highest order interaction effects between design factors and estimation method for the SE of parameter estimates of the misspecified model.

| **SE of par.** | **F-statistic** | ***p*-value** | $\eta^2$ | **Figure** | **Sign. result** |
|---|---|---|---|---|---|
| $\lambda_{2,1}$ | 50.33, 91.02 | < .001, < .001 | .031, .037 | B18* | nfact × method, $N$ × method |
| $\lambda_{6,1}$ | 2.62 | .015 | .003 | B19 | nfact × $N$ × method |
| $\lambda_{8,2}$ | 6.79 | < .001 | < .001 | B20 | nfact × $N$ × method |
| $\lambda_{12,2}$ | 2.24 | .037 | < .001 | B21 | nfact × $N$ × method |
| $\theta_{2,1}$ | 16.72 | < .001 | .006 | B22 | nfact × $N$ × method |
| $\theta_{6,1}$ | 33.54 | < .001 | .014 | B23 | nfact × $N$ × method |
| $\tau_{8,2}$ | 80.01 | < .001 | .007 | B24 | nfact × $N$ × method |
| $\tau_{12,2}$ | 52.09 | < .001 | .003 | B25 | nfact × $N$ × method |
| $\psi_{1,2}$ | 331.58 | < .001 | < .001 | B26 | $N$ × method |

*Note* * Plot (a) and (b) respectively. SE of par. = Standard Error of parameter, Sign. result = Significant ANOVA result, nfact = number of latent variables, $N$ = sample size.

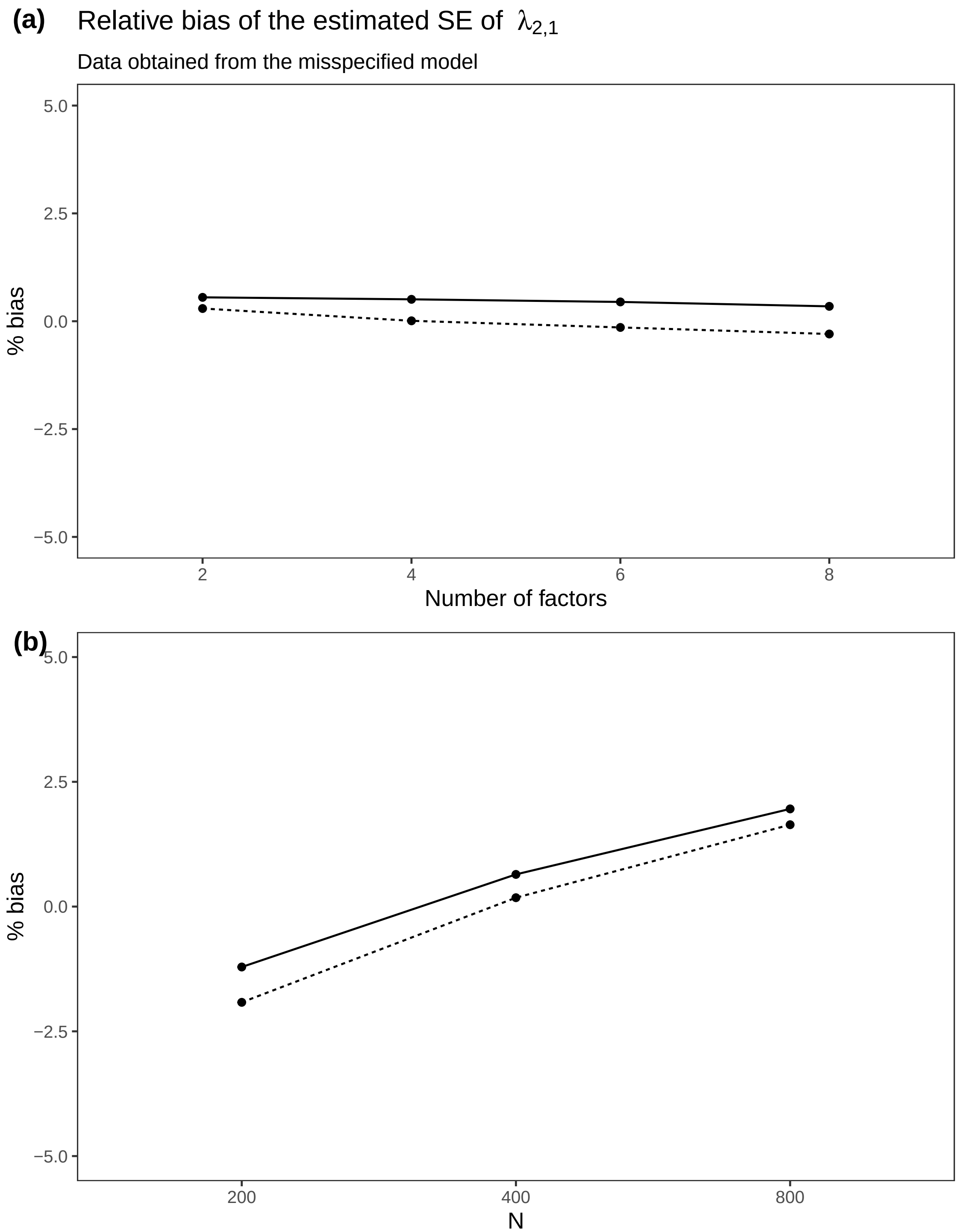


Figure B18: Two two-way interaction effects; plot (a) displays interaction effect between the number of latent factors and the estimation method for $\lambda_{2,1}$; plot (b) shows the interaction effect between sample size and estimation method for the SE of $\lambda_{2,1}$. The dashed line and solid line respectively represent the data sets with PML and DWLS as estimation method.

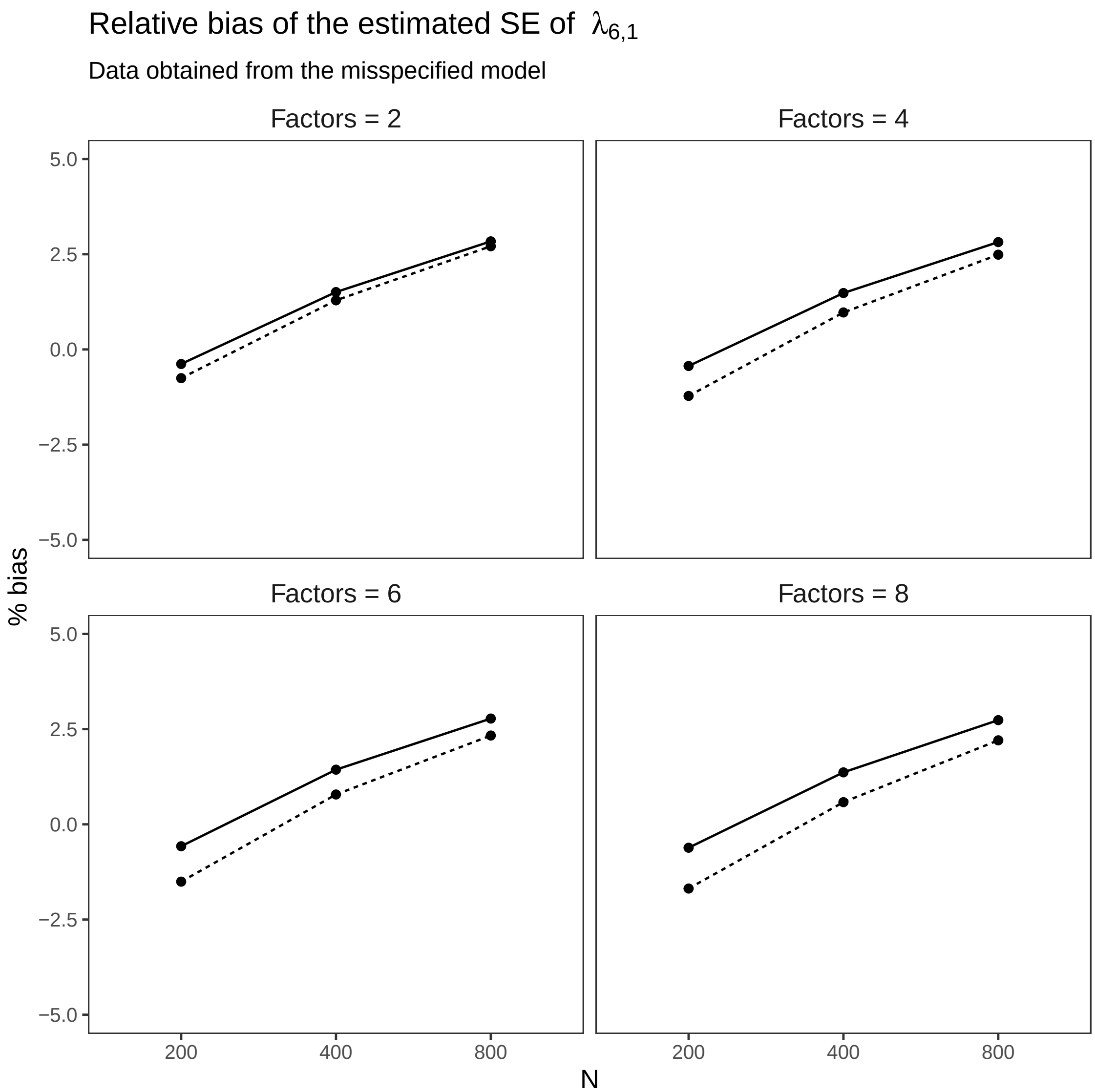


Figure B19: Three-way interaction effect between number of latent factors, sample size, and estimation method for the SE of $\lambda_{6,1}$. The solid line and dashed line respectively represent the datasets with DWLS and PML as estimation method.

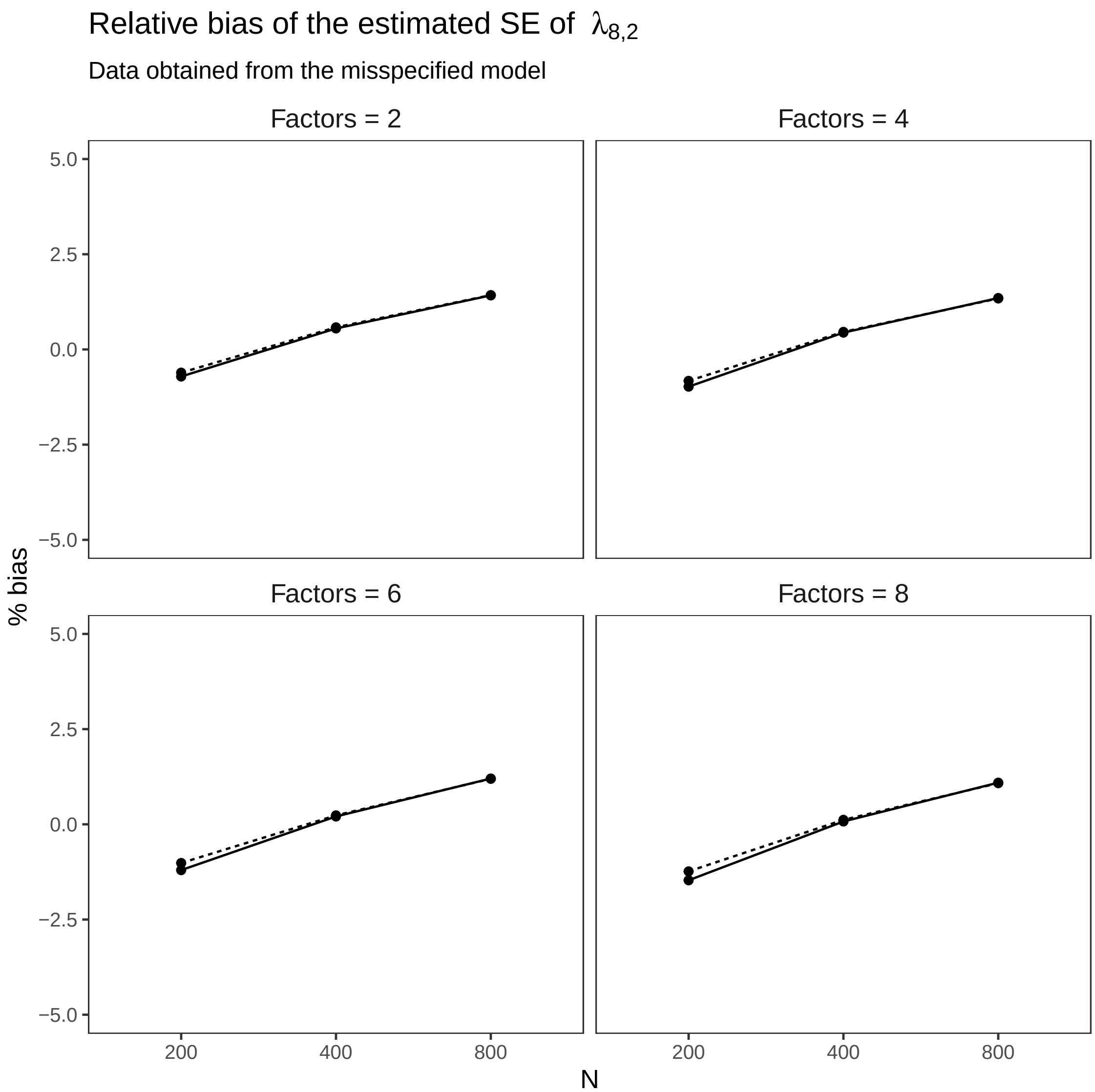


Figure B20: Three-way interaction effect between number of latent factors, sample size, and estimation method for the SE of $\lambda_{8,2}$. The solid line and dashed line respectively represent the datasets with DWLS and PML as estimation method.

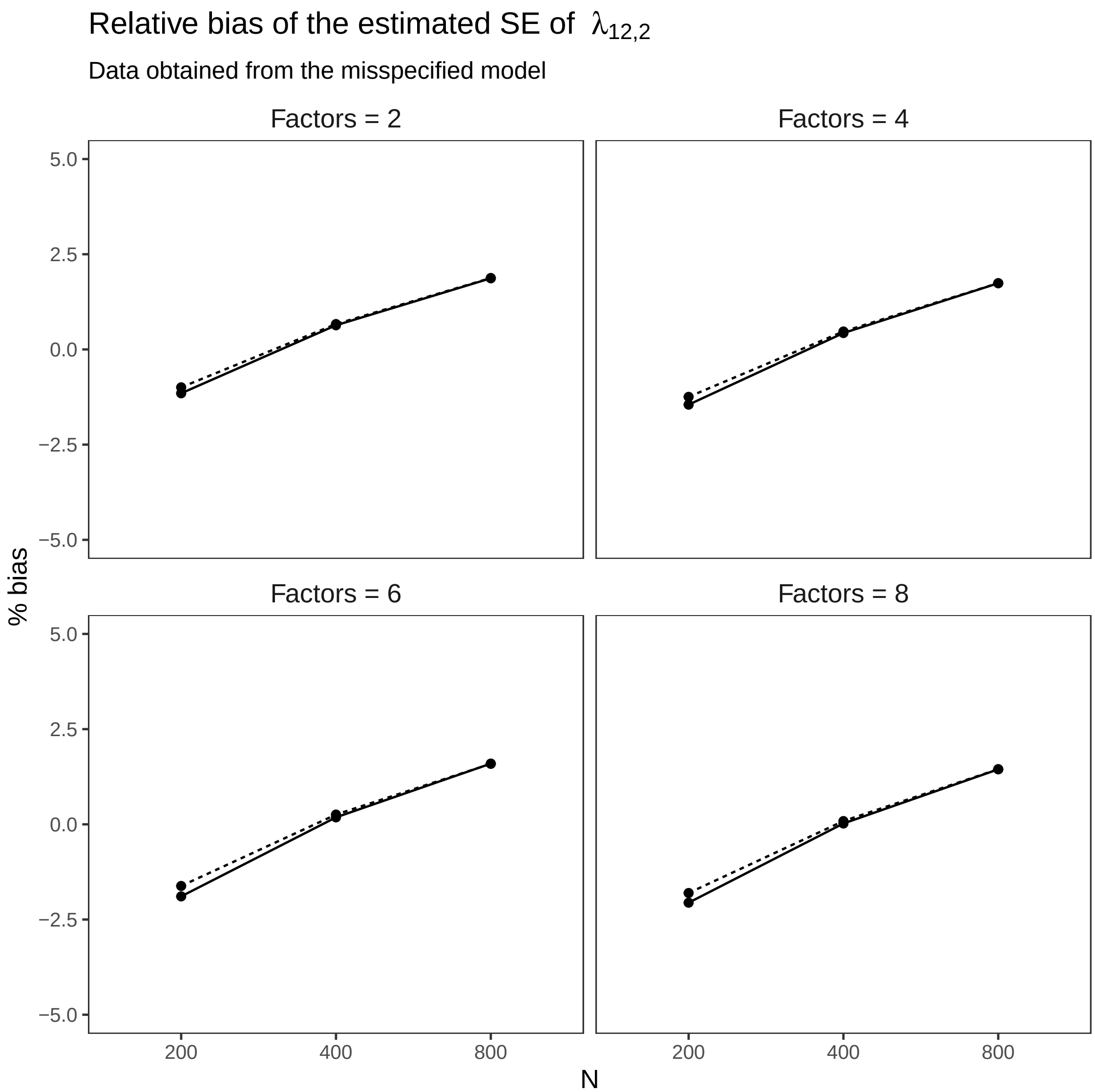


Figure B21: Three-way interaction effect between number of latent factors, sample size, and estimation method for the SE of $\lambda_{12,2}$. The solid line and dashed line respectively represent the datasets with DWLS and PML as estimation method.

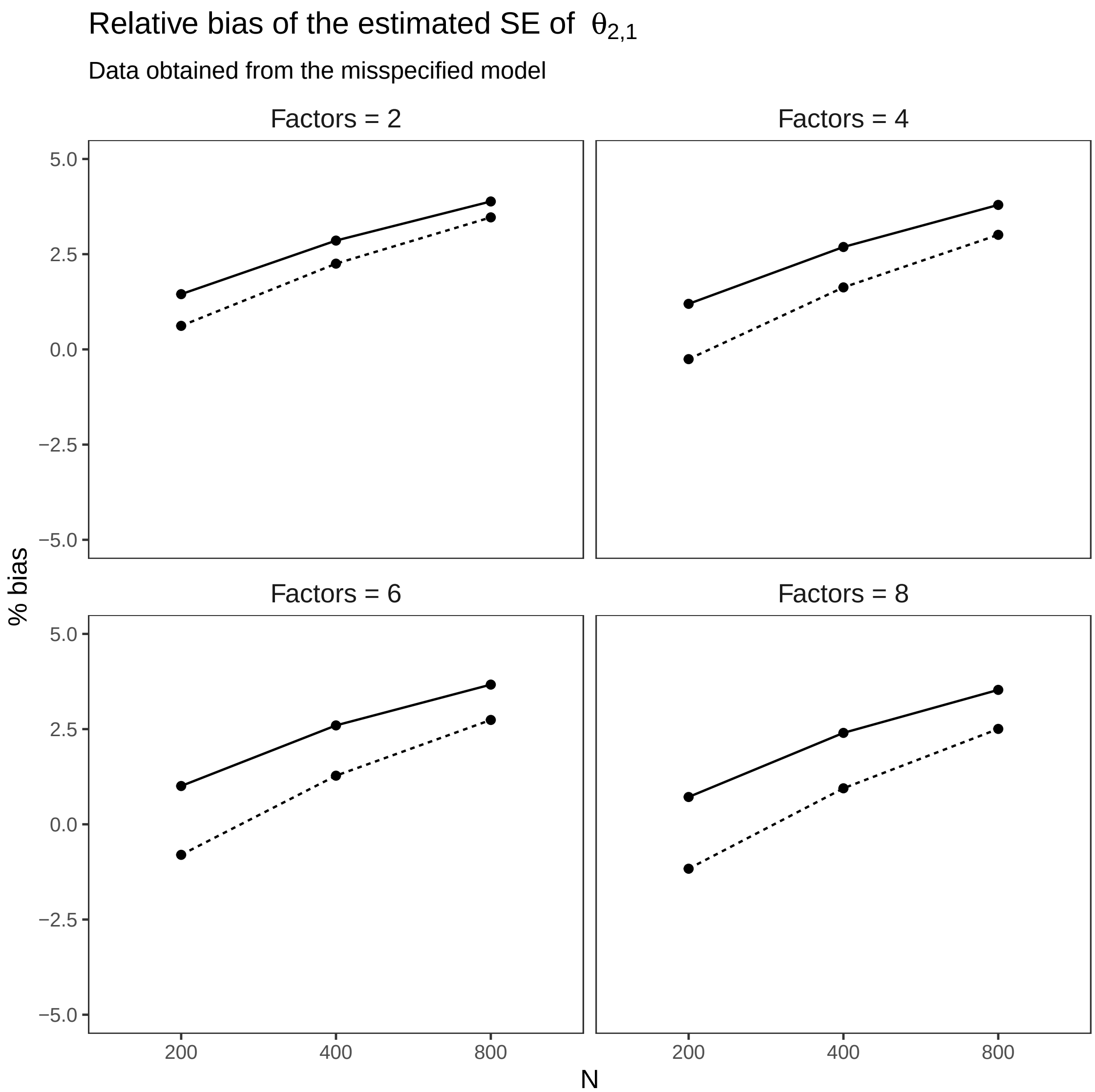


Figure B22: Three-way interaction effect between number of latent factors, sample size, and estimation method for the SE of $\theta_{2,1}$. The solid line and dashed line respectively represent the datasets with DWLS and PML as estimation method.

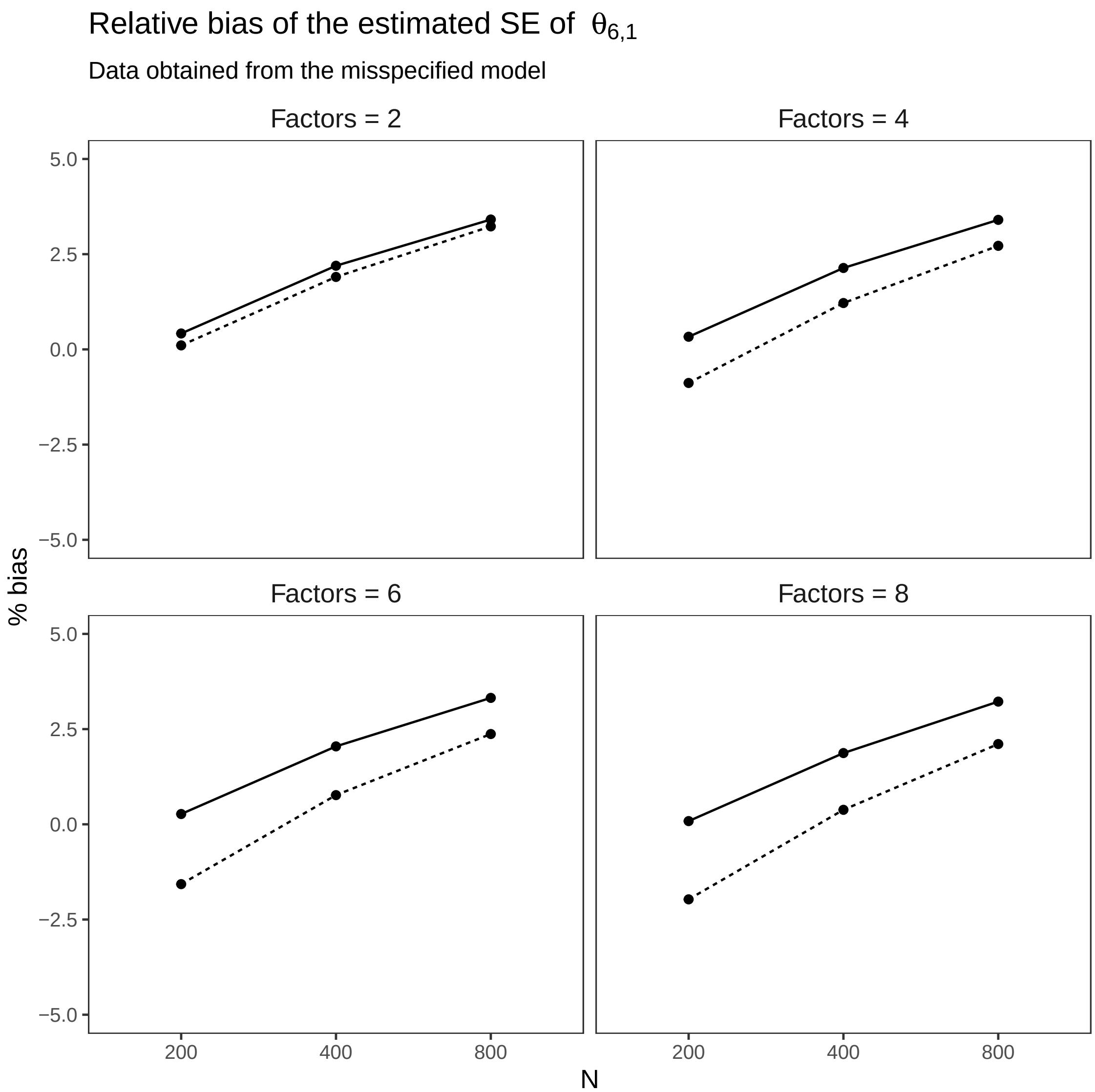


Figure B23: Three-way interaction effect between number of latent factors, sample size, and estimation method for the SE of $\theta_{6,1}$. The solid line and dashed line respectively represent the datasets with DWLS and PML as estimation method.

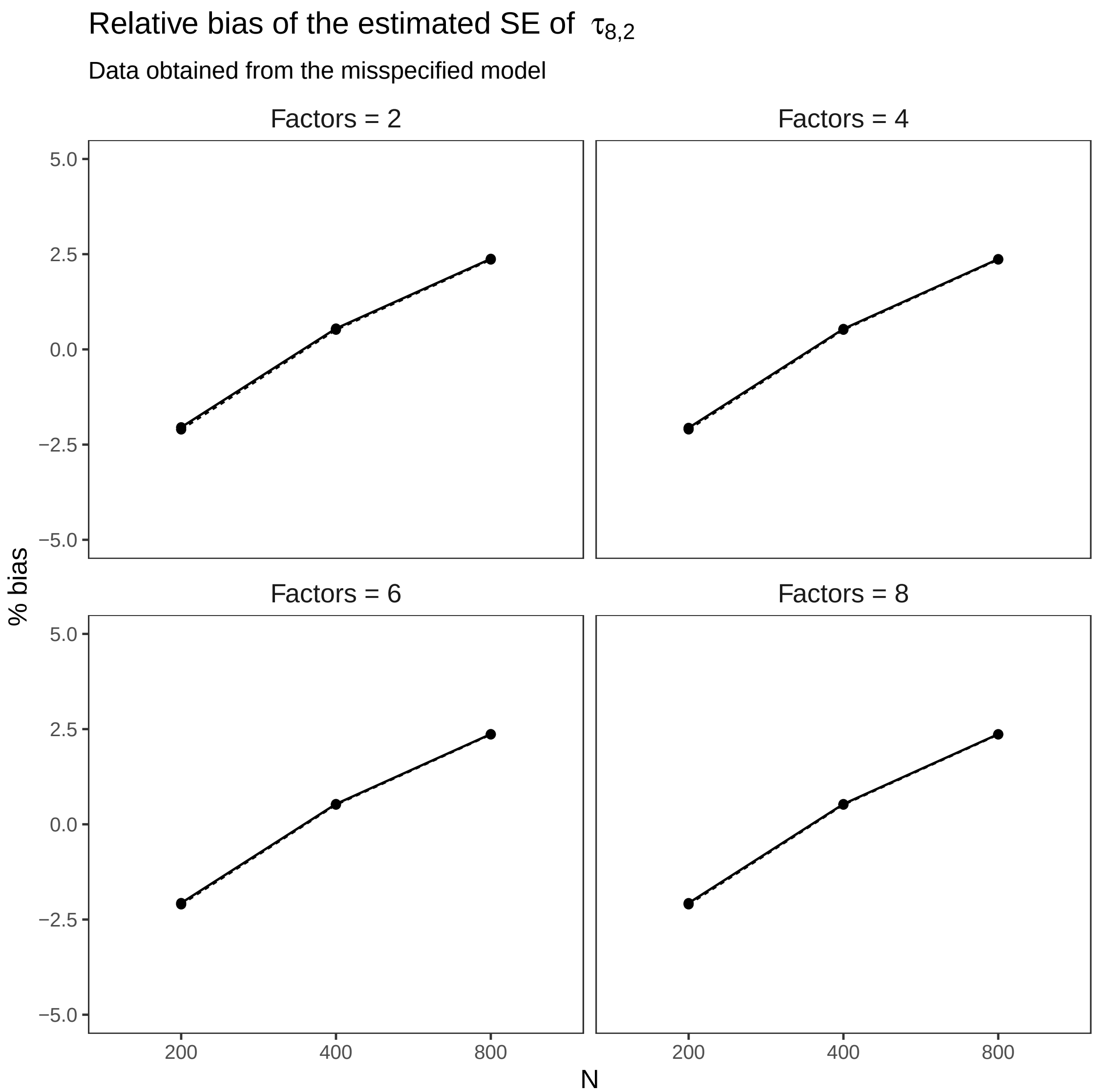


Figure B24: Three-way interaction effect between number of latent factors, sample size, and estimation method for the SE of $\tau_{8,2}$. The solid line and dashed line respectively represent the datasets with DWLS and PML as estimation method.

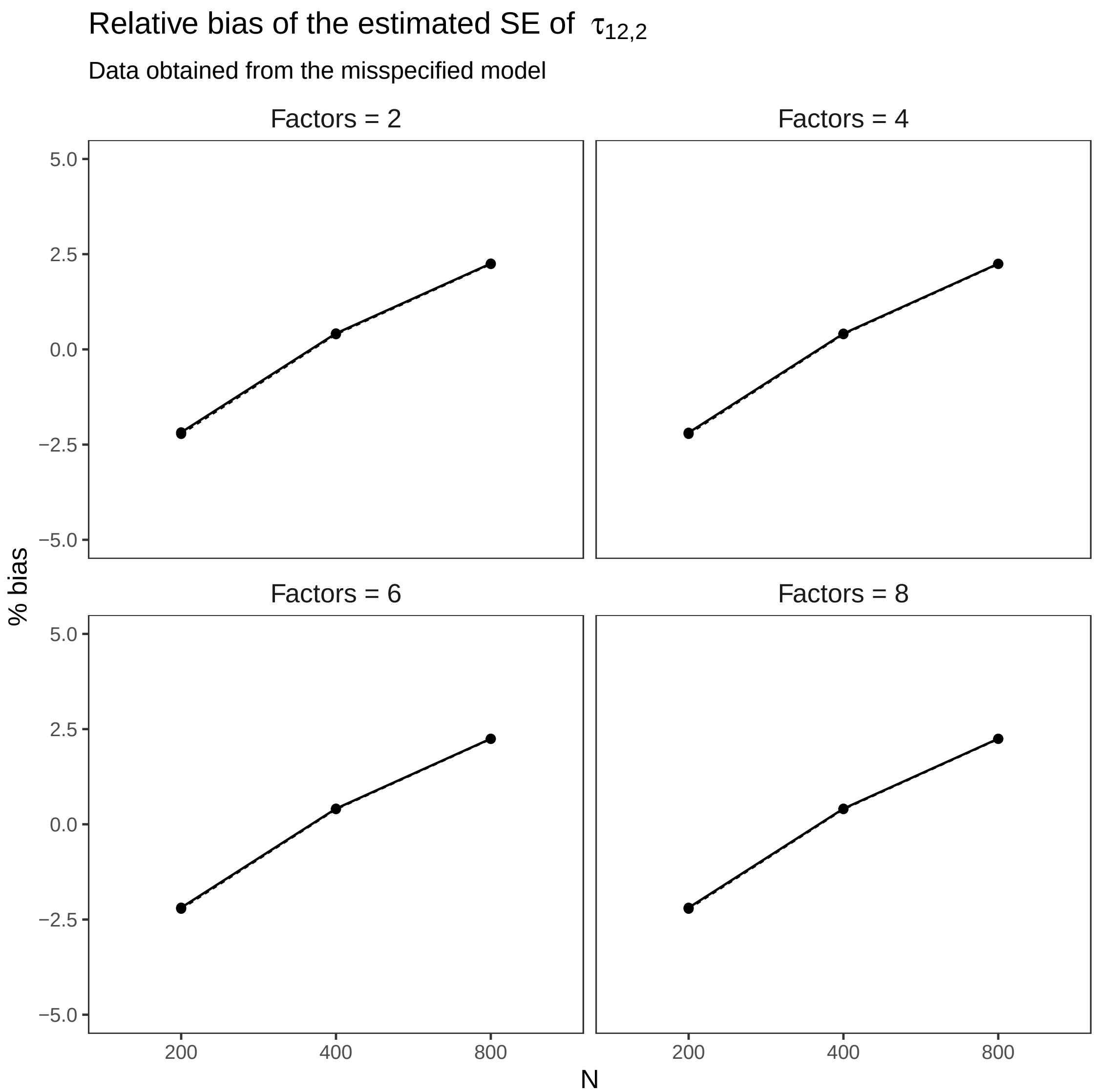


Figure B25: Three-way interaction effect between number of latent factors, sample size, and estimation method for the SE of $\tau_{12,2}$. The solid line and dashed line respectively represent the datasets with DWLS and PML as estimation method.

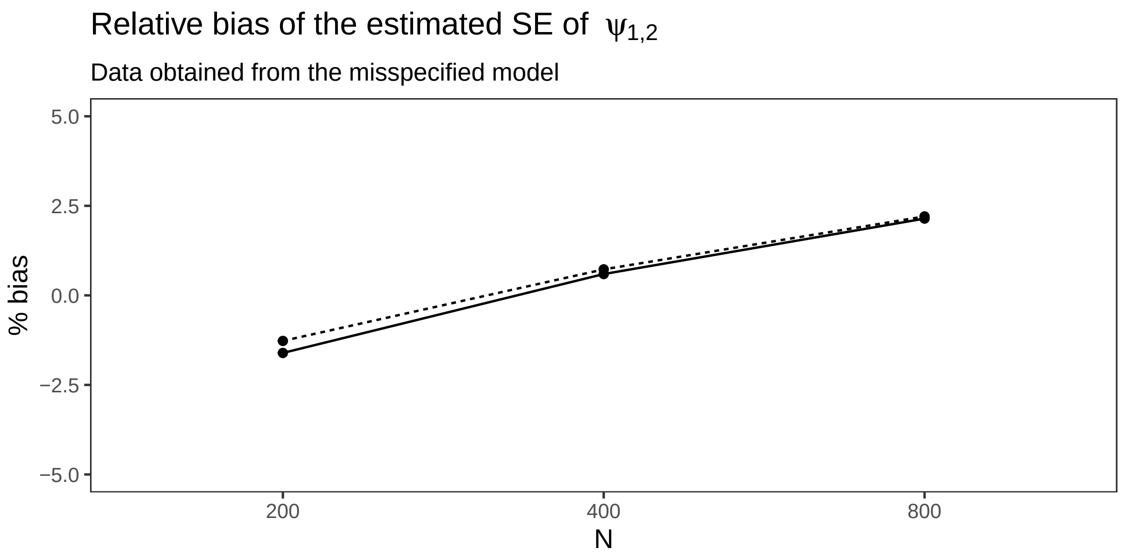


Figure B26: Two-way interaction effect between sample size and estimation method for the SE of $\psi_{1,2}$. The solid line and dashed line respectively represent the datasets with DWLS and PML as estimation method.

Table B6: Table with true values, raw means, and raw bias for the SE of the parameter estimates for each of the 12 conditions separately. Values are obtained from the misspecified model.

| | | | **Experimental condition** | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Value** | **Parameter** | **Method** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **Mean*** |
| SD of parameter estimate | $\lambda_{2,1}$ | PML | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | - |
| Estimate | SE of $\lambda_{2,1}$ | PML | 0.07 | 0.07 | 0.07 | 0.07 | 0.05 | 0.05 | 0.05 | 0.05 | 0.03 | 0.03 | 0.04 | 0.04 | 0.05 |
| Raw bias | SE of $\lambda_{2,1}$ | PML | -0.02 | -0.02 | -0.02 | -0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.01 | 0.01 | 0.01 | 0.00 |
| SD of parameter estimate | $\lambda_{2,1}$ | DWLS | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | - |
| Estimate | SE of $\lambda_{2,1}$ | DWLS | 0.06 | 0.06 | 0.06 | 0.07 | 0.04 | 0.04 | 0.05 | 0.05 | 0.03 | 0.03 | 0.03 | 0.03 | 0.05 |
| Raw bias | SE of $\lambda_{2,1}$ | DWLS | -0.01 | -0.01 | -0.01 | -0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 | 0.02 | 0.01 |
| SD of parameter estimate | $\lambda_{6,1}$ | PML | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | - |
| Estimate | SE of $\lambda_{6,1}$ | PML | 0.07 | 0.07 | 0.08 | 0.08 | 0.05 | 0.05 | 0.05 | 0.06 | 0.03 | 0.04 | 0.04 | 0.04 | 0.06 |
| Raw bias | SE of $\lambda_{6,1}$ | PML | -0.02 | -0.02 | -0.02 | -0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.02 | 0.01 | 0.01 | 0.00 |
| SD of parameter estimate | $\lambda_{6,1}$ | DWLS | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | - |
| Estimate | SE of $\lambda_{6,1}$ | DWLS | 0.06 | 0.07 | 0.07 | 0.07 | 0.05 | 0.05 | 0.05 | 0.05 | 0.03 | 0.03 | 0.03 | 0.03 | 0.05 |
| Raw bias | SE of $\lambda_{6,1}$ | DWLS | 0.00 | 0.00 | -0.01 | -0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.03 | 0.03 | 0.03 | 0.03 | 0.01 |
| SD of parameter estimate | $\lambda_{8,2}$ | PML | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | - |
| Estimate | SE of $\lambda_{8,2}$ | PML | 0.04 | 0.04 | 0.05 | 0.05 | 0.03 | 0.03 | 0.03 | 0.03 | 0.02 | 0.02 | 0.02 | 0.02 | 0.03 |
| Raw bias | SE of $\lambda_{8,2}$ | PML | -0.01 | -0.01 | -0.01 | -0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.01 | 0.01 | 0.00 |
| SD of parameter estimate | $\lambda_{8,2}$ | DWLS | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | - |
| Estimate | SE of $\lambda_{8,2}$8 | DWLS | 0.04 | 0.05 | 0.05 | 0.05 | 0.03 | 0.03 | 0.03 | 0.03 | 0.02 | 0.02 | 0.02 | 0.02 | 0.03 |
| Raw bias | SE of $\lambda_{8,2}$ | DWLS | -0.01 | -0.01 | -0.01 | -0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.01 | 0.01 | 0.00 |
| SD of parameter estimate | $\lambda_{12,2}$ | PML | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | - |
| Estimate | SE of $\lambda_{12,2}$ | PML | 0.06 | 0.06 | 0.06 | 0.07 | 0.04 | 0.04 | 0.05 | 0.05 | 0.03 | 0.03 | 0.03 | 0.03 | 0.05 |
| Raw bias | SE of $\lambda_{12,2}$ | PML | -0.01 | -0.01 | -0.02 | -0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.02 | 0.02 | 0.02 | 0.01 | 0.00 |
| SD of parameter estimate | $\lambda_{12,2}$ | DWLS | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | - |
| Estimate | SE of $\lambda_{12,2}$ | DWLS | 0.06 | 0.06 | 0.07 | 0.07 | 0.04 | 0.04 | 0.05 | 0.05 | 0.03 | 0.03 | 0.03 | 0.03 | 0.05 |
| Raw bias | SE of $\lambda_{12,2}$ | DWLS | -0.01 | -0.01 | -0.02 | -0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.02 | 0.02 | 0.02 | 0.01 | 0.00 |
| SD of parameter estimate | $\theta_{2,1}$ | PML | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | - |
| Estimate | SE of $\theta_{2,1}$ | PML | 0.06 | 0.07 | 0.07 | 0.08 | 0.04 | 0.05 | 0.05 | 0.05 | 0.03 | 0.03 | 0.04 | 0.04 | 0.05 |
| Raw bias | SE of $\theta_{2,1}$ | PML | -0.01 | -0.02 | -0.02 | -0.03 | 0.01 | 0.00 | 0.00 | -0.01 | 0.02 | 0.01 | 0.01 | 0.01 | 0.00 |
| SD of parameter estimate | $\theta_{2,1}$ | DWLS | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | - |
| Estimate | SE of $\theta_{2,1}$ | DWLS | 0.05 | 0.05 | 0.05 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.03 | 0.03 | 0.03 | 0.03 | 0.04 |
| Raw bias | SE of $\theta_{2,1}$ | DWLS | 0.01 | 0.01 | 0.01 | 0.01 | 0.03 | 0.02 | 0.02 | 0.02 | 0.04 | 0.04 | 0.03 | 0.03 | 0.02 |
| SD of parameter estimate | $\theta_{6,1}$ | PML | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | - |
| Estimate | SE of $\theta_{6,1}$ | PML | 0.06 | 0.07 | 0.08 | 0.08 | 0.05 | 0.05 | 0.06 | 0.06 | 0.03 | 0.04 | 0.04 | 0.04 | 0.06 |
| Raw bias | SE of $\theta_{6,1}$ | PML | 0.00 | -0.01 | -0.02 | -0.02 | 0.01 | 0.01 | 0.00 | 0.00 | 0.03 | 0.02 | 0.02 | 0.02 | 0.01 |
| SD of parameter estimate | $\theta_{6,1}$ | DWLS | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | - |
| Estimate | SE of $\theta_{6,1}$ | DWLS | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.05 | 0.03 | 0.03 | 0.03 | 0.03 | 0.04 |
| Raw bias | SE of $\theta_{6,1}$ | DWLS | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.02 | 0.02 | 0.01 | 0.03 | 0.03 | 0.03 | 0.03 | 0.02 |
| SD of parameter estimate | $\tau_{8,2}$ | PML | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | - |
| Estimate | SE of $\tau_{8,2}$ | PML | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.06 |
| Raw bias | SE of $\tau_{8,2}$ | PML | -0.02 | -0.02 | -0.02 | -0.02 | 0.01 | 0.01 | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 |
| SD of parameter estimate | $\tau_{8,2}$ | DWLS | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | - |
| Estimate | SE of $\tau_{8,2}$ | DWLS | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.06 |
| Raw bias | SE of $\tau_{8,2}$ | DWLS | -0.02 | -0.02 | -0.02 | -0.02 | 0.01 | 0.01 | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 |
| SD of parameter estimate | $\tau_{12,2}$ | PML | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | - |
| Estimate | SE of $\tau_{12,2}$ | PML | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.06 |
| Raw bias | SE of $\tau_{12,2}$ | PML | -0.02 | -0.02 | -0.02 | -0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 |
| SD of parameter estimate | $\tau_{12,2}$ | DWLS | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | - |
| Estimate | SE of $\tau_{12,2}$ | DWLS | 0.09 | 0.09 | 0.09 | 0.09 | 0.06 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.06 |
| Raw bias | SE of $\tau_{12,2}$ | DWLS | -0.02 | -0.02 | -0.02 | -0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 |
| SD of parameter estimate | $\psi_{1,2}$ | PML | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | - |
| Estimate | SE of $\psi_{1,2}$ | PML | 0.07 | 0.07 | 0.07 | 0.07 | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 | 0.05 |
| Raw bias | SE of $\psi_{1,2}$ | PML | -0.03 | -0.03 | -0.03 | -0.03 | -0.01 | -0.01 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| SD of parameter estimate | $\psi_{1,2}$ | DWLS | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | - |
| Estimate | SE of $\psi_{1,2}$ | DWLS | 0.08 | 0.08 | 0.08 | 0.08 | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 | 0.06 |
| Raw bias | SE of $\psi_{1,2}$ | DWLS | -0.02 | -0.02 | -0.02 | -0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.02 | 0.02 | 0.02 | 0.00 |

*Note*. Values $< 0.01$ are rounded to 0.00. The experimental conditions correspond to those specified in Table 3.2. * calculated across all conditions.

## 6.3 Appendix C

### Overview Github files

The following link provides access to all files used to replicate the current study:

`https://github.com/mmcstorm/PML.git`

The tables below show an overview of the files in the folders of the Github repository.

Table C1: File overview of folder: 'Simulation scripts (study 1)'

| Filename | Description |
|---|---|
| ORDINAL_MainSimulationScript_owncomputer.R | File contains simulation script for testing the code without using the cluster computer |
| ORDINAL_all_functions_script.R | File contains all functions used for data generation |
| ORDINAL_MainSimulationscriptSLURM.R | File contains simulation script for the cluster computer |
| ORDINAL_RunMySimulationSLURM4GB.sh | File used to run simulation on the cluster computer |

Table C2: File overview of folder: 'Simulation scripts (study 2)'

| Filename | Description |
|---|---|
| MIXED_MainSimulationScript_owncomputer.R | File contains simulation script for testing the code without using the cluster computer |
| MIXED_all_functions_script.R | File contains all functions used for data generation |
| MIXED_MainSimulationscriptSLURM.R | File contains simulation script for the cluster computer |
| MIXED_RunMySimulationSLURM4GB.sh | File used to run simulation on the cluster computer |

Table C3: File overview of folder: 'Simulation data (study 1)'

| Filename | Description |
|---|---|
| ORD_aov_withC_errR1_24_ID.csv | Simulated data with SE of the parameter estimates obtained from the correctly specified model |
| ORD_aov_withC_estR1_24_ID.csv | Simulated data with parameter estimates obtained from the correctly specified model |
| ORD_aov_withoutC_errR1_24_ID.csv | Simulated data with SE of the parameter estimates obtained from the misspecified model |
| ORD_aov_withoutC_estR1_24_ID.csv | Simulated data with parameter estimates obtained from the misspecified model |

Table C4: File overview of folder: 'Simulation data (study 2)'

| Filename | Description |
|---|---|
| MIX_aov_withC_errR1_12_ID.csv | Simulated data with SE of the parameter estimates obtained from the correctly specified model |
| MIX_aov_withC_estR1_12_ID.csv | Simulated data with parameter estimates obtained from the correctly specified model |
| MIX_aov_withoutC_errR1_12_ID.csv | Simulated data with SE of the parameter estimates obtained from the misspecified model |
| MIX_aov_withoutC_estR1_12_ID.csv | Simulated data with parameter estimates obtained from the misspecified model |

Table C5: File overview of folder: 'ANOVA (study 1)'

| Filename | Description |
|---|---|
| ORDINAL_ANOVA_results_(RAW).R | File used to create tables with true values, raw means, and raw biases |
| ORDINAL_Analysing_ANOVA_results.R | File used to conduct the ANOVAs |

Table C6: File overview of folder: 'ANOVA (study 2)'

| Filename | Description |
|---|---|
| MIXED_ANOVA_results_(RAW).R | File used to create tables with true values, raw means, and raw biases |
| MIXED_Analysing_ANOVA_results.R | File used to conduct the ANOVAs |

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