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Citation

Bijl, J. (2024). *Prof. dr. ir.*

The influence of an external magnetic field on an MRFM cantilever

THESIS

submitted in partial fulfillment of the
requirements for the degree of

BACHELOR OF SCIENCE
in
PHYSICS

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Leiden, The Netherlands, July 12, 2024

The influence of an external magnetic field on an MRFM cantilever

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July 12, 2024

Abstract

The effects of an external magnet on a cantilever and the cooling of a cantilever are presented. There appears to be a linear connection between the current through the external coil and the resonance frequency of the cantilever. The resonance frequency of the cantilever does jump between linear fits at undetermined intervals. The linear fit of the resonance frequency against the current through the coil gives a gradient of (4.874 $\pm$ 0.005) Hz/mA at a height of 20 μm above the sample. The fit at a height of 10 μm gives a gradient of (9.65 $\pm$ 0.04) Hz/mA. The jumps in the resonance frequency at non determined intervals are on average (7.35 $\pm$ 0.04)Hz at a height of 20 μm above the sample and (18.9 $\pm$ 0.4)Hz at a height of 10 μm above the sample. There was no strong connection between the current through the external coil and the quality factor of the cantilever.

The cantilever was cooled, with the help of adiabatic magnetic cooling, from (29 $\pm$ 2) mK to a temperature of (4.1 $\pm$ 0.4) mK. This was done by reducing the current through a coil from 40 A to 2 A.

Chapter 1

Introduction

Throughout history, humanity has always been driven to push the boundaries of what is known. This drive for understanding the unknown is at the core of physics. The largest challenge in modern day physics is the incompatibility between quantum mechanics and general relativity[1]. These two theories are the most successful theories in physics yet. Quantum mechanics is the field of physics that describes the world at the smallest scale. It explains how the smallest objects can simultaneously be a wave and a particle. General relativity describes the motion and behaviour of macroscopic objects, such as the motion of a star of a planet. The difference in scale between both theories presents a major challenge in their unification. Another important reason is that there is not a theory for quantum gravity. Gravity is the only fundamental force that cannot be described by quantum mechanics. A unified theory may provide insights into how gravity works at the quantum level and it may solve long standing problems such as the singularities in black holes. So far there is not a widely accepted theory of everything.

The historical evolution of our understanding of these grand theories starts with the foundations of modern physics in the 17th century. This was the development of classical mechanics by Isaac Newton. In the early 20th century a new theory began to form. The theory of quantum mechanics formed, which deepened our understanding of phenomenon as the wave-particle duality and quantum energy levels. In 1915 Albert Einstein introduced the theory of general relativity. General relativity successfully explained space time and the bending of light around a star. A few ideas exist about the theory of everything, but nothing has been proven jet. The Oosterkamp group investigates these theories at their limits. One way of researching this is to enlarge the scale of a quantum experiment to the point where quantum rules don't apply anymore. This thesis will focus on exactly that. The ultimate goal of MRFM in the Oosterkamp group is to create a massive spatial superposition. This is attempted by coupling the cantilever to a single particle spin. To enable us to do so, it is imperative that we improve the precision of our measurements every time.

1.1 MRFM

MRFM means magnetic resonance force microscopy. This technique is used to obtain our results in this thesis. This idea was first proposed in 1991[2]. MRFM combines the concepts of magnetic resonance imaging(MRI) and atomic force microscopy(AFM). MRI uses strong magnetic fields and radio waves to influence the protons in hydrogen atoms within the body. AFM moves a probe across a samples surface and measures the forces on the probe. MRFM uses a ferromagnet attached to a cantilever. When this magnet is close to the sample, the spins in the sample can generate a small force on the cantilever changing its resonance frequency. The resonance frequency of the cantilever changes if the spins flip. When a spin is aligned with the B0 magnetic field, it exerts a force on the cantilever, causing the cantilever to experience a slight displacement towards the center. When a spin is anti aligned with the B0 field it exerts a force on the cantilever pushing it away from the center. MRFM is a form of scanning probe microscopy. In this case, however, it is not used for microscopy, but to study the boundaries of quantum mechanics. The technique is usually used to study the dynamic magnetic properties of samples or structures. The method is so sensitive that it can be used to detect a single electron spin[3].

1.2 The endgoal of NV-MRFM

The reason for continuously improving the MRFM experiments is to complete one specific measurement. It is an experiment that is on the boundaries of classical and quantum mechanics. The measurement that we are working towards is to couple the cantilever to one electron spin system in the sample[4]. This has already been done before, but the lower temperature will allow us to complete a very specific experiment[5]. This experiment of J. Pereira Machado(2019) is to send three $\frac{\pi}{2}$ pulses with an RF-wire to the spin system in the diamond sample. After sending these three pulses we could then measure the spin state of this electron system. We could then establish if the electron obeyed the laws of quantum mechanics or the laws of classical mechanics. The difference is in the final state of the electron system. The probability distribution for the state of the electron after these three pulses is different for both mechanics.

In order to conduct this experiment two major factors have to be improved significantly[6]. The temperature has to be approximately 10^{-4} k and the quality factor has to be between $10^5 - 10^6$. This thesis will be about the effects of an external magnet on the cantilever and about cooling the cantilever to low temperatures. It is thought that an external magnet will polarise the spins in the sample and by doing so it may increase the Q-factor. This experiment tries to reach low temperatures with the help of adiabatic magnetic cooling. These two improvements bring us closer to the final experiment mentioned by J. Pereira Machado[5] and thus to the boundary of the quantum and classical world.

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Chapter 2

Theory

This chapter will discuss the theory needed to understand the experiment of this thesis. In this section the B_0 field is the magnetic field off the magnet on the cantilever.

2.1 Fundamentals of MRFM

In this section the fundamentals of MRFM are discussed. MRFM is a technique which uses a small magnet on a mechanical resonator to measure the magnetic moment in a sample[7]. Due to the shape its displacement from the center in the x-axis is used in the equations to complete all the calculations. The displacement in the y-axis is negligible due to its shape. The data for this thesis is obtained trough the resonance frequency of the cantilever. The resonance frequency of a harmonic oscillator is given by

$$f_{res} = \frac{1}{2\pi}\sqrt{\frac{k_0}{m_{eff}}}.$$

k_0 is the spring constant and m_{eff} is the effective mass of the harmonic oscillator. In this case that is the mass of the crystal cantilever and the magnet.

The magnet on the mechanical resonator induces a magnetisation in the sample due to the magnetic moment of the spins. These magnetic moments exert a force on the magnet of the cantilever. This can change the resonance frequency of the cantilever slightly[8]. This method is so sensitive it can be used to detect the spin of a single electron[9]. The spin

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systems in the sample exert a force on the cantilever due to their magnetic moment.

$$\vec{F} = \vec{\nabla}(\vec{\mu} \cdot \vec{B_0})$$

This force has an effect on the spring constant of the cantilever. This coupling with spins can be associated with a stiffness k_s. If the spins are aligned with the B_0 field then the force pulls the cantilever to the center. If the spins are misaligned with the B_0 field, then it is the opposite. This force results in a different spring constant.

$$k_s = \mu \cdot \frac{\partial^2 B_0}{\partial x^2}$$

This new spring constant changes the resonance frequency of the cantilever[4]. The change in frequency is given by

$$\Delta f = \frac{1}{2} \frac{k_s}{k_0} f_0$$

2.1.1 Spin dynamics

Spin is a property of angular momentum for particles or systems. Spins obey the laws of quantization. Spin quantum numbers can only be half-integer or integer values. Fermions have half-integer values for spin. Bosons have integer values for spin. The direction of spin can be changed. The magnitude of spin however can not be changed for particles. The magnetic moment of individual spins in the sample can be aligned with the B field(spin up) of anti-aligned(spin down) with the B field. The magnetic moment of an individual spin is

$$\mu_i = S_i \hbar \gamma.$$

γ is the gyromagnetic ratio and $\hbar$ is the reduced plack constant. The gyromagnetic ratio relates the magnetic moment of a particle or system to its angular momentum. S is the spin quantum number. S is +1/2 for spin up and -1/2 for spin down in the most common N-centers in diamond. S is ± 1 for the NV-centers in diamond[10]. The two spin states have different energies. The energy gap between these two states is given by the Zeeman equation. The energy gap scales with magnetic field.

$$\Delta E = \hbar \gamma \left| \vec{B_0} \right| .$$

The spin state with $S = -\frac{1}{2}$ is in the higher energy state. This spin is anti aligned with the magnetic field. The spin with $S = +\frac{1}{2}$ is aligned with the magnetic field and in a lower energy state.

2.1.2 Spins in diamond

The perfect lattice structure of a diamond has a theoretical net spin of zero, but due to impurities such as vacancies, defects or dislocations a diamond sample can have a net spin[11]. Loubser and Wyk provided a review about the electron spin resonance in the study of diamond[10]. It mentions many defects and impurities in diamonds important to this experiment. The impurity which is important for this experiment is the nitrogen vacancy impurity.

Neutral nitrogen atoms have an extra electron, with respect to the carbon atom they replace. A vacancy in the carbon roster with a nitrogen atom next to it create a spin system. This spin system consists of the extra electron in the nitrogen atom and a extra atom that is in the vacancy. This is different from a free electron spin. According to literature the P1 and P2 centers will contribute the most to the total tip sample interaction. A P1 center is a single nitrogen atom in the diamond structure that shares its electron with a adjacent carbon atom. This creates a simple 1/2 electron spin system. A P2 center is a vacancy surrounded by three nitrogen atoms. This creates a free electron. This electron is usually between the vacancy and the adjacent carbon atom. This system can be treated as a free electron spin. This also creates a 1/2 spin system.

2.1.3 Energy coupling

The oscillating magnet on the cantilever induces a changing flux on the pick-up coil. This induces a current in a superconducting circuit. This circuit consists of a pick-up coil, a input coil to the SQUID and a calibration coil. The coupling is a measure of how much of the energy in the cantilever is transferred to the energy in the electrical system.

$$\beta^2 = \frac{E_{system}}{E_{kin}}$$

β is the energy coupling, E_{system} is the energy in the electrical system and E_{kin} is the energy of the cantilever. There is a derivation in the thesis of D. Uitenbroek (2021) of the energy coupling expressed in terms of the changing flux in a pickup loop.

$$\beta^2 = (\frac{d\Phi}{dx})^2 \frac{1}{L_{tot} m \omega^2}$$

Φ is the flux of the pickup loop that is induced due to the cantilever. This can be seen in figure 2.1. $L_{tot} = L_{calibration} + L_{det} + L_{in} + L_{SQUID}$ this is the total impedance of the electrical system. m is the mass of the cantilever with the magnet attached. ω is the resonance frequency.

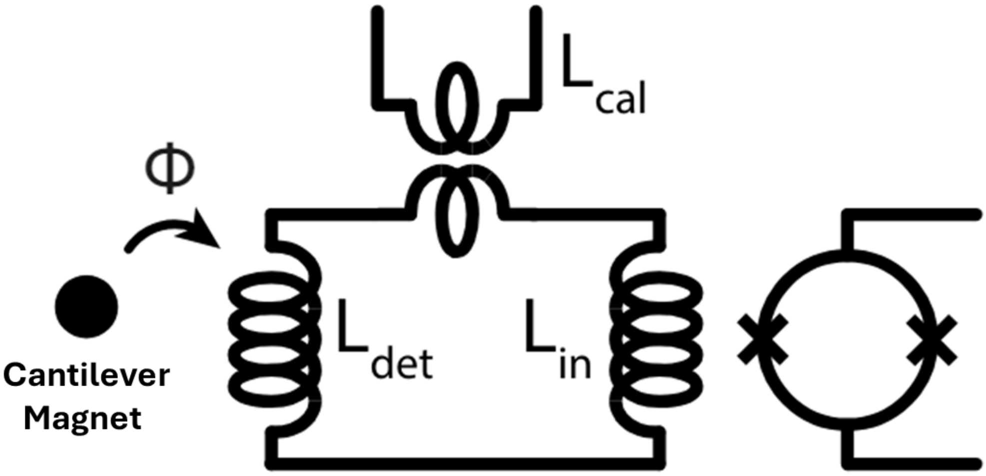

Figure 2.1: *The electric circuit is shown that is used for the readout of the cantilevers magnet. The changing flux of the magnet induces a current in the pickup coil. In this figure that is* L_{det}*. This current produces a changing flux in the input coil, which has an effect on the SQUID. Figure adapted from the thesis of D. Uitenbroek.*

2.2 Magnetic cantilever interactions

This section will be about the theory needed to understand the interactions between the cantilever and the external magnet. It will also mention the Meissner effect. This can be important because the pickup loop and RF-wire are both superconducting. So this can have an effect on the cantilever.

2.2.1 Meissner effect

The Meissner effect was first discovered by W. Meissner and R. Ochsenfeld in 1933[12]. They discovered this phenomenon by measuring the magnetic field outside lead and tin. These materials were cooled below their superconducting-transition temperature. When a magnetic field was applied to the superconducting material, it expelled the magnetic field from inside the material.

This phenomenon is based on the London equation. This states that the electromagnetic free energy in a superconductor is minimized given the equation below. This equation states that the magnetic field inside a superconductor must decay exponentially from the surface inwards to the material.

$$\nabla H = \lambda^{-2} H.$$

In this equation H is the magnetic field and λ is the London penetration depth. When a weak magnetic field is applied to a superconductor in superconducting state the superconductor expels the magnetic flux by inducing a current in the London penetration depth below the surface. These currents cancel out the magnetic field lines inside the material. The field cancellation is not dependent on time, so the current that cancels the magnetic field in the London penetration depth does not decay. When the magnetic field that is applied to the material is too strong the Meissner effect breaks down. So the material goes back to normal conductivity. The magnetic field then prevents a transition to the superconducting state of the material.

The reason that the Meissner effect can play a part in this thesis is the superconducting wires on the sample. These superconducting wires are during our experiments in superconducting state. So these wires will cancel the magnetic field inside the wires. This changes the magnetic field slightly. The magnetic field has an direct effect on the resonance frequency of the cantilever. So the Meissner effect of the superconducting wires can change the resonance frequency. This effect becomes stronger when the cantilever is close to the superconducting wires. This is the reason that positing the cantilever is so important.

2.3 Cooling of the cantilever

This section will be about the theory needed to understand the experiment of cooling the cantilever to low temperatures with the help of adiabatic magnetic cooling.

2.3.1 Equipartition theorem

The equipartition theorem connects the temperature (T) to the average energy of a system with N degrees of freedom. This can be seen in the following equation

$$E = \frac{1}{2} N k_b T.$$

In this situation it can be used to determine the temperature of the cantilever at extremely low temperatures. The cantilever oscillates, because of its shape mostly in one direction. The motion in the other directions can be neglected[13]. The cantilever can be approximated as an harmonic oscillator with one degree of freedom. The position of the cantilever is given by $x(t) = Acos(\omega t + \phi)$ and the velocity is $v(t) = -A\omega sin(\omega t + \phi)$. A is the maximum displacement of the cantilever. The total energy of the harmonic oscillator oscillates between the kinetic and potential energy. $E_{total} = U + K = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$. Using these equations and $\omega = \sqrt{\frac{k}{m}}$ the total energy of the system can be found:

$$\begin{aligned} E_{total} &= \frac{1}{2}kA^2cos^2(\omega t + \phi) + \frac{1}{2}mA^2\omega^2 sin^2(\omega t + \phi) \\ &= \frac{1}{2}kA^2(cos^2(\omega t + \phi) + sin^2(\omega t + \phi)) = \frac{1}{2}kA^2 \end{aligned}$$

The energy of the cantilever is given by both the equipartition theorem and the energy of a harmonic oscillator. These two formulas can be equated, yielding the following equation.

$$\frac{1}{2}kA^2 = \frac{1}{2}k_b T$$

This equation can be rewritten in terms of the maximum displacement due to thermal motion (A). This yields:

$$A = \sqrt{\frac{k_b T}{k}}$$

2.3.2 Adiabatic magnetic cooling

This method was first reported in 1881 by E. Warburg. The discovery can be attributed to P. Weiss and A. Piccard in 1917[14]. Adiabatic magnetic cooling was the first cooling method, which was able to cool below 0.3 K[15]. Adiabatic magnetic cooling uses the properties of magnetic fields to cool to very low temperatures. When a magnetic field is applied in a metal the entropy decreases, because the spins in the metal are more polarised. So there are fewer total options for the spins to be aligned. Due to the decrease in entropy the metal will heat up. To achieve cooling below its original temperature the material must radiate its heat away while in the magnetised state. When the magnetic field is removed, the entropy increases so the metal will cool below its original temperature.

Chapter 3

Methods

In this chapter the setup and methods used to obtain the data is explained. The NV MRFM setup used in this thesis uses a magnet on tip setup. The magnet is attached to the cantilever. An overview of the setup can be seen in figure 3.1 and the schematic overview can be seen in figure 3.2. The setup is placed inside a dry dilution refrigerator. The setup is attached to the mixing chamber plate. The setup uses various methods for vibration isolation. The data used in this thesis was obtained with two different setups. These setups are cooled in different cryostats. Both are similar setups. The difference is that the setup in the ″Yeti″ cryostat utilises adiabatic magnetic cooling with the help of a magnet to cool the cantilever to even lower temperatures. The setup in the ″Marshmallow″ has a working external magnet underneath the sample. Unless mentioned otherwise the ″Marshmallow″ cryostat is the cryostat that is written about.

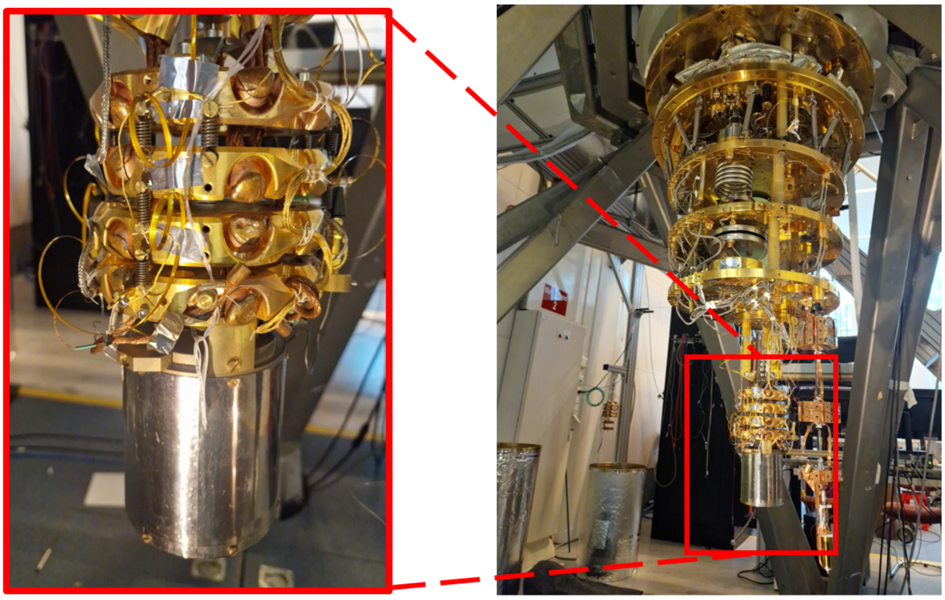

Figure 3.1: *On the left a close up of the experiment can be seen. On the right the total open dry dilution refrigerator without shielding can be seen. This is the "Marshmallow" refrigerator.*

3.1 MRFM setup

A schematic overview of the setup can be seen in figure 3.2. There is an external magnet underneath the diamond sample. This magnet is a flux concentrator coil. On the sample is a pickup loop which is used to measure the changing flux through it. On the sample there is also an RF-wire attached which can be used to send microwaves. Above the sample is the cantilever with a magnet attached at the end. This is the probe in the setup. The cantilever is attached to a piezo. The magnet generating the B_0 field is attached to the end of the cantilever. This magnet has a diameter of 3.6 μm. Its remnant magnetisation is 1.4 T and it is polarised in the z-direction. The spring constant of the cantilever is 30 μN/m. This makes the cantilever sensitive to small forces. A piezo is attached to the base of the cantilever. When a voltage is set over the piezo it can drive the cantilever at resonance frequency. When the cantilever oscillates it induces a current in the pickup loop due to the changing magnetic flux. This current is detected with a SQUID that is read out by a Zurich Instruments lockin amplifier. The external magnet is a niobium coil with a diameter of 11.9 mm. A detection chip is placed on the external magnet. This chip consists of the pickup loop. The feature in the setup which is used for spin manip-

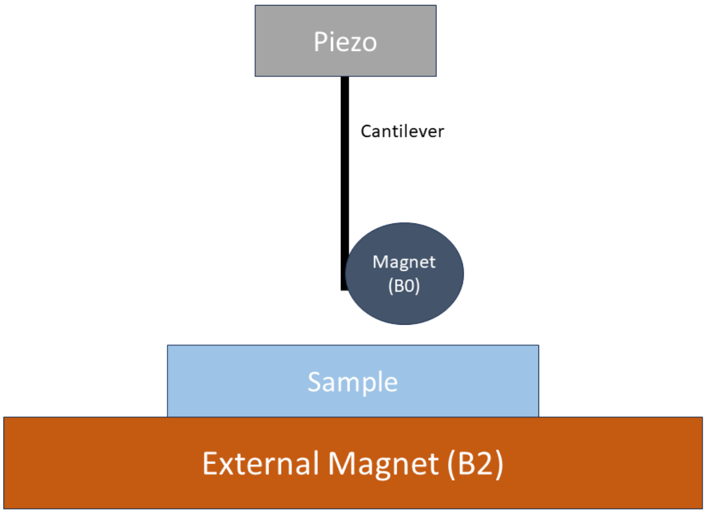

Figure 3.2: *A schematic overview of the setup*

ulation is a RF-wire over the diamond. This wire is able to transmit about 10 mA up to a frequency of 5 GHz. The effects of this microwave line are beyond the scope of this thesis. It is however important to mention the existence of this microwave wire, because when the cantilever is close to the microwave line it can behave strangely. The wire is superconducting, so it tries to counteract the magnetic field lines of the cantilever. The pickup loop is also superconducting, so this effect is the same for the pickup loop and the microwave line. So it is important to choose the location of the measurements correctly. Otherwise the superconducting properties of the wires can have a significant effect on the measurements.

3.1.1 Dry dilution refrigerator

To cool our experiment to extremely low temperatures a dry dilution refrigerator is used. In 1962 H. London published a paper about a refrigerator that could reach temperatures below 1 K[16]. The first experimentally realised refrigerator was realised in 1964[17]. It was build at the Kamerlingh Onnes Laboratorium in Leiden. Nowadays the lowest temperature that can be reached with a conventional dillution refrigerator is 1.75 mK[18].

The dry dilution refrigerator used in the setup is able to cool to approximately 20 milikelvin. It is a vacuum and uses a pulse tube cooler to cool the 50k and the 4k plate. To reach even lower temperatures it mixes He3 and He4 in the mixing chamber. The resulting mixed gas has a higher total entropy then the total entropy of the two individual gasses. By mixing the gasses in the mixing chamber energy is absorbed by the mixture.

3.1.2 Positioning of the cantilever

The coordinate system of the cantilever is not the same for every run. When cooling a cryostat to such low temperatures objects may decrease in size slightly. This causes the cantilever to not be in the exact same position for each run. However, it is important to exactly know the position of the cantilever with respect to the pickup loop and the diamond sample. This to ensure that the measurements are done at the optimal position.

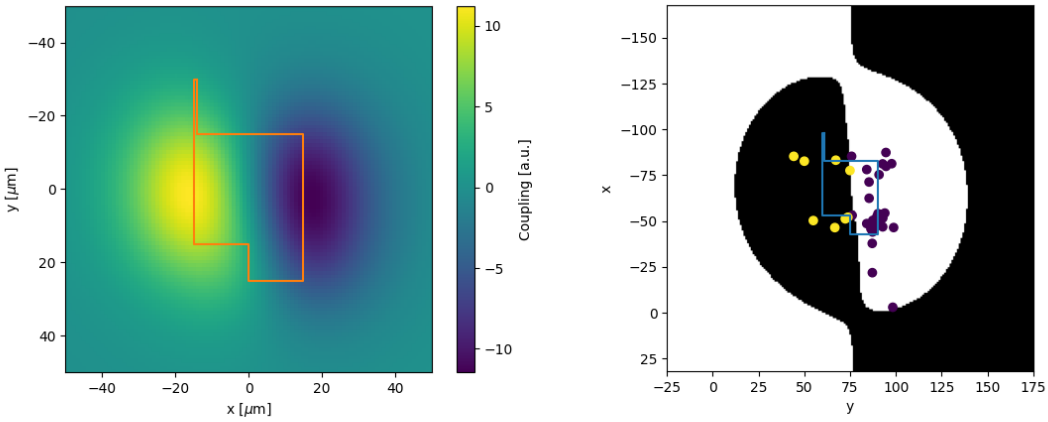

Figure 3.3: *A figure of the coupling at a height above the sample of 30 μm. On the left you see the value of the coupling. On the right you see the sign of the coupling. A positive sign of the coupling is the white area and the black area is a negative sign for the coupling of the cantilever. On the right you also see all the sweeps that were done to position the cantilever. A yellow dot is a positive sign and a purple dot is a negative sign.*

So for every cool down the position of the cantilever has to be established again. The coupling between the pickup loop and the cantilever is crucial in this process. The coupling is defined as the magnetic flux change in the pickup loop due to the movement of the cantilever. The coupling is influenced by the height of the cantilever and the position relative to the

X and Y position of the sample. So before the actual measurements can be taken, a X and Y sweep has to be done in order to estimate the position. In figure 3.3 on the left the coupling of the cantilever can be seen for different positions. A sign switch takes place as you can see in figure 3.3 on the right. The coupling is plotted in the left plot. When frequency sweeps were done the sign of the coupling could be established. The opening of the circle in the polar plot determines the sign of the coupling. In the right plot you see the measurements that were done to position the cantilever. You can clearly see a sign change, so that is crucial to determine where the sample is exactly.

3.1.3 Flux concentrator coil

The external magnet is a flux concentrator coil. This coil uses the Meissner effect to transform a current in a wire to a current in the coil. The slit forces the Meissner current to go through the center. This concentrates the magnetic field in the center of the coil. In figure 3.4 a sketch of a flux concentrator coil can be seen and the direction of the current. The MRFM experiment happens on a very small scale, so big coils are not necessary for this experiment. It is therefore beneficial to concentrate the magnetic field in the core. Therefore less current has to be used to create the magnetic field that is needed just above the flux concentrator coil. A normal coil has a magnetic field that is more spread out. It therefore needs more current to create the same magnetic field as a flux concentrator coil just above the center.

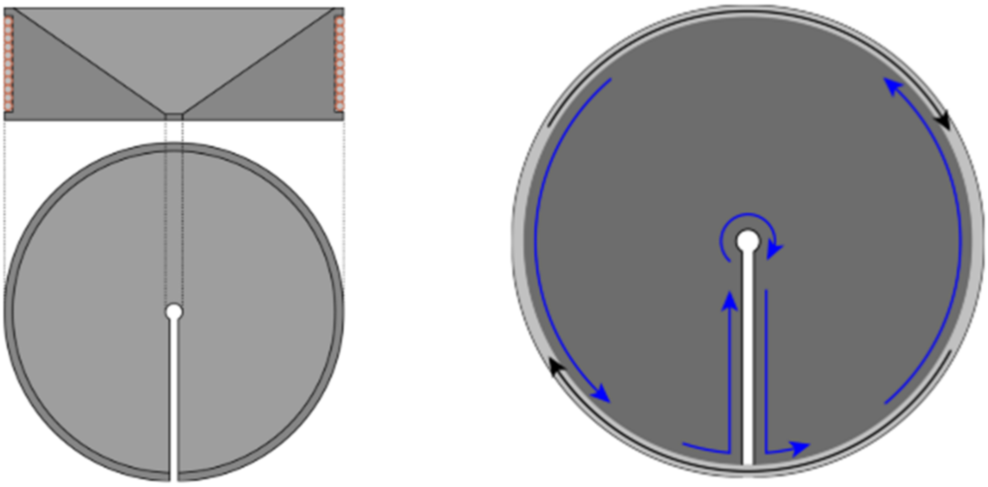

Figure 3.4: *On the left a sketch of a flux concentrator can be seen. On the right the direction of the current through the coil can be seen.*

3.2 The effects of an external magnet

Under the sample an external magnet is placed. This is a flux concentrator coil. This coil can directly and indirectly influence the magnet on the cantilever. The coil can directly influence the cantilever magnet trough the interaction between its magnetic field and the polarisation of the magnet. The coil indirectly influences the magnet by polarising the spins in the sample. These spins then have an effect on the cantilever. The idea is that this external magnet polarises the spins in the sample and thus increases the Q-factor in of the measurements. This experiment researches the effects that an external magnetic field has on the cantilever. The external magnet is positioned under the sample. This magnet has a direct effect on the cantilever. This effect is due to the magnetisation of the cantilever in the magnetic field of the external magnet. This force is given by the following formula

$$\vec{F} = \nabla(\vec{\mu}_{canti} \cdot \vec{B}_{ext}).$$

The $\vec{\mu}_{canti}$ is the magnetisation of the cantilever. $\vec{B}_{ext}$ is the magnetic field of the external magnet. The external magnet also has an indirect effect on the cantilever. The external magnet has an effect on the spins in the diamond sample. The magnetic field can polarise the spins in the sample. This creates a magnetic field, which influences the cantilever. This effect is characterised with the ”de Voogd” approach[13]. This approach describes the interaction between the magnetisation of the cantilever with the magnetic field of the external magnet with more terms. It also takes the spins in the sample into account.

3.2.1 Measuring procedure

The goal of this experiment is to determine the effects of an external magnet has on the cantilever. This is achieved by doing a frequency sweep around the resonance frequency of the cantilever at a constant temperature, with different currents through the external magnet and at different locations.

The voltage set on the coil by the Zurich Instruments was between -4 V and 4 V with 0.5 V increments. The setup was in series with a total resistance of 1024 Ohm. So this voltage resulted in a current through the coil between -3.9 mA and 3.9 mA. These measurements were done at two different positions. These positions were 10 μm and 20 μm above the sample. On the height of 20 μm the current through the coil was swept from 3.9 mA to -3.9 mA and again back to 3.9 mA. On a height of 10 μm the current through the coil was swept from -2.9 mA to 3.9 mA.

To perform the measurements a frequency sweep around the resonance frequency is taken at different currents through the coil. In such a sweep the piezo drives the cantilever at a given frequency for 2.3 seconds per point for a total of 101 frequency points with a 8 mHz spacing between each point. The sweep will sweep back and forth for a total of 202 data points. This is done to decreases non-linear effects. The temperature of the sample is kept constant at 85 mK during the measurement. This is done with the help of a heater and thermometer on the sample. A PID feedback system is used to control the heater. This to ensure that the difference in frequency shift and quality factor is not due to a difference in temperature, because the temperature of the sample can change the resonance frequency.

The location of the cantilever also has a effect on the resonance frequency and quality factor of the cantilever. To understand these effects the measurements were conducted at two different locations. The pickup loop and magnet both influence the cantilever so both should be taken into consideration when choosing the locations. The pickup loop is superconducting, so when the magnet is turned on the pickup loop will counteract this field. This will change the resonance frequency significantly when the cantilever is close to the superconducting wires, so for now the location is chosen at a distance were this counteracting field does not have a significant effect anymore.

3.2.2 Analysis

The frequency sweep will result in a magnitude, phase and polar plot of the cantilever. The magnitude plot will be a peak at the resonance frequency containing 202 data points. Fits on these plots will result in the resonance frequency and quality factor. The data will then be fitted with a linear fit.

3.3 Cooling the cantilever to extremely low temperatures

To complete the experiment described by J. Pereira Machado[5] a temperature of approximately $10^{-4}k$ is needed. So not only a better quality factor is needed, but a submilikelvin temperature as well. This section will explain the methods used to cool the cantilever to temperatures of a few milikelvin. The cooling of the cantilever was done in the "Yeti" cryostat. This cryostat has an extra cooling method to further cool the cantilever. This cooling method is adiabatic magnetic cooling.

Adiabatic magnetic cooling is used in the Yeti cryostat to further cool the setup to temperatures not achieved with only dry dilution. There is a coil attached to the 4k plate. This coil has a metal core made of $PrNi_5$. When a current flows through the coil, the coil produces a magnetic field. This magnetic field polarises the spins in the metal core, thus lowering the entropy. This causes the metal core to heat up. The core is then cooled with a heat-conductor switch to the mixing chamber plate. When the metal core is cooled down again the current flowing through the coil is turned down. The spins in the metal core are less polarised with the decreasing magnetic field. This causes the entropy to rise and this process requires energy. So the temperature of the coil drops to about 1mK[8].

3.3.1 Measuring procedure

The objective is to measure the energy stored in the cantilever and then calculate its temperature with a calibration. The thermal excitation results

in an oscillation of the cantilever. This oscillation can be measured with the help of the pick up loop and the SQUID.
The energy through the coil is first turned up to 40 A which corresponds to 2 T. Then the coil is cooled again in its magnetised state for as long as possible. This is due to the fact that the heat switch is broken, so the coil slowly radiates its heat away. When the coil has radiated its heat away as much as possible the current through the coil will be turned down. It will take approximately half a week to radiate its heat away. Then the current through the coil will be turned down to 2 A with intermediate steps of 20 A and 5 A.

3.3.2 Analysis

The signal is continually measured to know the temperature over time. The continually measured signal is split into files of ten minutes each. On this signal a Fast Fourier Transform is taken. This will result in a peak around the resonance frequency. On this data in frequency space a digital energy lockin is performed. This will result in the energy of the resonance peak. The background noise contributes to the energy of the cantilever. So a background reduction is done to remove the background from the actual signal.

The energy of the signal is then converted to mode temperature. The equation of the energy coupling and the equation for the equipartition theorem are combined to form the following equation

$$T = \frac{k_s}{k_b} * \frac{E}{\beta^2}.$$

In this equation k_b is the Boltzmann constant. k_s is the spring constant of the cantilever. β^2 is the energy coupling of the cantilever. E is the energy of the cantilever.

Chapter 4

Results

In this chapter the results of the experiment will be presented. The results were obtained using the methods described in the methods chapter. The first section is about the effects of an external magnet on the cantilever. The second section is the cooling of the cantilever to low temperatures.

4.1 The effects of an external magnet

The resonance frequency and quality factor were determined for different currents through the flux concentrator coil. These were determined with the help of a frequency sweep as discussed in the methods chapter. These measurements were done at two different locations. The locations were 10 μm and 20 μm above the sample.

4.1.1 Sequence of measurements

In figure 4.1 the sequence of the frequency sweeps is drawn. The arrows indicate the order in which the measurements were done. Unfortunately due to a lack of measurement time the measurements at a height of 10 μm were done with 102 data points instead of 204 data points. The order in which the measurements were done is important for us to understand were the jumps in resonance frequency take place. The colorbar indicates which measurements were done first. A 0 on the colorbar indicates the first measurement and measurement points with a lighter color were done later. There is not a clear connection between the order of the measurements and the jumps that occur in the resonance frequency.

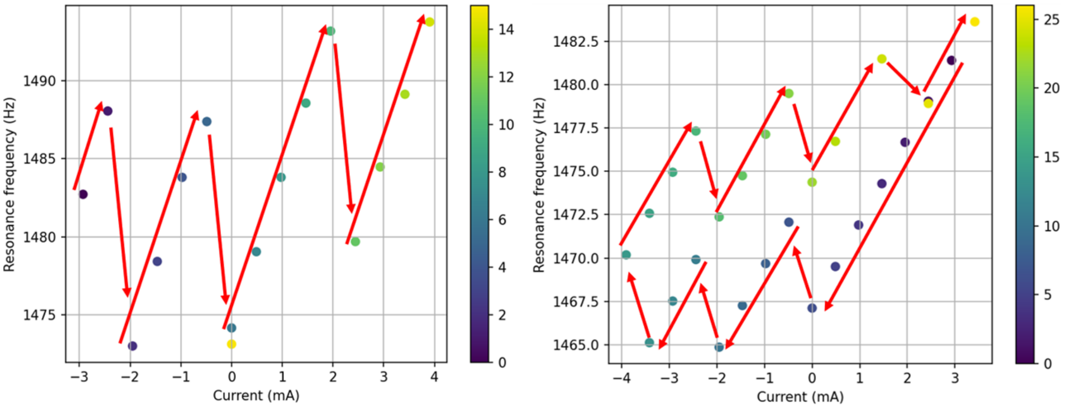

Figure 4.1: *On the left the measurements of the resonance frequency are plotted against the current through the coil at a height of 10 μm above the sample. On the right the measurements of the resonance frequency are plotted against the current through the coil at a height of 20 μm. The color index of the points indicates the sequence in which the points were measured. 0 being the first point measured. The arrows also indicate in what order the measurements were done.*

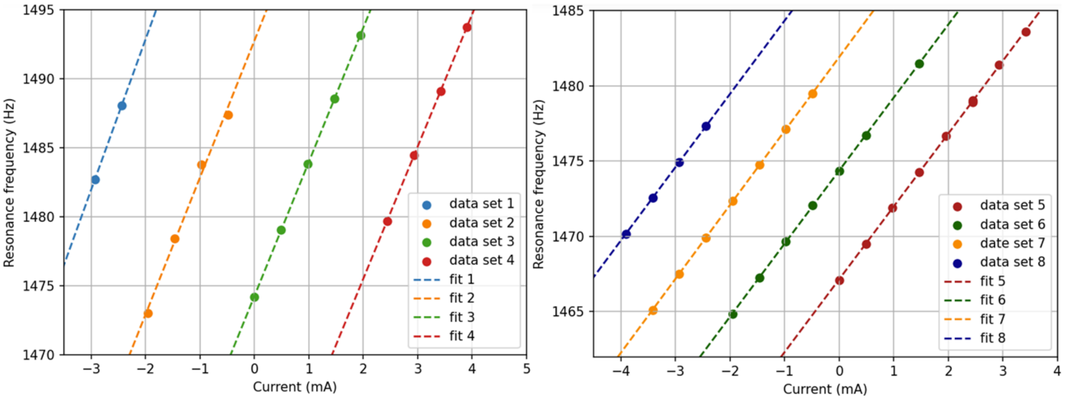

Figure 4.2: *On the left the measurements of the resonance frequency at a height of 10 μm above the sample are fitted. On the right the measurements of the resonance frequency 20 μm above the sample are fitted. In each case four data selections were made.*

4.1.2 Resonance frequency

We expect a linear connection between the current through the coil and resonance frequency. However a translation in resonance frequency of these linear fits can be observed. These are called the "Jumps" in resonance frequency. That's why a selection of the data set was fitted with a linear fit. All the data points that were on a line are grouped together. The formula that was used to fit the data is: $f_{res} = a * I + b$. The resonance frequency is f_{res} and I is the current through the coil. a and b are both parameters that were calculated using the scipy.optimize.curvefit function. All the calculated parameters are listed in table 4.1. No error was calculated for the first fit, because this fit only contains two data points. So the error could not be estimated correctly. That is the reason why there is a x at the place of the error.

Each fitted line has a gradient that is similar with the other fits at the same position of the cantilever. The closer the cantilever gets to the external magnet the steeper the fit becomes. All the data sets and their fits can be seen in figure 4.2. The data of 20 μm above the sample is less accurate. This is the left plot of figure 4.2. This is because fewer data points were used in the measurements.

Fit	a (Hz/mA)	b (Hz)
Fit 1	10.917 ± x	1514.710 ± x
Fit 2	9.9 ± 0.6	1492.8 ± 0.9
Fit 3	9.73 ± 0.06	1474.25 ± 0.07
Fit 4	9.59 ± 0.05	1456.3 ± 0.2
Fit 5	4.84 ± 0.02	1467.16 ± 0.04
Fit 6	4.85 ± 0.01	1474.38 ± 0.02
Fit 7	4.91 ± 0.01	1481.92 ± 0.03
Fit 8	4.871 ± 0.009	1489.22 ± 0.03

Table 4.1: *This table contains the different fitted parameters. The function that was used to fit the data is $f_{res} = a * I + b$. Fit 1 through 4 are for 10 μm above the sample. Fit 5 through 8 are for 20 μm above the sample.*

4.1.3 Jumps in resonance frequency

Quantised resonance frequency jumps can be observed when the external magnetic field changes. This can be observed in figure 4.1. These jumps were calculated based on the fits of the start and endpoint.

$$\Delta f_{res} = f_{res,after}(I_{av}) - f_{res,before}(I_{av}) \quad \text{with} \quad I_{av} = \frac{1}{2}(I_{before} + I_{after})$$

All the jumps are listed in table 4.2. In the measurement at 10 μm above the sample only three jumps took place so that is why jump 4,5 and 6 are empty. The closer the cantilever gets to the sample the greater the jumps become.

Jump	20 μm above the sample	10 μm above the sample
Jump 1	(7.2 ± 0.1) Hz	(18.2 ± 0.7) Hz
Jump 2	(7.4 ± 0.1) Hz	(18.6 ± 0.7) Hz
Jump 3	(7.4 ± 0.1) Hz	(19.8 ± 0.7) Hz
Jump 4	(7.4 ± 0.1) Hz	x
Jump 5	(7.5 ± 0.1) Hz	x
Jump 6	(7.2 ± 0.1) Hz	x

Table 4.2: *This table contains the frequency jumps in resonance frequency of the current sweep. Jump 1 is the first jump that took place during the measurements and 6 is the last. Figure 4.1 gives insight into which jump is which.*

4.1.4 Quality factor

Unfortunately the quality factor between the different heights can not be compared. This is due to the fact that the frequency sweeps at both heights do not have an identical amount of data points. The frequency sweeps at a height of 20 μm above the sample have 202 data points per sweep. The frequency sweeps at a height of 10 μm above the sample have 101 data points per sweep. So the piezo at a height of 20 μm had more time to drive the cantilever at that exact frequency. On a height of 10 μm the piezo had less time at each frequency point, so the cantilever did not have enough time to be fully resonant at each frequency point. So the cantilever was driven to quickly and did not have enough time to fully reach its maximum peak at each frequency. This caused the peak to be lower and more spread out. So the quality factor at a height of 20 μm above the sample is almost 1.5 times as high.

The measurements of the quality factor are plotted against the current through the coil in figure 4.3. The data sets that were grouped for the fits for the resonance frequency are again grouped together. There is no clear

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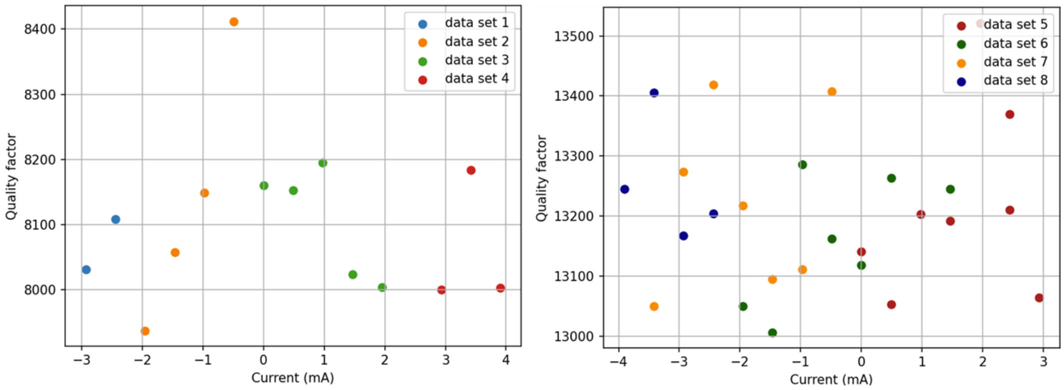

Figure 4.3: *On the left the measurements of the quality factor are plotted against the current through the coil at a height of 10 μm above the sample. On the right the measurements of the quality factor are plotted against the current through the coil at a height of 20 μm. The selection of data sets used for the resonance frequency is used again*

pattern for the quality factor. After frequency jumps the quality factor does not change significantly.

4.2 Cooling of the cantilever

In this section the results of the cooling of the cantilever are discussed. The results were obtained using the methods discussed in section 3.3. The current through the coil with a $PrNi_5$ core was first set to 40 A. After cooling down again with the help of the mixing chamber plate, the current through the coil was reduced from 40 A to 2 A with two intermediate steps of 20 A and 5 A. In figure 4.4 on the left, the current can be seen with the energy of the cantilever plotted. The reduction in current through the coil closely matches the reduction in energy of the cantilever. In the right plot in figure 4.4 the different data points are averaged. The data points that were grouped together are for the different currents through the coil. These data points were within each other's error margin.

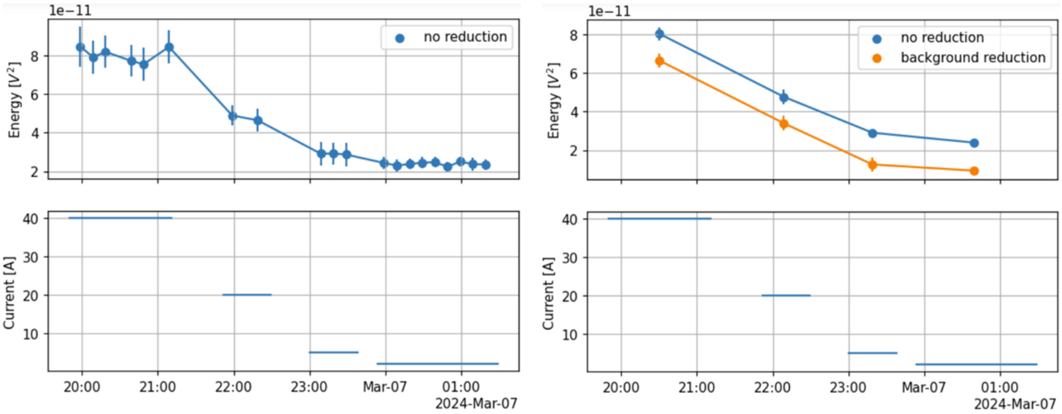

Figure 4.4: *On the left: The top graph are the individual tdms files and the bottom graph is the current through the coil over time. On the right: The top graph are the averaged tdms files for each current and the bottom graph is the current through the coil.*

There is always background noise that is also measured, but this background noise does not contribute to the temperature of the cantilever. So to calculate the temperature more accurately the background noise has to be removed from the signal. It is important for this background reduction that you only reduce the signal with the background and not some mechanical or electrical peak. This will reduce the signal to much and lower the mode temperature to a point below its actual value. Two intervals were chosen for the background reduction the intervals were -5.5 Hz to -4.5 Hz and 4.5 Hz to 5.5 Hz. there were no peaks in these intervals. There was also a peak close to the signal so to ensure only the signal of the resonance frequency was used the integration over the signal was reduced to -0.5 Hz and 0.5 Hz around the resonance frequency.

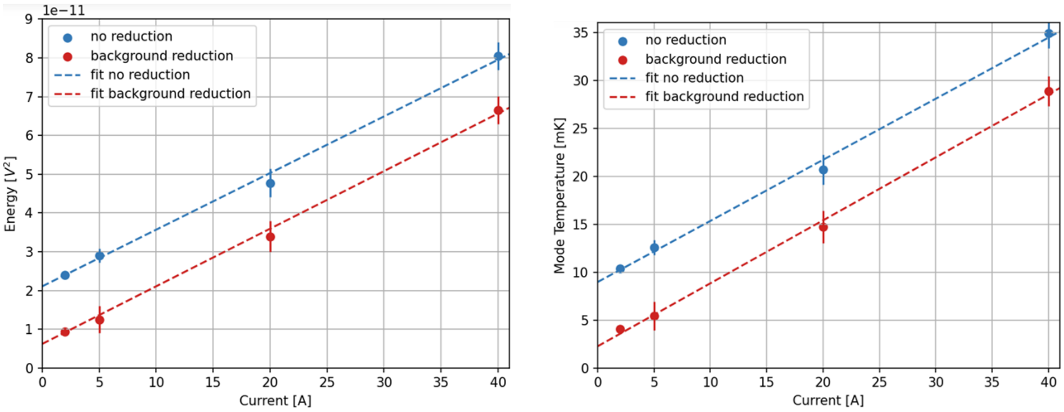

Figure 4.5: *On the left the energy[V^2] of the cantilever can be seen. On the right the energy is converted to the temperature of the cantilever.*

When the energy of the cantilever is measured it has to be converted from V^2 to temperature. Unfortunately during the measurements there was not a thermometer present close to the cantilever to measure its temperature. There were however past runs with a thermometer present. So with these runs our analysis method could be checked for its validity.

The expectation is that there is a linear correlation between the current through the coil and the temperature. This can be seen in figure 4.5. The data has been fitted with a linear function. That function is: $T = a * I + b$. In this function I is the current through the coil and T is the mode temperature. a and b are parameters which were fitted. The calculated parameters are listed in the table below.

Fit	a	b
Energy no BR	(0.146 ± 0.005)$*10^{-11}V^2/A$	(2.11±0.05)$*10^{-11}V^2$
Energy BR	(0.149 ± 0.004) $*10^{-11}V^2/A$	(0.62 ± 0.05)$*10^{-11}V^2$
Temperature no BR	(0.64±0.03) T/A	(9.0±0.6) T
Temperature BR	(0.66±0.02) T/A	(2.3±0.5) T

Table 4.3: *These are the different parameters of the fits that can be seen in figure 4.5. BR means background reduction. The fitted parameters for the temperature are for the plot on the right in figure 4.5. The fitted parameters for energy are for the plot on the right in figure 4.5*

Chapter 5

Discussion

In this chapter the data from the previous chapter is interpreted and discussed. Additionally, improvements and suggestions for future research are presented. First the measurements of the cantilever with an external magnet are discussed, then the measurements where the cantilever is cooled

5.1 The effects of an external magnet

This section is split into three sections. Section 5.1.1 will discuss the effects the external magnet had on the resonance frequency. Section 5.1.2 will be about the jumps in the resonance frequency. Section 5.1.3 will discuss the effect the external magnet had on the quality factor.

5.1.1 The resonance frequency

To measure the effect that an external magnet has on a cantilever the current through the coil was varied at two locations. Theory and simulations suggested a linear connection between the current through the coil and the resonance frequency of the cantilever. This connection can also be seen in the data that is obtained. However an additional translation in resonance frequency of these linear fits can be observed. The discussion of those jumps in resonance frequency is in section 5.1.2. The data that was fitted can be seen in figure 4.2.

The gradient of the fits that were done at a height of 10 μm are at least one order of magnitude close to each other. The fits that were done at a height of 20 μm were even closer to each other. This smaller error is due to the fact that those data points were taken with a smaller spacing between the frequency points. This resulted in a smaller error for each data point in figure 4.2. There were also a greater number of data points at a height of 20 μm, which resulted in the fit being more accurate.

Grouping the data points for different linear fits seems to be the best way to analyse the data. The fits are accurate and provide small errors for the different fits. The data supports the theory of a linear fit. The height above the sample has a direct effect on the slope of the fit. When the sample was approached from 20 μm to 10 μm the slope of the fit got steeper with a factor of 2.06. This is in line with theory. The closer the cantilever is to the external magnet the greater the gradient of the external magnetic field is. This means that a larger frequency shift is expected of the resonance frequency.

Further research could focus on data for more locations. Due to a lack of measurement time the only factor that was changed in the location was the height. Further research can focus on the different locations with respect to the pickup loop. The pickup loop also has an effect on the cantilever and with an external magnet these effects change. So to see what different x and y positions do to the resonance frequency with a current through the external magnet can be an interesting research topic to fully understand the effect an external magnet has on the cantilever.

For our experiments it is not known if the resonance frequency was changed due to the direct effect of the external magnet or the indirect effect. The direct effect is the effect that the external magnet has on the cantilever. The indirect effect is the effect of the external magnet polarising the spins in the sample. These spins can change the resonance frequency of the cantilever. So for future measurements it can be beneficial to measure the resonance frequency with a known spin density at different heights with different temperatures and different currents through the external magnet. These measurements can give an insight into the effect of the spins and the effect of the magnet on the cantilever. This is due to the fact that the effect of the spins is related to the temperature of the sample. So by changing the temperature the effect of the spins will change, but the effect of the external magnet will not.

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5.1.2 Jumps in resonance frequency

For the different heights of the measurements there were jumps in the data of the resonance frequency. That is why multiple fits had to be used to fit the data. The jumps in the resonance frequency at non determined intervals are on average (7.35$\pm$0.04) Hz at a height of 20 μm above the sample and (18.9$\pm$0.4) Hz at a height of 10 μm above the sample. The value of the jumps also depends on the height above the sample. When the sample was approached from 20 μm to 10 μm the jumps increased with a factor of 2.57. The jumps are on different moments and times in the current sweep, so that is not the critical factor. When sweeping the current back the jumps also occurred at different moments and times in the current sweep.

The resonance frequency at a height of 20 μm was 1467.11 Hz with a current through the coil of 0 mA. When sweeping back the resonance frequency was 1474.37 Hz at a current through the coil of 0 mA. Three jumps happened when sweeping forth, but when sweeping back only two jumps happened. This resulted in a changed resonance frequency of 7.26 Hz.
The direction of the current sweep does have an effect on the jumps. When sweeping the current through the coil up, the jumps only occur to a lower resonance frequency. When sweeping the current through the coil down, the jumps only occur to a higher resonance frequency. So we know that the direction of sweeping the current and the height above the sample are critical factors in the jumps.

At this moment this phenomenon is still unexplained, but it may be due to flux trapping. The pickup loop is a superconducting circuit which tries to expel the magnetic field according to the Meissner effect, but if the thickness of the material is very thin the magnetic field lines may be able to penetrate. They will only penetrate the superconductor in flux vortices. At different magnetic field strengths the superconductor will allow magnetic flux to penetrate in quantised packets. These penetration places are known as flux tubes. This theory does fit the data of the quantised resonance frequency jumps at different magnetic field strength.

At this time it is not exactly known where or when these resonance frequency jumps happen. The current can be increased and decreased with a much smaller margin to research where these jumps exactly occur. This can be an improvement to the current research. More locations can also be helpful to fully understand this phenomenon. The effect of the x and y location on the value of the jumps can be determined in further research.

This can give insight into what the location does to this unexplained phenomenon.

5.1.3 Quality factor

The quality factor is plotted against the current through the coil in figure 4.3. There was no fit possible which accurately fitted the data. The data does not show an increase in Quality factor for an increase in current through the coil. This is not unexpected. At a height of 10 μm and 20 μm the spins in the sample have a relative small effect on the cantilever. The jumps in resonance frequency do not appear to have an effect on the quality factor of the cantilever.

More research is necessary to know what effect the external magnet has an the quality factor. The cantilever can be brought closer to the sample. This would increase the effect the spins in the sample have on the cantilever. So when the spins are then polarised the effect would be larger. So in that instance there could be a significant change in quality factor.

5.2 Cooling of the cantilever

This section is split into two parts. The first will discuss the way in which the background reduction was done and the second will be about the final temperature that was reached.

5.2.1 Background reduction

The background reduction was done with data from 0.5 Hz around ±5 Hz. In these two intervals no visible peaks were visible on any of the TDMS files. This is important to ensure that the final temperature is indeed the final temperature and not lower due to our analysis. If any peaks were visible in the intervals with which the data reduction was done, then the final calculated temperature would have been lower than its actual value.

The interval of integration of the resonance peak was also reduced to ±0.5 Hz. This was because on some TDMS files there was a small peak at -0.8 Hz. So to ensure that this peak did not contribute to the final temperature the interval was reduced. The resonance peak was steep enough to choose an interval of ± 0.5 Hz.

5.2.2 Temperature

The energy in the cantilever was measured at four different current values through the coil. The current was each time turned down. The final temperature achieved according to the data is (4.1 ± 0.4) mK. This temperature was averagde from all the data points with a current through the coil of 2 A. The coil was cooled to a temperature of (29 ± 2) mK in its magnetised state. Unfortunately the heat switch in the setup was broken during these measurements. So the magnetised coil was not able to radiate away its heat very quickly. That is the reason why the magnetised coil was cooled for an entire weekend. This was done to ensure that the magnetised coil was able to radiate its heat away.

The temperature should have a linear connection with the current through the coil. The linear fit that was done closely matches the data obtained during the measurements. So it is believed that the temperature of (4.1 ± 0.4) mK was indeed reached. 5 days before this measurement this experiment was also done and at that time the data showed that a temperature of approximately 5 mK was reached. We consider these values to be similar

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Chapter 6

Conclusion

Two current sweeps through a flux concentrator coil were done at two locations of the cantilever. The resonance frequency of the cantilever was measured for different currents through the coil. The fits that were done on the data show that there is a linear connection between the current through the external coil and the resonance frequency of the cantilever. The closer the cantilever is to the sample the higher the gradient is. However, there are jumps in resonance frequency when the current is turned up or down. The reason behind these jumps is still unknown. The height of the jumps is also depended on the distance to the sample and magnet. The closer the cantilever is to the sample the higher the jumps are. Unfortunately no pattern could be observed in the jumps. At a height of 20 μm and 10 μm there was no connection observed in the data of the quality factor and the current through the external coil.

The cantilever reached a mode temperature of (4.1 ± 0.4) mK with the help of adiabatic magnetic cooling. This temperature is the average over a period of 90 minutes. This happened when the current through the coil with a PrNi5 core was turned down from 40 A to 2 A. Theory states that there is a linear connection between the current through the coil and the temperature of the cantilever. The fits prove that this indeed is the case.

6.1 Acknowledgements

The past months were the months from my bachelor in which I learned the most physics and research skills. These past months were also the most exciting and fun months from my bachelor. I would firstly like to thank my supervisor Tjerk Oosterkamp for providing me with a spot in his research group. I never got bored listening to all your stories and physics advice. A big thanks to both of my daily supervisors Jaimy and Loek. Thanks for guiding me through my bachelor research project with such enthusiasm and dedication. I learned a lot and had a great time while doing so. Whenever I needed help one of you was always nearby to help me out. I will miss the great time we had. Also thanks to my fellow bachelor students: Thijmen, Job, Mart and Matthijs. I had a great time with you all. I also want to thank the fine mechanical service and the electronics department. I also need to thank my second reader Martina Huber. A special thanks to the rector magnificus of Leiden University Hester Bijl to provide the funding for our extremely high electricity bill.

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