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Defended on January 29, 2025

MASTER THESIS
STATISTICS AND DATA SCIENCE
UNIVERSITEIT LEIDEN

Abstract

In recent years, the analysis of longitudinal (social) network data with Stochastic Actor-Oriented Models (SAOMs) has become increasingly popular. SAOMs offer insight into the dynamics that drive network evolution by modeling multiple, discrete observations of sociocentric network data as coming from a continuous-time Markov chain. However, as with all survey-based data, social network data are susceptible to measurement errors. It was found that respondents can fail to recall 20% or more of their friends (Brewer, 2000). Such error is known to influence other types of social network analysis, such as descriptive statistics of networks and Exponential Random Graph Model results. This study aims to extend this body of research by assessing the sensitivity of SAOMs to measurement errors using a simulation study. It evaluates the effects of the false exclusion (negative error) and false inclusion (positive error) of relationships on SAOM parameters. The effects of negative error are explored more thoroughly due to the importance of recall bias in the context of social network data.

The Glasgow *Teenage Friends and Lifestyle* data (Michell & West, 1995) are used to determine a "true model", which is used to simulate 1000 network trajectories data that fit this true model. Two types of negative error are introduced into these network trajectories. The *random error scenario* introduces negative error into network trajectory data at random. The *embeddedness-based scenario* is a systematic error scenario, where negative error is introduced in a pattern informed by social and cognitive sciences. Ties that are more embedded into networks are modelled to be less likely to be forgotten. Errors are introduced into the same 1000 network trajectories in varying amounts, with 7 levels of negative error for the random scenario and 10 for the systematic scenario. Each scenario is assessed with and without a fixed degree of positive error. SAOMs are re-estimated on the error-induced data, after which the resulting SAOM parameters are compared to the true model by assessing the relative bias.

Results show that conclusions based solely on parameter significance and effect direction can be reliably used to answer research questions. However, measurement error does bias the sizes of parameters on a logit scale. Negative error (both random and systematic) tends to moderately overestimate most SAOM parameters, while positive error moderately underestimates SAOM parameters. Differences between the random and systematic scenario are concentrated in the parameters that are related to the systematic error pattern, such as transitivity and homophily. Related effects tend to be more strongly overestimated, since the systematic type of negative error tends to selectively preserve their network structures.

Results are specific to the data and parameter selection used in this study, and cannot be fully generalized to other types of networks and SAOM specifications. However, they do indicate that (i) as long as exact numerical results are unimportant, SAOM parameters remain informative about network dynamics, and (ii) researchers should be careful when basing conclusions on sizes of SAOM parameters, since they are likely to be biased. When assessing parameter sizes in SAOMs, researchers should address the potential for parameter biases to contextualize results and conclusions.

Contents

Glossary

3-Cycles effect An effect that can be part of the objective function of a Stochastic Actor-Oriented Model. It reflects the tendency of actors to create ties resulting in triadic, cyclic structures. A higher parameter value means an actor is more likely to create a tie to complete a cycle with two other actors. Such a tie creates a closed loop between the three actors.

Actor An individual or entity that is part of a social network, and that can form ties with other actors. Actors can represent various types of entitites, such as people, organizations, or companies.

Alter Alters are the other actors that ego identifies as having a given type of relationship with. This can be any type of relationship; alters can be friends, coworkers, organizations, etc.

Bias A measure that indicates systematic error in the estimation of a parameter. The expected value of the estimates of the parameter differs from the true value of the parameter.

centrality A social network analysis-metric that quantifies the importance of an actor within a network. It can be calculated using various measures. An example is the degree centrality metric, which is the number of ties an actor has. Centrality measures can also be calculated on a network level, to quantify the concentration of influence in a network.

Components A subset of a network where all actors are connected to each other - directly or indirectly through other actors.

Continuous time Markov chain A stochastic process where transitions between states occur continuously over time. The probability of transitioning to a new state depends only on the information in the current state (memoryless property).

Convergence An approximation algorithm convergences when the estimates become closer and closer to a specific value as more iterations go by. In the context of Stochastic Actor-Oriented Models and the Robbins-Monroe algorithm, it means that the expected values of the sensitive statistics calculated from the simulated networks become very close to the statistics calculated from the observed networks.

Coverage In a simulation study, the coverage is the proportion of simulations that the true parameter falls within the 95%-confidence intervals of the estimating statistic.

Density A measure of the tendency of actors to have ties. It is the proportion of the number of observed ties compared to the number of ties that could be created.

Density effect An effect that can be part of the objective function of a Stochastic Actor-Oriented Model. It reflects the tendency of actors to form ties. A higher parameter value for this effect indicates that actors are more likely to create ties, leading to denser networks over time. A lower parameter value indicates that actors prefer fewer connections, leading to less dense networks over time.

Directed network A type of network where the ties between actors are represented as directed edges, which indicates a one-way connection. In a directed network, an edge from A to B shows that A indicated to have a relation to B. However, B may not have a relation to A.

Edge A relation or tie between two actors in a social network. In images visualizing network data, edges are shown as lines connecting the different circles that represent the nodes/actors.

Ego The central actor or individual in an egocentric network, whose relationships and connections with others (alters) are the topic of analysis.

Egocentric network data A type of network data collected from the perspective of a single actor (ego) and their immediate social connections (alters). A dataset typically constitutes of a random sample from a given population.

Exponential Random Graph Models (ERGMs) A type of model that is used to analyze static social network structures. It models the probability of observing a given social network configuration based on the prevalence of specific tie configurations in the observed network data.

Hamming distance The size of the difference between the union and intersection of the ties in two states of the same network. This is equal to the number of observed changes between the two network states: the number of mini-steps that is needed to transfer from one observed network state to the next observed network state. Therefore, it is a measure of change within a network.

Homophily The extent to which all actors in a network have ties with others that are similar to them in a given way. This can be related to all kinds of characteristics, such as sex, age, behaviour or interests. It is quantified as the number of in-group ties divided by the total number of ties in a network.

Homophily effect An effect that can be part of the objective function of a Stochastic Actor-Oriented Model. It reflects the tendency of actors to form ties with others who are similar to them based on a specified characteristic, such as age, sex, interests or behaviour. A higher value for this parameter means that actors are more likely to form ties to actors who resemble them.

Indegree The number of incoming ties of an actor in a directed network. It represents how many other actors have reported a directed relation to the given actor.

Indegree activity effect An effect that can be part of the objective function of a Stochastic Actor-Oriented Model. It reflects the tendency of actors with many incoming ties to create more ties. A higher parameter value means that actors who are popular in terms of receiving ties from other actors, are also more likely to create new ties to others.

Indegree popularity effect An effect that can be part of the objective function of a Stochastic Actor-Oriented Model. It reflects the tendency of actors with many incoming ties to become even more popular by receiving additional ties. A higher value for this parameter means that actors are more likely to create ties to actor with a high indegree value. It reflects the process of preferential attachment, or the Matthew effect (Merton, 1968), where individuals with advantages earlier on gather more benefits over time.

Jaccard index A similarity measure between two sets. It is the size of the intersection between the sets divided by the size of the union. In networks, it is the proportion of ties that is measured in both observed networks, compared to the number of ties that is measured in at least one of the observed networks. It is a measure of stability in the network over time.

Learning rate A tuning parameter that controls the step size of optimization algorithms. It influences how quickly or how slowly an algorithm can converge to its solution.

Measurement error The difference between the true value of a parameter, and the value that is retrieved through a measurement instrument for this parameter.

Median distance The distance between two actors is the fewest number of steps an actor has to take through the network to reach the other actor. Here, the median distance is the median of all distances in the largest component of the network. When there multiple components with the same, largest size, the median of the distances within those components is used. The measure shows how easy it is for actors to reach one another.

Method of (simulated first) moments The method of moments is a statistical estimation method. The theoretical moments (the expected values of the assumed probability distribution) , are equated to the sample moments (calculated from the data). The k^{th} moment from random variable X is $E[X^k]$. In the context of Stochastic Actor-Oriented Models, only the first moment (mean) is typically used. The sample moments are equated to the expected values obtained from simulations, instead of theoretical moments.

Mini-step In Stochastic Actor-Oriented Models, mini-steps refer to the smallest possible change in a network, and are used to model the evolution of the network over time. They are modelled as the results of actors' decisions to remove, maintain, or create ties.

Name generators A survey tool used in social network research to identify individuals in a person's network. Respondents are asked to list names of people they interact with based on specific criteria (e.g., friends, colleagues, etc.).

Node An actor within a network. In images visualizing network data, nodes are shown as circles, and they are connected by edges to represent relationships with other nodes.

Objective function In Stochastic Actor-Oriented Models, the Objective function is used to model the changes actors make when they have the opportunity to make a change to their ties. Their options are to create or dissolve a tie, or maintain the current network. Various kinds of effects, such as transitivity and homophily, are part of this function, and influence the probability an actor decides to change a certain tie. The function is a type of random utility function. Individuals are modelled to make choices based on their preferences. More specifically, choices are based on the utility/preference values for a collection of discrete options, combined with a random component. The aim is to maximize this random utility.

Outdegree The number of outgoing ties from a node in a directed network. It represents how many other actors the node has indicated to have a directed relation with.

Percentile rank A measure that indicates the relative standing of a value in a distribution. In the context of this study, it is used to assess the position of the true value of a SAOM parameter in the distribution of values of that parameter across simulation runs.

Power The probability of correctly rejecting a null hypothesis.

Preferential attachment Preferential attachment is a type of popularity-effect. It refers to the tendency of actors to form ties to popular actors. It can be modelled by the indegree-popularity effect.

Random measurement error Measurement error is the difference between the true value of a parameter, and the estimated value that is retrieved through a measurement instrument. Random measurement error refers to error that occurs in a way that is random and cannot be predicted. It does not follow a systematic pattern.

Rate function In Stochastic Actor-Oriented Models, the Rate function is used to model when which actor in the network gets the opportunity to make a change to their ties.

Rate parameter A parameter that is part of the rate function in a Stochastic Actor-Oriented Model. The basic rate parameter reflects how many opportunities actors on average have to make changes to their ties. The rate function can be modelled more extensively, but this option is not considered here.

Reciprocity effect An effect that can be part of the objective function of a Stochastic Actor-Oriented Model. It reflects the tendency of actors to form ties to those actors that have reported a tie to them. A higher reciprocity parameter indicates that when an actor receives a tie, they are more likely to create a reciprocal tie. This leads to mutual relationships between pairs of actors, and a higher reciprocity index in the network.

Reciprocity index A measure of the extent to which ties in a directed network are reciprocated. In this thesis, it is quantified as the proportion of connections that are mutual compared to the total number of connections in the network.

Relative bias A standardized measure of systematic error in the estimation of a parameter, that mainly shows the magnitude of the bias. It is the bias of an estimate expressed as a proportion of the true value of the parameter.

Robbins-Monro algorithm An iterative stochastic approximation algorithm. It is used to find extreme values for functions that can only be estimated through noisy observations, and cannot be computed directly. It is used in the estimation of parameters in Stochastic Actor-Oriented Models.

Sensitive statistic A statistic for which the expected value is sensitive to changes in the value in the corresponding parameter.

Small-world phenomenon The small-world phenomenon is a concept in the field of social networks. It entails that "shortcut" paths (Watts & Strogatz, 1998) reduce the median distance between actors. These shortcuts are ties that connect different network clusters to each other. Because of this, even in large networks, actors in different parts oft the network can often reach each other through only a few intermediate actors. This promotes the quick spread of information.

Sociocentric network data A type of social network data that captures relationships within a pre-defined group of actors. Data are collected for all actors in the network, which creates a full view of the network structure.

Standardized Variance Variance as a statistical measure indicates the dispersion of data points around their mean value. In this simulation study, variance is used as a statistic to assess the spread of SAOM parameter estimations across runs for each combination of positive and negative error rates. To compare the variance of the estimates, and its relation to error rates, between the different SAOM parameters, the variance measures are standardized for every parameter.

Stochastic Actor-Oriented Models (SAOMs) A type of model that is used to analyze the evolution of social networks over time. It considers the observed network data to come from a continuous-time Markov chain, and models changes between observed network states as the decisions of actors to form, maintain, or dissolve ties.

Subgraph count Subgraph counts are counts of specific tie-configurations within the network. This can refer to reciprocal ties, triads configurations, or more complex structures. In the context of SAOMs, these configurations are counted in the neighbourhood of a given actor, e.g. the number of reciprocal ties the focal actor is involved in.

Systematic measurement error Measurement error is the difference between the true value of a parameter, and the estimated value that is retrieved through a measurement instrument. Systematic measurement error refers to error that follows a consistent pattern. For example, in the context of social network surveys, respondents may tend to forget naming friends they are less close to.

Transitive reciprocated triplets effect An effect that can be part of the objective function of a Stochastic Actor-Oriented Model. It reflects the tendency of actors to have reciprocal ties within transitive triplets. A higher value for this parameter means that, if an actor is part of a transitive structure with two other actors, they are more likely to reciprocate ties within that triplet. In this thesis, this effect is also referred to as the "reciprocated triplets" effects for short.

Transitivity effect An effect that can be part of the objective function of a Stochastic Actor-Oriented Model. It reflects the tendency of actors to create ties that transitively close triplets in the network. Consider actor A, who has a tie to actor B, who is connected to actor C. A higher transitivity parameter means that actor A is more likely to create a tie to actor C, to close the triplet. Higher parameters lead to more clustering in the network over time.

Transitivity index A measure of the extent to which two-paths in networks are transitively closed. Transitive closure means that when there is a tie from actor A to B, and from B to C (a two-path), A also has a tie to C. It is quantified as the proportion of closed two-paths amont all two-paths in the network.

Triad census The count of all possible triadic configurations (subgraphs of three actors) within a network. There are 16 different types of triads, which are categorized based on the number and directionality of the ties. They are named with a sequence of numbers in the MAN-format: the first number refers to the number of Mutual (reciprocated) sides in the triad, the second number stands for the number of Asymmetric sides in the triad, and the third number shows the number of Null sides: sides with no relations. For example, if in a subgraph of actor A, B, and C, A has a tie to C, and C to A, the characterization would be 102. A letter is added to some subgraphs to reflect the directions of ties: U for Up, D for Down, T for triadic, and C for cyclic. The triad census of a network indicates how often each unique triad configuration occurs in the network.

Two-path In this thesis, a two-path is considered to be a subgraph involving actors A, B and C. It contains an undirected tie between A and B, and an undirected tie between B and C. In a closed two-path, there is also an undirected tie between A and C. In an open two-path, there is no tie between A and C.

Undirected network A type of network where the relationships between actors are represented as undirected edges, which indicates a mutual connection. In an undirected network, an edge between A and B signifies that A is connected to B and B is connected to A.

Variance A measure that indicates to what extent data differ from the mean value. It reflects the spread of the data points. A high value indicates that the data are widely spread out.

Chapter 1

Introduction

In recent years, the analysis of longitudinal social network data using Stochastic Actor-Oriented Models has become increasingly popular, with more and more papers using this type of analysis (Scopus, 2024). The ability to analyze network dynamics over time yields important and interesting insights. This ranges from analyzing the formation and evolution of drug trafficking networks (Bright et al., 2019), to seasonal changes in badger interactions (Silk et al., 2017). SAOMs have created new opportunities to understand the way actors interact with others in their networks, and how both individual attributes and structural network characteristics influence this. However, as in all empirical, and especially survey, research, the validity of the results of these models depends on the quality of the input data. Measurement errors can potentially distort the findings of SAOMs, which can lead to incorrect conclusions. Hence, the present thesis aims to analyze the effects of measurement errors on the reliability of the results of SAOMs.

Stochastic Actor-Oriented Models (SAOMs) (Snijders, 2001; Snijders, van de Bunt, & Steglich, 2010) are models that use multiple waves of sociocentric network data to model changes in social networks over time. Sociocentric network data contain the social relationships between all individuals, or nodes, in a pre-defined group. A SAOM model analyzes the way relationships between actors in this group change over time. It uses a simulation-based approach to estimate the effects of several types of network dynamics on the changes in the network data. The model assumes the network data to come from a continuous-time Markov chain, which is observed at two or more discrete points in time. The changes in the network between these time points are modelled as a stochastic process driven by the decisions of individual actors. The total amount of change between observations is decomposed into a sequence of the smallest possible, unobserved mini-steps. Each mini-step is a one-unit change in the current network state, which means that one tie is either newly formed or deleted. These changes are modelled as decisions by actors to make changes in their relations to other actors. The decision can be influenced by various factors, such as individual characteristics of the actor, and attributes of peers. It can also consider characteristics related to the network neighbourhood of an actor, and the positions of the actor and peers within the network, as effects driving network change. Examples include reciprocity (the tendency to form mutual ties), transitivity (the likelihood of forming connections with friends of friends), and interactions of such effects with other mechanisms or covariates (Steglich & Snijders, 2022; Snijders, van de Bunt, & Steglich, 2010). The flexibility of the model allows researchers to draw conclusions about the role of various types of complex network dynamics in the emergence and evolution of network structures.

SAOMs assume the network data to be an accurate representation of reality, and to be free of measurement errors. However, *this is an optimistic and often incorrect assumption, which may lead to faulty conclusions*. Network data are mostly collected using name generators: survey questions that let respondents generate a list of people they have a specific kind of social tie to. Survey data are in general prone to measurement error (Weisberg, 2005). For example, random errors may be introduced into survey answers by data entry mistakes, or respondents can be randomly distracted while answering a survey question and make a mistake. In addition, there are systematic errors, which are errors in survey answers that follow a pattern across respondents. For example, network data acquired by the use of name generators are prone to recall bias. Brewer (2000) provides a systematic review of studies into recall bias for network data. This shows that, across groups and types of relationships, respondents of network surveys tend to forget a considerable number of relationships, although the exact amount of forgetting varies. For example, in a study among dormitory students Brewer & Webster (2000) found that students forgot on average 20% of their friends. The amount of forgetting did depend on the strength of the relationship, with 3% of best friends, and 9% of close friends being forgotten. Other studies confirm that recall bias seems to be more apparent for weak ties than for strong ties (Bell et al., 2007; Marin, 2004). On the other hand, respondents may mention more friends than they in reality have to seem more popular (social desirability bias). Or, as suggested by Feld & Carter (2002), they may specifically overreport ties to specific social groups. In general, social network data are likely to contain both random and systematic errors.

Earlier studies have shown that the results of other types of network analyses are sensitive to measurement errors. For example, Almquist (2012) found that there are significant changes in network measures such as density in ego networks, starting at 5% of ties being randomly removed. From 20%, network characteristics like density and triad census become heavily biased. In addition, variation in tie-forgetting between respondents both distorts attributes of networks by individuals, as well as network-level characteristics such as transitivity and clustering (Feld & Carter, 2002). Kim et al. (2016) studied the effects of measurement errors on the results of Exponential Random Graph Models (ERGMs), which analyze the network structures cross-sectionally. They found that the introduction of random errors causes classic attenuation bias: with more negative random errors, coefficients for reciprocity and homophily effects are biased towards zero. While the robustness of several network analyses to both random and systematic measurement errors has been investigated before, *no research has been done on the effects of these measurement errors on SAOMs*.

Given the sensitivity of other types of network analyses to measurement errors, it is likely that SAOMs are also affected by such errors. This is an important concern when it comes to interpreting and using the results from SAOM analyses. In the extreme case, the results of SAOMs are mostly a reflection of changes in measurement error over time instead of actual network dynamics. In less extreme scenarios, results are likely to be biased by the presence of measurement errors. This is a potentially important limitation to the use of results from SAOM analyses. Given how detailed the insights that SAOMs can give us into social processes and network dynamics are, the investigation of the effects of measurement errors on SAOM results is a highly relevant topic.

The present study aims to fill the gap in the literature regarding the performance of Stochastic Actor-Oriented Models under measurement error. A simulation study is conducted, where the effect of various scenarios of measurement error on SAOM results is investigated. The study has to goal of identifying the effects of false omission and false inclusion of ties (labelled *negative* and *positive* error respectively) on SAOM results. Since the primary focus of existing literature on measurement errors in social network data is on negative error (Brewer, 2000; Marsden, 1990, 2005; Butts, 2003; Brashears, 2013; Marin, 2004; Bell et al., 2007; Wright & Pescosolido, 2002), this is the main focus of this simulation study. The effects of positive and negative measurement error are assessed in two types of scenarios. The first scenario focusses on measurement error that is randomly and independently introduced into network data. The second assesses the effects of a more systematic introduction of measurement error, taking into account relevant social and cognitive biases.

The rest of this thesis will have the following structure. Chapter 2 will give an introduction into the mathematical background of Stochastic Actor-Oriented Models. Chapter 3 covers an overview of the literature on measurement errors in network data and their effect on various types of social network analysis. It also introduces the way the literature is incorporated into the error scenarios considered in the simulation study. In Chapter 4, the data and methods for the simulation study are introduced, and the details of the assessed error scenarios are explained. Chapter 5 focuses on the results of both measurement error scenarios. In the final chapter, the results of the study are summarized, and their limitations and implications are discussed. Recommendations for future research are also presented. Lastly, it is important to note that this thesis contains a substantial amount of terminology that is specific to the field of social network analysis. For clarity and accessibility, definitions and short explanations of these concepts, as well as other relevant terms, are listed in a glossary at the beginning of the thesis.

Chapter 2

Mathematical Description of Stochastic Actor-Oriented Models

Stochastic Actor-Oriented Models, as developed by Snijders (2001), are models that use panel data on sociocentric social networks to draw conclusions about the network dynamics underlying network change. Their aim is to estimate a model that describes what effects drive the changes between the first and subsequent network observations, using a simulation-based aproach. The estimated parameters from this model can be used to test hypotheses regarding the effects driving network change.

The model uses simulations to assess the way the network changes between the observed instances of the network. While the network data considered by SAOMs are only observed at two or more discrete points in time, the underlying time variable is assumed to be continuous. The total change between the discrete observations is decomposed into a sequence of small, unobserved mini-steps. A mini-step is a one-unit change in the current state of the network, where one extra tie is added, or an existing die is removed. Such a mini-step is modeled as a choice by a given actor to either create a social relationship to someone else in the network, or dissolve one of their existing social relationships. It is also possible for the actor to leave the network as is - the mini-step then reflects the decision by the actor that the current network state is their most attractive network state. The changes between observed network states are thus specifically attributed to choices by the actors in the network. Actors are assumed to control their outgoing ties, which gives the model its actor-oriented perspective.

The change between the observed data is assumed to follow a continuous-time Markov chain (Tolver, 2016), where the current state of the network determines the evolution to the next state in a probabilistic manner. The mini-steps are dependent on the state of the network at that moment in time. They use the information available on the attributes of alters, their position in the network, as well as the general structure of the network. Based on their preferences, actors decide to take a mini-step that is optimal for them. Which actor makes which decision at what time is determined by the *rate function* and the *objective function*. The mathematical details of these functions are described in the sections below. The general idea is that the rate function is used to determine a "waiting time" for each actor. Then, the actor with the shortest waiting time gets to make a change. The objective function is used to specify what change the given actor will make. It calculates the attractiveness of each potential change given the preferences of the focal actor. The network state with the highest attractiveness to the focal actor is chosen. Some randomness is added to this process to account for deviations between the theoretical expectation and the observations, which gives the model its stochastic nature. In the end, all mini-steps made by the actors add up to the overall change in the network. The aim of the SAOM model is to estimate the parameters in the rate and objective functions, so that they accurately describe the changes between the observed network states. These parameters can be tested to draw conclusions about network dynamics.

In the following sections, the mathematical details (Snijders, 2001; Steglich et al., 2010; Snijders, 2017; Ripley et al., 2024) of Stochastic Actor-Oriented Models are outlined. This includes the description of the rate and objective functions, the simulation algorithm, the estimation of parameters using the method of moments, and model convergence.

2.1 Model Definition

Consider a fixed set of actors $\mathcal{N} = \{1, \dots, n\}$ for which a directed network is measured at $M \geq 2$ discrete points in time. Ties between the actors can at each timepoint be represented by an adjacency matrix $x = (x_{ij}) \in \{0, 1\}^{n \times n}$, where each element indicates if there is a tie going from i to j ($x_{ij} = 1$) or not ($x_{ij} = 0$). Self-ties are assumed to be meaningless, so the diagonal of each matrix is set to zero by default. In total, there are M observed adjacency matrices $x(t_m)$ at time points $t_1 < \cdots < t_M$, with $m = 1, \dots, M$.

2.1.1 The Rate Function

The evolution of the network is modelled as a continuous-time Markov chain $X(t)$, where small network changes (mini-steps) occur at random moments in time. At these moments, a stochastically chosen actor i chooses to change one outgoing tie $X_{ij}(t)(j \in \mathcal{N}, j \neq i)$. The frequency of these changes is governed by the rate function with the rate parameter ρ. The parameter ρ indicates how many opportunities actors have, on average, to make a change to their relationships. At each point t, each actor draws a waiting time from the exponential distribution with parameter ρ. The actor with the shortest waiting time can make a change to their network configuration. Hence, at any given time point in the stochastic process t with $X(t) = x$, the waiting times until the next actor can make a choice are exponentially distributed with rate parameter

$$\lambda(\rho) = \rho n. \tag{2.1}$$

The memoryless property of the exponential distribution ensures that the probability of an actor initiating a change is only dependent on the current state, and not on how much time has already elapsed since the last change. This allows the stochastic process to fulfill the Markovian property.

Since the rate parameter ρ is a constant, the probability that actor i is allowed to make the change at timepoint t is given by $\pi_i(\rho) = \frac{1}{n}$. This means that actors all have the same probability of making changes to their relationships at any given point in time. Although this thesis only considers these constant rate parameters ρ, SAOMs also include possibilities to model the rate function more extensively. For example, one can account for actor-specific activity levels by including effects based on covariates or positional characteristics in the rate function. These more extensive rate specifications fall outside of the scope of this thesis. For more details on non-constant rate functions, the reader is referred to Snijders (2017).

2.1.2 The Objective Function

The direction of the change made at a given mini-step depends on the choice made by the actor i, which drew the shortest waiting time from the rate function. Actor i can choose to move towards a network state that differs at most one tie from the current network (denoted by x^0), and thus either create a tie or dissolve one of their ties. They can also choose to keep the current network as is. The set of possible outcome states for actor i, where the network state differs at most one tie from the current network state, is reflected by $A_i(x^0)$. To choose the optimal network state among all options, the objective function is used to calculate the attractiveness of each potential network state x for actor i.

Attractiveness values are calculated using the objective function

$$f_i(\beta, x) = \sum_k \beta_k s_{ik}(x) + \epsilon. \tag{2.2}$$

This function is made up of a deterministic and a random component. The deterministic part is a linear combination of effects ($s_{ik}(x)$), which are based on theory and knowledge about the subject matter. From the totality of possible effects, a selection of effects is made that are (potentially) meaningful and relevant in explaining network change in the particular dataset. Effects can be related to the network structure as seen from the perspective of actor i, but they can also contain actor attributes, and interactions between effects. The weights β_k indicate whether each effect in the network x in the neighbourhood of actor i are desired ($\beta_k > 0$), and thus increase the attractiveness of a given network state, or disfavoured ($\beta_k < 0$), and decrease a network state's attractiveness.

The weights corresponding to the effects in the objective functions are where hypotheses regarding the processes underlying the observed network evolution are tested. For example, one can test to what extent network dynamics are driven by a tendency of actors to form relationships with the friends of their friends (transitivity). This effect can be expressed as $s_i = \sum_{j,h} x_{ij} x_{jh} x_{ih}$. It is the subgraph count in the current network state for subgraphs where i has a tie to both j and h, and j also has a tie to h. If its weight is positive, it means that a mini-step leading to a network state with more transitivity is favourable. All effects, and their parameters, together determine the overall attractiveness of each potential network state. A couple of other examples of effects can be found in Table 2.1. For a more extensive overview and explanation of effects, the reader is referred to Ripley et al. (2024).

The objective function also contains a random component ϵ to introduce stochasticity in the attractiveness values of the potential network states. The values are drawn from a Gumbel distribution with mean 0 and scale parameter 1. This is similar to the random utility-type models often used in econometrics: choices are driven by the utility that actors attribute to a collection of discrete choices, combined with a random component (McFadden, 1973). The choice of this random disturbance makes that the maximizing probabilities of the objective function coincide with a multinomial logit model (McFadden, 1973). The probability of actor i choosing a given network configuration can then be expressed as follows:

$$p_i(\beta, x) = \begin{cases} \dfrac{exp(f_i(\beta, x))}{\sum_{x \in A_i(x^0)} exp(f_i(\beta, x))} & x \in A_i(x^0) \\ 0 & x \notin A_i(x^0) \end{cases} \tag{2.3}$$

where the denominator sums the network attractiveness scores over all potential network states for actor i $x \in A_i(x^0)$. It reflects the conditional probability that x is the next state from the set of potential new states $A_i(x^0)$, given that actor i gets to make a change.

The network state for which the objective function's attractiveness score is the highest thus ends

Table 2.1: Examples of effects that can be specified in the objective function of SAOMs

Formula	Effect	Description
Outdegree	$\sum_j x_{ij}$	The tendency to have ties
Reciprocity	$\sum_j x_{ij}x_{ji}$	The tendency to reciprocate ties
Transitive triplets	$\sum_{j,h} x_{ij}x_{jh}x_{ih}$	The tendency to have transitive groups - to *be friends with the friends of your friends*
Indegree related popularity effect	$\sum_j x_{ij} \sum_h x_{hj}$	The tendency to form ties to alters with high indegrees
Covariate-related identity	$\sum_j x_{ij} I\{v_i = v_j\}$	The tendency to form ties to alters with the same value on covariate v

up being the next network state. This way of choosing the next states fits the Markovan property. The objective function only considers information from the current network, and the potential next configurations that can be created by changing at most one tie in the current network.

There are various extensions of the Stochastic Actor-Oriented Model. One example is the differential valuation of the costs and rewards that belong to creating a new tie versus losing an existing tie. Another possibility is to analyze changes in behaviour together with changes in the network with a co-evolution SAOM. However, these extensions are not considered in this thesis. For more information, the reader is referred to Snijders (2017) and Snijders et al. (2007).

2.1.3 The Simulation Algorithm

The previously described rate and objective functions are used to simulate the network change for the time between observation t_m to t_{m+1}. The specifics of the simulation algorithm that is used for this, are described in the steps below (Snijders, 2017; Snijders & Steglich, 2023).

1. Set $t = t_m$ and $x = x(t_m)$;
2. For all actors i , generate a waiting time ΔT_i based on the exponential distribution with parameter ρ;
3. Select the actor i with the smallest generated waiting time, and set the time until the next change ΔT to ΔT_i.
4. If $t + \Delta T > t_{m+1}$, set $t = t_{m+1}$, and end the simulation. The total network change has already been achieved.
5. Otherwise, randomly select the next network state $x' \in \mathcal{A}_i(x^0)$ with the probabilities $p_i(\beta, x)$ based on Equation 2.3;
6. Set $t = t + \Delta T$ and $x = x'$;
7. Return to step 2.

2.2 Model Estimation

In the end, the aim is to find values for the parameters in the rate and objective function that describe the changes between the observed network states. Exact calculations of the parameter estimates are not possible in SAOMs. For exact calculations, the state space of all theoretically possible network trajectories needs to be defined, which can be very large. This is because, in a longitudinal setting, the possible data-connectivity trajectories between the observed networks can be extended indefinitely. As a result, there is no bounded number of potential network trajectories. Since this makes it computationally infeasible to account for all potential network trajectory configurations, it becomes problematic to assess the likelihood of the data given certain parameter estimates. The computational difficulty to evaluate this likelihood makes analytical solutions for estimating SAOM parameters not viable.

However, as highlighted above, it is relatively easy to simulate the processes underlying network change. Therefore, simulation-based methods for estimation and inference can be used. There are several ways in which the estimation procedure can be performed, such as Bayesian (Koskinen & Snijders, 2007) and likelihood-based (Snijders, Koskinen, & Schweinberger, 2010) methods. The most common estimation procedure is the method of moments (Snijders, 2001), which is described here. The efficiency of the method of moments can be further improved by using the generalized method of moments (Amati et al., 2019).

2.2.1 Sensitive Statistics

Before the estimation method itself is discussed, some elaboration is needed on the relevant *sensitive statistics* that the method of moments uses to estimate the parameters. A sensitive statistic is a statistic for which the expected value is sensitive to changes in the values of the corresponding parameter (Snijders, 2001). In the context of SAOMs, sensitive statistics are statistics that respond to changes in the relevant dynamics in the network. For example, a sensitive statistic belonging to the objective function parameter for a transitivity effect should be sensitive to the amount of transitivity in the network.

The parameter to be estimated in SAOMs is $\theta = (\rho, \beta)$. For each component θ_h a sensitive statistic can be heuristically defined. ρ_m indicates the total amount of change between observations between network $x(t_m)$ and $x(t_{m+1})$. The Hamming distance

$$D(X(t_{m+1}), X(t_m)) = \sum_{i,j} \mid X_{ij}(t_{m+1}) - X_{ij}(t_m) \mid \tag{2.4}$$

is a sensitive statistic for this parameter. The larger the rate of change between two networks, the larger the absolute difference (Hamming distance) is between the two networks.

In the objective function (Equation 2.2), the parameter β_k is used to indicate the importance of effect $s_{ik}(x^0)$ in determining the change actor i will make when it is their turn. When β_k is higher, the effect is important in driving network change, which leads to networks with higher values of $s_{ik}(x)$ for all actors. Therefore, the sensitive statistics is the sum of the effect $s_{ik}(x)$ across all actors:

$$s_k(X(t_m)) = \sum_i s_{ik}(X(t_m)). \tag{2.5}$$

2.2.2 The Method of Moments

The method of moments uses the sensitive statistics to estimate SAOM parameters. In general, the technique of the method of moments is used to estimate parameters in probability distributions. Typically, sample moments are equated to population moments derived from theoretical distributions. In the case of SAOMs, the sample moments refer to the values of the sensitive statistics in the observed networks. The population, or theoretical moments, are the expected values of the sensitive statistics calculated in the simulated networks. This estimation process aims to find a SAOM model specification that generates simulations where theoretical moments closely match the values observed from the data. Although the general method of moments often uses multiple moments, the SAOM specification only uses the first moment. The method of moments equations are the following:

$$D(X(t_{m+1}), X(t_m)) = E_\theta\{D(X(t_{m+1}, x(t_m)) \mid X(t_m) = x(t_m)\} \tag{2.6a}$$

$$\sum_{m=1}^{M-1} s_k(x(t_{m+1})) = \sum_{m=1}^{M-1} E_\theta\{s_k(X(t_{m+1})) \mid X(t_m) = x(t_m)\} \tag{2.6b}$$

with $k = 1, \ldots, K$. K refers to the number of effects in β.

The solutions to the equations in 2.6 are found using the Robbins-Monro algorithm (Robbins & Monro, 1951). This is a stochastic approximation algorithm that iteratively updates parameter estimates based on differences between expected and observed values of the sensitive statistics. The goal is to find parameter values for $\theta = (\rho, \beta)$ that solve Equations 2.6a and b. The difference between the expected values of the sensitive statistics in the simulated networks and those in the observed networks then approach 0. Estimates are therefore adjusted in the direction that reduces the difference between observed and expected values most, which causes the estimates to gradually converge to the true parameters. The algorithm uses a learning rate with decreasing step sizes to ensure convergence. The iterative updating rule is

$$\hat{\theta}_{l+1} = \hat{\theta}_l + a_l D_0^{-1}(S_k^{sim} - S_k^{obs}) \tag{2.7}$$

where l refers to the iteration step, and a is a sequence with decreasing step size that converges to 0 at rate l^{-c}, with $0.5 < c < 1$. D_0 is the Jacobian matrix, which indicates the sensitivity of the expectations of simulated sensitive statistics S_k^{sim} to changes in the parameter values θ. Its entries are the partial derivatives of each sensitive statistic with respect to each parameter in θ. S_k^{sim} refers to the values of the sensitive statistics based on simulated networks, and S_k^{obs} refers to their values in the observed data.

2.3 Convergence

The estimation procedure converges when the difference between the simulated values of the sensitive statistics S_k^{sim} and their observed (or target) values S_k^{obs} approaches zero. In the last part of the estimation procedure, extra simulations are run using the final parameter values, to determine the standard deviations. These runs are also used to determine whether the algorithm converged properly. Convergence is assessed using the overall maximum convergence ratio, and parameter-specific t-ratios (Ripley et al., 2024). For each parameter, a convergence t-ratio is calculated by dividing the average deviation between simulated and observed values by the standard deviation of the deviation. The absolute value of this t-ratio should be below the threshold of .1 for each parameter. In addition, the overall maximum convergence ratio is the maximum value of the convergence t-ratio for any linear combination of the parameters. A properly converged solution has a value below .25 for this statistic. These criteria are not strict rules, but more overall indications of proper convergence, and they are the recommended convergence criteria by developers of the RSiena software (Ripley et al., 2024, p. 73).

Chapter 3

Measurement Errors in Social Network Analysis

That social network data are prone to measurement errors, is a long-established result. Research on the accuracy of informants concerning their social interactions took off in the 1970s, with a series of papers by Bernard and Killworth (Killworth & Bernard, 1976; Bernard & Killworth, 1977; Bernard et al., 1979; Killworth & Bernard, 1979). "What people say, despite their presumed good intentions, bears no resemblance to their behavior" (Bernard et al., 1982, p.63), is what they concluded. They studied the ability of informants in different groups to recall their communications over a certain period, (with the question "Who do you talk to?"), which indicated that people claim to have talked to people they never talked to and vice versa. They also cannot rank their communications, even when it only concerns people they talk to a lot. Reports by informants on their interactions were, all in all, concluded to be unreliable.

The work of Bernard and Killworth challenged the assumptions about the reliability of social network data. Since then, a lot of additional research has been done into the reliability of this type of data. This extends beyond the communication interactions studied by Bernard and Killworth, and is concerned with many different types of relationships, ranging from friends, to schoolmates and neighbours. Studies agree that measurement error is a problem in social network data, but the results are not all as negative as those presented by Bernard and Killworth. A study by Lee & Butts (2020) shows that, although recall of relations by respondents is error-prone, it is a valid reflection of the underlying social reality.

In this section, the existing research on measurement errors in the context of social network analysis is discussed. It starts with a definition of measurement error, and a discussion of the existence of a real "groud truth" network. In addition, it covers types and patterns of measurement errors in social network data, and the influence of such errors on the results of social network analysis. The section ends with a description of how the existing research is incorporated into the simulation study using two distinct error scenarios.

3.1 Measurement Error

Measurement error is "any deviation from the true value of the concept in measurement" (Suen & Cerin, 2014). It can be the result of various sources, ranging from mistakes in the data entry process, to the tendency of respondents to give socially desirable answers in survey research.

Measurement error can be split up into random error and systematic error (Weisberg, 2005). Systematic measurement error usually follows a specific pattern, and therefore introduces bias into the concepts that are to be measured. An example of this would be the inclination of adolescents to underreport their drinking behaviour (since drinking less is more socially desirable). This would lead to an overall underestimation of drinking behaviour, because survey answers tend to be biased in the same direction. Random errors do not follow such a systematic pattern, and occur randomly and independently. This could happen when an existing relationship is, completely by mistake, recorded as a non-relationship in the collection of social network data.

In the context of social network data, various types of errors can be introduced into the network data. Wang et al. (2012) distinguish the following types of such error in sociocentric network data. First of all, actors can be completely missing from the data, for example due to non-response of a specific actor. Actors can also be erroneously included, possibly when respondents mention actors that fall outside of the boundary specification of a given sociocentric network. These are respectively false negative and false positive nodes. Similarly, existing relationships between nodes can be missing in the data (false negative edges), or non-existing relations can be falsely included (false positive edges). Lastly, multiple nodes can be falsely treated as one node (*falsely aggregated node*), or one node can be mistakenly treated as multiple separate nodes (falsely disaggregated node). This can happen when names of different actors are similar, leading to them mistakenly being treated as the same actor. Similarly, things like name misspellings can lead to one actor being treated as multiple disstinct actors. This thesis focusses on the effects of false negative and false positive edges. Hereafter, these types of error are referred to as *negative error* and *positive error* respectively.

3.2 The True Network

Before research on error in social network data is discussed, the concept and discussion of a "ground truth" in social network data need to be addressed. Any study into the effects of measurement errors requires a clear definition of what constitutes as measurement error, and what does not. Therefore, this simulation study requires a reflection on what can be considered the "ground truth network", and what can be defined as measurement error - but this consideration is not straightforward. Before, measurement error was defined as any deviation in a measurement of a concept from the true value. However, it can be questioned whether there even is such a thing as one natural true value of a social network, since respondent-reported social relations are inherently subjective. And if there is such a ground truth - which is it? This also raises the question which inconsistencies in data can be treated as measurement errors, and which cannot.

In the papers by Bernard and Killwordth, observed interactions were treated as the truth. However, the perceived communication relationships may be conceptually different relations, instead of a reflection of measurement error. For example, Freeman et al. (1987) showed that respondent recall tends to be biased in the direction of long-term patterns, instead of an accurate representation of specific events. There have been attempts to theorize how respondent-reported networks differ conceptually from interaction data observed by external parties (Corman & Scott, 1994). Characterizing differences between different data sources as conceptually different, instead of attributing them to error, can be extended to friendship networks. People can have different thresholds for certain types of relationships: some people need to feel closer to someone before they call them a friend than others (Feld & Carter, 2002). Although they may report different networks and relationships, the reported data may reflect their truth. Hence, the perceptions of respondents may be a more accurate representation of the true network than observations by an external observer. After all, actors act based on their own perceptions and interpretations of social reality, and not on those of an external observer.

As shown in the study conducted by Lee & Butts (2020), the recall of relations by respondents is a reflection of the underlying social reality, and thus potentially a proxy to a "ground-truth" network. Lee & Butts (2020) used cognitive social structure (CSS) data, where all informants report on all possible relationships within the network, and not just their own. They found that random subsets of actors agree more often in their reports of friendship and advice-seeking relationships than can be explained by chance. In addition, the study shows that the mistakes in reports seem to be consistent with cognitive models of social perception. However, the study also acknowledges that self-reports of relationships do involve measurement errors. Other studies using a comparison of recall and recognition and test-retest studies also found inconsistencies in reports of respondents on their interactions and relations (Brewer, 2000).

This thesis does not aim to draw a conclusion about the existence of a true, underlying "ground-truth" network; that is beyond the scope of this project. However, the project does need a base network to treat as the "real network" to determine the effects introduced by measurement error. The presented research suggests that (i) self-reported relationships do reflect a social reality, and (ii) measurement errors in self-reported relations are common and pervasive enough to study the potential impact on statistical network analyses. In this thesis, self-reported relationships will therefore serve as a base network, for which the effect of measurement errors is assessed. Differences in self-reported relations and other data sources (e.g. an external observer) are not considered to be measurement errors. Only the differences that respondents themselves would recognize as errors are considered measurement errors. This is relevant when considering what type of systematic measurement errors to assess in the simulation study.

3.3 Measurement Errors in Social Network Data

Measurement errors can be introduced into network data in various ways. The exact source of measurement error is in itself not relevant for this simulation study. However, the potential sources or error, and therefore the existing research on these sources, are noteworthy. Insight into the specific error patterns in measurement error in network data can inform relevant error scenarios to consider in this simulation study, so that the introduced error mimics the patterns of real measurement error. Below, the available literature on both random and systematic errors is discussed. The specific scenarios based on the available literature are presented in a subsequent section.

Before exploring the existing literature, it should be noted that there are various types of study designs that can be used to collect social network data - from social media data to paper-based surveys and interviews. Of course, the data collection method highly influences what types of measurement error are possible and likely to occur. For example, in a sociocentric network study that uses a fixed roster with all names of actors in the network to choose from, recall bias is much less likely than in a free-recall design. Due to the magnitude of the potential error caused by recall bias (Brewer, 2000) the remainder of this thesis assumes that recall bias is possible and likely. Results are therefore mostly applicable to sociocentric studies with a free-recall design.

3.3.1 Random Error

Random positive and negative errors are introduced into network data without following a specific pattern. False inclusion or exclusion of ties is not dependent on the importance of the tie to the sending actor, the embeddedness of the tie in the network, or any other relevant internal or external factor. There are a couple of error sources that can be (partly) characterized as random measurement error.

Most importantly, humans are fallible and make mistakes. Mistakes in entering data, such as typography errors and interpretation errors, are a common occurrence. These errors reflect the cognitive limitations of human minds, and are not obviously related to any outside factors. Data entry rates range from 3.1% of data entry fields for electronic data capture during face-to-face interviews, to 5.1% for paper-based data recording (Tate & Smallwood, 2021). In addition, random error may be introduced due to mistakes made by respondents. Random distractions, mood fluctuations, and respondent fatigue may impact the cognitive skills of respondents, leading them to randomly misreport relations (Marsden, 2005).

It is important to note that the error sources presented above are unlikely to be completely random. Interviewer and respondent characteristics, and the clarity of interview/survey questions may introduce some structure into these errors. However, the error types also contain clear stochastic components, such as unpredictable cognitive states of respondents and random lapses in concentration. It is reasonable to assume that at least a part of these errors can be approximated as random errors.

3.3.2 Systematic Error

Systematic errors contain specific patterns that could systematically bias the data in specific directions. Research on the sources of these errors and their specific patterns is introduced in this section. The main type of systematic bias is *recall bias*, which refers to errors caused by the fallible memory and cognitive skills of respondents. The patterned errors caused by recall bias, as well as the cognitive structures that underlie these errors, are discussed below. In addition, the systematic effects of interviewer effects are mentioned.

Recall Bias

Research on recall bias reports the forgetting of ties by respondents (*negative* error). Results on degrees of recall bias vary a lot when considering different types of relationships. Brewer (2000) summarized levels of recall across multiple studies. High school students were found to recall 16-22% of their high school classmates (Bahrick et al., 1975). Among neighbours, 61% of the relations with acquaintances, 91% of relations with friends, and 92% of relations with close friends were recalled (Sudman, 1988). 80% of overall friendships were recalled among dormitory residents in Australia, which increases to 91% for close friends and 97% for best friends (Brewer & Webster, 2000). 75%-86% of sexual partners tend to be recalled, and 61%-78% of drug injection partners are recalled among people at high risk for sexually or parentally transmitted diseases (Brewer et al., 1999).

Keeping track of all relations one has with others is a cognitively demanding task. Because of this, humans use schemata to condense social information into a reduced form. Schemata are mental frameworks that help us understand and remember information, and that influence the way we recall information from memory (Brashears, 2013). Brashears & Quintane (2015) performed an experimental study to detect what kind of compression heuristics are typically used to store social network information. They found that people do not tend to store this information in dyads. The primary structure to remember social networks consists of triads. When triads are not available in the network, people have the flexibility to start using larger groups as a way to recall network structure. This indicates that triads, and to a lesser extent groups, may be more easily recalled and less prone to forgetting. At the same time, triads may also be more likely to be erroneously closed since they are easier to remember. Within these types of network structures, false negative ties may therefore be relatively less common, while false positive ties may be more common. The study also indicates that the exact form of recall bias is dependent on network characteristics; the presence of triadic patterns influences whether people use this as a compression heuristic. The notion that social groups are a relevant structure to remember social relationships is consistent with other research. Successive nominations in name generators are often clustered by social groups, and time between listed names that are part of the same social groups is often shorter (Marsden, 2005).

Freeman et al. (1987) found that the organization of human memory leads to recall biases in the direction of long-term patterns of social structure. "Typical" relationships, which a respondent interacts with frequently, are more embedded into the mental structure of a respondent, and therefore more likely to be remembered. From this, one can infer that less typical friendships may be forgotten more often in the data collection process. On the other hand, "old" friendships that have been neglected in the recent past may still be named as relationships, since they are still embedded in memory as a typical friendship.

In addition to research on schemata, there are several studies on other aspects influencing the accuracy of recall by respondents. Firstly, strong ties are generally less likely to be forgotten than weak ties, in terms of emotional closeness, frequency of interaction, and duration of the relationship (Brewer, 2000; Marin, 2004; Roth et al., 2021; Brewer & Webster, 2000). This pattern aligns with the effects of the earlier presented schemata on recall tendencies. People with stronger relationships tend to share overlapping network neighbourhoods, causing stronger relations to be involved in a larger number of triads and clusters (Granovetter, 1973, 1983; Friedkin, 1980; Burt, 2000). One explanation for this is that stronger relationships tend to involve larger time commitments, leading to more interaction between a person's contacts (Granovetter, 1973). Noting that larger networks contain larger fractions of weak ties, it is also not surprising that network size has also been found to play a role in recall bias. In larger networks, more relationships are forgotten than in smaller networks (Brewer & Webster, 2000; Bell et al., 2007). The amount of forgetting is also dependent on the type of relationship being studied. For example, relationships with more behavioural specificity, such as sex partners and needle-sharing partners, are more likely to be recalled than more ambiguous relationships such as friends or acquaintances (Bell et al., 2007). Social roles that involve more emotional intimacy, such as romantic partners and immediate kin, are also less likely to be forgotten than friends or schoolmates (Fischer & Offer, 2020). Lastly, Brashears et al. (2016) found that women are more efficient in processing social information, and more accurate at encoding and recalling the information than men.

Interviewer Effects

Interviewer effects are a common occurrence in research involving interviewers to collect data. Due to differences in interviewer characteristics, experience, and interviewer behaviour (such as variation in probing), results to survey questions can be correlated for respondents interviewed by the same person (West & Blom, 2017). Interviewer effects on eliciting responses to name generators, questions where respondents are asked to list their ties to people that fit a certain characteristic, seem to be larger than for typical survey questions (Marsden, 2003). Van Tilburg (1998) performed a study on interviewer effects in estimating the size of personal networks. It was found that the network sizes differed for respondents from different interviewers; for some interviewers, respondents named more alters than for other interviewers. The effect was partly explained by differences in education and interviewing experience among the interviewers. Other explanations for the interviewer effects can be variation in answers of interviewers to clarifying questions by respondents, or variation in the amount of probing (Marsden, 2003). Interviewer fatigue (Cornwell & Hoagland, 2014) may cause some interviewers to put more effort into probing for additional names than others. This means that respondents interviewed by one interviewer may make more recall errors than those interviewed by another interviewer, since they are probed for relationships to different extents.

3.4 Measurement Errors and Types of Social Network Analysis

Various studies have investigated how measurement errors affect different types of social network analysis. Out of these studies, most focussed on the effect of *random* measurement errors on the outcomes of the analyses.

Borgatti et al. (2006) studied the robustness of centrality measures in random networks under random measurement errors. They found that, under randomly introduced false positive and negative edges and nodes, the accuracy of the centrality measures declines predictably with the amount of error. In general, with up to 10% error, the centrality measures remain relatively robust. The decrease in accuracy is more pronounced for networks with higher density. This research was extended by Frantz et al. (2009), who found that the topology of the network has an important effect on the robustness of various network measures. Wang et al. (2012) found that robustness to error is highest in networks with low clustering, and less positively-skewed degree distributions.

False negative and false positive ties by themselves are not the only concerns for social network analysis. The *variation* in under- and/or overreporting of relationships by respondents can introduce additional bias into the results of social network analyses. Feld & Carter (2002) found strong indicators for the presence of such variation. The most apparent type of bias this leads to is the inflation of the variation in outdegree. It also influences other measures, like the amount of transitivity and centralization in a network. It also has the potential to distort relations between network size and other variables (Feld & Carter, 2002).

Kim et al. (2016) studied the effects of random measurement errors on the outcomes of Exponential Random Graph Models (ERGMs). These models are used to test hypotheses regarding network structures cross-sectionally. They found that randomly introduced measurement error leads to attenuation bias; coefficients of homophily and reciprocity effects tend closer to 0 with higher amounts of negative error. The effect is stronger for more complex effects such as reciprocity, where more ties are involved in creating the specific network structures. The effect is smaller for homophily, where only one tie is needed to construct an instance of homophily in a network.

Neuhäuser et al. (2021) assessed the influence of *systematic* types of measurement errors on the centrality-based ranking of minority nodes. They included various sources of systematic edge noise, including group label congruence noise (error related to intra- or inter-group edges), group label specific noise (error related to edges to or from specific groups), Jaccard noise (error related to the similarity of neighbourhoods of tied nodes) and centrality noise (edges related to the centrality of tied nodes). They conclude that the various types of errors lead to significant bias in the representation of minority nodes in centrality-based rankings. The exact effects vary depending on the amount of homophily in networks, but it is a clear indication that the introduction of systematic measurement error can bias the results of social network analyses.

3.5 SAOMs and Measurement Error

No research has been done on the effect of network-related measurement errors, whether random or systematic, on the effects of Stochastic Actor-Oriented Models. However, there are important indications that measurement errors can seriously influence the outcomes of these models.

First of all, it is important to consider what Stochastic Actor-Oriented Models focus on: the evolution of networks over time. In case a large proportion of the changes between observed networks are caused by measurement errors, the models may be modeling measurement errors instead of true data. It is plausible that the amount of measurement errors in changes between networks is larger than in a network at one time point, because measurement errors can simply be introduced at more moments. For the state of a given tie from i to j between time points t_1 and t_2 to be measured correctly, the existence of the tie needs to be measured accurately at both time points. If actors randomly forget an average of 20% (Brewer & Webster, 2000) of their friends at each time point, there is only a .64 probability that a stable tie's state is measured correctly at both time points. However, there is a .8 probability of the tie being measured correctly at each of the individual time points. This indicates that stable observations of networks may show more instability in relations than there is in reality. The higher amount of measurement error and larger instability likely make it more difficult to accurately capture the true processes underlying network dynamics with a SAOM.

A couple of studies has assessed the turnover in relations over multiple waves of ego network data, concerning important personal relations. They asked respondents why they did not name previously named alters again. The amount of turnover due to forgetting varies substantially across the studies - ranging from 5% (Wright & Pescosolido, 2002) and 7.9% (Mollenhorst et al., 2014) up to 41% (Fischer & Offer, 2020). This confirms that changes in observed networks can, for a considerable amount, be due to recall errors instead of actual network development. However, note that these data stem from egocentric network data and not from sociocentric network data. For sociocentric studies, smaller relationship turnover due to forgetting may be expected when data collection methods are used that do not require free recall, such as a roster design where all members of the network are listed. However, the numbers do indicate that measurement errors do have the potential to seriously disrupt SAOM results.

The direction in which SAOM results are biased is more difficult to pinpoint. Research in other areas of social network analysis suggests dependence of the effects on the specific characteristics of the network (Frantz et al., 2009; Wang et al., 2012; Neuhäuser et al., 2021; Brashears & Quintane, 2015). Different network configurations can respond in unique ways to the introduction of errors, and therefore the response of SAOM results may also be dependent on the specific network structure.

It can be expected that the results of Kim et al. (2016) for ERGMs are also applicable to SAOMs. They found that random measurement errors caused classic attenuation bias, and that more complex effects, such as reciprocity, are more strongly affected by measurement errors than others. ERGMs make use of subgraph counts to draw conclusions about network structures. Complex effects usually involve a relatively large number of ties to form the network structure. When introducing random error, there are more opportunities to break down complex existing network structures than to build up new ones (Kim et al., 2016). To break down a structure, the removal of only one edge in the structure is needed, while (multiple) specific edges are needed to build up new structures. Random error will therefore likely decrease the subgraph count of the effect, and therefore bias the estimate towards 0. Since SAOMs also use subgraph counts to determine the role of the effects in network evolution, it is likely this will also influence the results of this model. The effect may be less pronounced when introducing more systematic errors instead of random errors, since social ties are often remembered based on triads and groups (Brashears & Quintane, 2015). Triadic and group structures are therefore less likely to "lose" ties, causing network structures to be relatively resistant to measurement error.

3.6 Error Scenarios

Based on the discussions and research presented before, two error scenarios have been set up to test the effect of measurement errors on the results of Stochastic Actor-Oriented Models. The patterns of real error are unknown, but the goal is to investigate two informed scenarios that cover a variety of potential error patterns. The first scenario focuses on various amounts of random error. The second one introduces a systematic type of error, that captures patterns of forgetting that are reported in the presented literature. Both scenarios focus mostly on introducing varying amounts of negative error, due to the emphasis on recall bias in the literature. Mathematical details of the scenarios are presented in the next chapter.

Previously, it was addressed that only tie inconsistencies recognized as measurement error by actors themselves are considered to be measurement error. This is captured in the study by Brewer & Webster (2000), who identified measurement errors by comparing recall and recognition of friends by respondents. It is therefore used as a base to inform relevant error rates in the random and systematic scenario.

3.6.1 Random Scenario

The random error scenario contains SAOM analyses on networks with various amounts of *random* negative error. These errors mimic the effects of random data entry errors, as well as recall errors, assuming they are introduced randomly. In essence, this random scenario serves as a simplified, baseline scenario that assesses simple random error. It is made more realistic in the systematic error scenario introduced in the next paragraph. It is likely that measurement error in network data lies somewhere on the scale between random and systematic error, and therefore these two scenarios together capture an informative range of potential effects of measurement error on SAOM outcomes.

Brewer & Webster (2000) reported that on average, 20% of friends tend to be forgotten. To capture the magnitude of this recall bias in the random scenario, the negative error rate in the random scenario is varied from 0% to 30% in steps of 5%. The negative error rates are introduced in combination with and without a fixed amount of positive error, to also capture the effect of, and potential interaction with, positive error. Positive errors are also introduced randomly and independently, and these mostly represent data entry errors.

In this study, the random errors refer to the false inclusion or exclusion of ties, and are unrelated to the random rewiring idea of Watts & Strogatz (1998). Ties are not randomly redirected to other actors in the network.

3.6.2 Embeddedness-based Scenario

The second error scenario is based on the results from the presented literature that some relationships are more like to be forgotten by respondents than others. As presented above, there is a large number of systematic effects that could be implemented in an error scenario. In this study, two important findings are combined into one systematic error scenario. It contains the effects that (i) stronger ties, in terms of emotional closeness, frequency of interaction and relationship duration, are less likely to be forgotten than weak ties (Brewer, 2000; Marin, 2004; Roth et al., 2021; Brewer & Webster, 2000), and (ii) people tend to recall social network information in triads (Brashears & Quintane, 2015). As presented before, these two effects are intertwined. People with stronger relationships tend to have a larger overlapping network neighbourhoods, causing stronger relations to be involved in a relatively large number of triads and clusters (Granovetter, 1973, 1983; Friedkin, 1980; Burt, 2000).

Together, these results are incorporated in a systematic error scenario where the probability of forgetting a tie is dependent on the embeddedness of a tie in the network structure. Ties that are embedded in relatively more triads, and thus reflect stronger relationships, are more likely to be rememberred by respondents than less-embedded ties. Given this relationship, this scenario again involves various combinations of error rates. The negative error rate is varied to represent various degrees of forgetting, with negative error-rates for the least-embedded ties ranging from 15% to 35%. This is approximately centered around the estimation of Brewer & Webster (2000) of 26% of regular friends being forgetting by respondents. Negative error rates are again introduced in combination with and without a set amount of randomly introduced positive error.

Chapter 4

Methods

To assess the effects of various amounts of systematic and random measurement errors on the outcomes of SAOMs, a simulation study is conducted. In this chapter, the specifics of this study are outlined. All analyses were conducted using R, version 4.4.0 (R Core Team, 2024). SAOM analyses were performed using the RSiena package (Ripley et al., 2024), version 1.4.7. The scripts for all analyses can be found through the GitHub link in Appendix A.

The set-up of this simulation study is based upon the set-up used by Huisman & Steglich (2008), who used a simulation study to assess the influence of missing data, and various missing data correction mechanisms, on the outcomes of SAOMs. The simulation study follows the following steps:

1. Determine a "true" SAOM model, based on existing data;
2. Generate new data based on the true model, and introduce measurement error in both the start and end network according to a pre-defined scenario;
3. Fit the SAOMs to the data with measurement errors;
4. Assess the effect of measurement errors on SAOM outcomes.

After a short introduction into the used social network terminology, each of these steps is further outlined in the paragraphs below.

4.1 Social Network Terminology

Before going into the details of the study setup, a short note regarding social network-related terminology is required to make the following sections, as well as the results, more comprehensible for the reader. In the rest of this thesis, multiple network attributes and dynamics are discussed, such as density (how many ties are in the network) or reciprocity (the extent to which actors form mutual connections). These network attributes are discussed in two different ways. Firstly, they are discussed on a *network level*. For example, the density on a network level is the number of ties there are in the full network, compared to the number of potential ties there could be if the network was fully connected. For reciprocity, this refers to the proportion of ties in the network that are mutual These measures are used as descriptive statistics for full network observations. On the other hand, these network attributes are considered as SAOM *effects* in the objective function, where they are treated as (potential) drivers of tie formation and dissolution. Here, the network attributes do not refer to the overall level of density or reciprocity in the network, but to the number of related subgraphs in the neighbourhood of a given actor. For example, for the density *effect*, the number of outgoing ties for that actor is counted. For reciprocity, it refers to the number of ties of an actor that are reciprocated. These effect values are therefore much more localized, and they are important to distinguish from the overall network indices. To avoid confusion between the two types of measures, the SAOM effects are always referred to as effects (e.g. the density *effect*).

4.2 Determining a "True" Model

In this section, as well as the subsequent sections, the set-up of the simulation study and the analysis of the results are described. This starts with the definition of a "true" SAOM model, which describes the network dynamics that are present in the simulated data. This study aims to mimic the introduction of errors into networks in a way that resembles errors in everyday research. Therefore, the true model is calibrated to observed social network data, and not based on simulated data. This way, the model and data contain the characteristics of real social networks. The data that are used to define the true model in this study, are the Glasgow *Teenage Friends and Lifestyle Data* (Michell & West, 1995).

4.2.1 The Data

The original aim of the data collection was to study the processes underlying attitudes towards smoking and smoking behaviour among adolescents. The data contain two cohorts, out of which the older one is included in this dataset, with 3 data collection waves and a total of 160 pupils. The collection of data started in 1995 and ended in 1997. At the first data collection wave, pupils were around 13 years old.

The network data in this study are friendship data. Pupils were asked to give the name and information of up to 6 friends. They also had to indicate what kind of friend each friend was: a "best friend", or "just a friend". In the first data collection wave there was the opportunity to ask for an extra questionnaire to include more than 6 friends, but this was hardly used by the pupils. Therefore, this option was discontinued in later data collection waves. The restriction of naming 6 friends is a notable limitation of this dataset. In addition to the network data, other information was gathered, including sex, age, usage of various kinds of substances, and lifestyle data. In this study, only the network data and sex variable are used.

To create the "true model" as the basis for the simulation study, the last two data collection waves were used to estimate a SAOM. Two waves were used instead of all three, because this limits the computing time of the simulation study and a SAOM model based on 2 network observations is sufficiently complex to yield relevant results. The last two waves were chosen because they contain a bit more stability in the relationships between the networks (Jaccard index = .340) than the first two waves (Jaccard index = .303). In addition, only the students that took part in both wave 2 and 3 were included in the model. This led to a total network size of 133 pupils. On average, the pupils were 13.32 years old on January 1st 1995 (SD = 0.301), and 43.61% of them were girls.

Basic descriptive statistics of both observed networks can be found in Table 4.1. For reference, visualizations of the two networks are contained in Appendix C. Between the two waves, the friendship network in this cohort has become more integrated and more connected. The number of relationships slightly increased, and relationships became more mutual (increased reciprocity) and more clustered (increased transitivity). While the network became more cohesive, with fewer components, the median distance between the nodes in the largest component became larger. The networks are very clearly

Table 4.1: Descriptive statistics of the wave 2 and 3 friendship networks of the Glasgow student life data.

	Density	Components	Transitivity	Reciprocity	Median distance (i)
Wave 2	.026	24	.308	.556	5
Wave 3	.028	22	.437	.635	6

Notes: (i) This variable refers to the median distance between actors in the largest components of the network.

separated by sex: 89% of the relations are between pupils of the same sex, and this remains constant over time. In general, there is a substantial amount of turnover in relations between the two networks. In total, 464 relationships changed between the two networks (Hamming distance = 464).

4.2.2 The "True" Model

Based on the two data collection waves, a Stochastic Actor-Oriented Model was fitted. The estimates of this model can be found in Table 4.2. The rate function only contains a simple rate parameter, assuming a constant rate of change across actors. The objective function contains the effects listed in the table. These particular effects were selected to have both a parsimonious and well-fitting model for these data. For an exploration of the fit of this model to the empirical data, the reader is referred to Appendix G. All included effects are also commonly used in SAOMs (Ripley et al., 2024), making it relevant to study how they behave under measurement error.

All parameters in the model are significant at $\alpha = .05$, and have an absolute convergence t-ratio below .1, which indicates the effects are well-converged. The overall maximum convergence ratio is .210, which is also below the convergence limit of .250. Actors had on average about 9 opportunities to make changes to the network between data Wave 2 and 3. This shows the network is quite dynamic. Overall, the parameters indicate actors are generally averse to having ties (Est. = -1.847, p < .001). At the same time, there are tendencies towards creating mutual relationships (*reciprocity*) (Est. = 2.620, p < .001), to becoming friends with the friens of one's friends (*transitivity*) (Est. = 2.379, p < .001), and creating ties to an actor of the same sex (*sex-based homophily*) (Est. = 0.581, p < .001). Actors do

Table 4.2: "True" SAOM results, based on wave 2 and 3 from the Glasgow teenage friends and lifestyle study.

	Est.	**SE**	**p**	**Convergence t-ratio**
Rate	9.228	1.074		0.003
Density	-1.847	0.255	<.001	0.029
Reciprocity	2.620	0.187	<.001	0.039
Transitivity (gwespFF)	2.379	0.200	<.001	0.065
3-cycles	0.514	0.130	<.001	0.064
Reciprocated triplets	-0.542	0.115	<.001	0.080
Indegree popularity	-0.228	0.039	<.001	-0.000
Indegree activity	-0.300	0.055	<.001	0.034
Homophily-Sex	0.581	0.115	<.001	0.057

Table 4.3: Descriptive statistics of wave 3 of the Glasgow student life data, and the average and SD (between brackets) of the statistics of the simulated end networks without error.

	Density	Components	Transitivity	Reciprocity	Median distance [i]
Wave 3	.028	22.00	.437	.635	6.00
Simulated networks	.028 (.001)	13.44 (4.09)	.426 (.028)	.635 (.027)	5.88 (0.50)

Notes: (i) This variable refers to the median distance between actors in the largest components of the network.

not tend to reciprocate ties within clusters (*reciprocated triplets*) (Est. = -0.542, $p < .001$), but do tend to form 3-cycles, where A forms a tie to B, B to C, and C back to A. In addition, actors with more ties tend to both send (Est. = -0.300, $p < .001$) and receive (Est. = -0.228, $p < .001$) fewer ties over time (*indgree activity and popularity*).

4.3 Generating New Data

To conduct the simulation study, a set of 1,000 network trajectories is generated to match the patterns outlined by the SAOM model estimated on the Glasgow data. This model is now the "true" model for these data. Each trajectory begins with Wave 2 of the Glasgow study as its initial state (the start network). Subsequently, new observations are generated to form the end network of each trajectory. The generation process of these end networks is equal to the simulation process used in the estimation of Stochastic Actor-Oriented Models. The values of the parameters in the "true model", as shown in table 4.2, are used to define an objective function and rate function. Based on these functions, the simulation process as described in Section 2.1.3 is used to simulate a new network state. By using the same SAOM parameters and starting network, these resulting network trajectories follow the same evolution patterns and rate of change as the original Waves 2 and 3 of the Glasgow data.

The two error scenarios introduced in Section 3.6 are applied to both the start and end network of each of the 1000 network trajectories. The mathematical details of the error scenarios are outlined in the subsequent sections.

4.3.1 Random Scenario

The first error scenario concerns errors that are introduced randomly into the network data. Here, negative random error refers to observed ties that are randomly excluded with a certain probability (negative error rate). Subsequently, positive random error refers to the random, false inclusion of non-existing ties with a set probability (positive error rate). The two types of error are introduced independently. The purpose of this scenario is to assess how well SAOMs deal with unsystematic errors.

To have a broad understanding of how various amounts of error influence the results, the effects on SAOM parameters are explored for various error rates. The literature mostly stresses the prevalence of false negative ties (Brewer, 2000), so this aspect is also explored in most depth in this study. The negative error rates p_n are varied from 0 to .30 in steps of .05. These rates include the recall bias results by Brewer & Webster (2000), who found that dormitory residents forget about 20% of their friends.

The effects of the varying negative error rate are assessed on their own, and in combination with a small amount of positive error. The possible values for the positive error rate p_p are 0, where no positive rate is considered, and .001. This error rate is chosen more arbitrarily, due to a lack of research on the occurence of positive error. It is a much smaller value than the negative error rates. Since the main focus of this study is to study the effects of negative error, only a small amount of positive error is considered. Its main aim is to shortly assess the potential efects of positive error, and its interaction with negative error.

Since the adjacency matrix of the considered networks is very sparse, the positive and negative error rates are not directly comparable. To illustrate the rates in terms of tie counts: the observed start network has 457 ties, and 17232 non-ties. In this networ, negative error rates ranging from 0.05 to 0.30 cause expected numbers of tie dissolutions ranging from 23 to 137. The positive error rate of 0.001 leads to an expected number of 17 extra ties. Overall, negative error rates will have larger effects on network density than can be "repaired" by positive reror.

Error Introduction

The error introduction procedure follows the steps outlined below. The steps are repeated for the start and end network in all 1,000 generated network trajectories, for each combination of negative and positive error rates. For every cell x_{ij} in adjacency matrix x, representing a tie from actor i to actor j where $i \neq j$, positive error rate p_p and negative error rate p_n:

1. If cell x_{ij} contains the value 1, there is an observed relationship between actor i and j. The value is replaced by a value 0, representing no relationship, with probability p_n;
2. If the value of cell x_{ij} is 0, there is no observed relationship between actor i and j. The value is replaced by value 1, representing a relationship, with probability p_p.

With 1,000 original network trajectories, 7 unique negative error rates p_n, and 2 unique positive error rates p_p, this leads to 14,000 unique network trajectories with varying amounts and types of error. These data form the basis for the random error scenario.

4.3.2 Embeddedness-based Scenario

The embeddedness-based scenario involves the introduction of error that is systematically related to the embeddedness of ties in the network structure. More specifically, the probability of forgetting a tie (the *negative* error rate) is related to the degree of embeddedness of a given tie in its local neighbourhood. Again, the overall negative error rates are varied, to capture various amounts of error. Since the negative error rates vary per tie, a given amount of overall negative error is characterized by its baseline negative error rate in the subsequent sections. This is the amount of negative error introduced when a tie is not embedded in the network (value 0), and it ranges from 0.165 to 0.368. When applied to the observed start network, this leads to a number of tie dissolutions ranging from 34 to 110. The range is thus smaller than in the random scenario.

Similar to the random error scenario, each baseline negative error rate is jointly assessed with and without a small amount of random positive error (a positive error rate of .001). Although there are indications that positive error may also be related to tie-embeddedness (Brashears & Quintane, 2015), systematic positive errors are not considered here. This is because much less research available to inform the magnitude and patterns of systematic positive error. To limit the scope of the study, positive errors are therefore kept to a small, fixed amount of random positive error.

Before introducing the mathematical details of the embeddedness-based scenario, it should be noted that there is not one best way to define tie-embeddedness and connect it to specific negative error rates. For that, there is not enough knowledge on the exact relation between friendship strength, tie embeddedness, and probability to forget ties. The approach introduced in the subsequent sections is just one way to introduce the overall shape of the desired error pattern into the network data; there are likely many other valid approaches that can achieve the same goal. However, the approach outlined here seems a reasonable one, and is informed by the results of Brewer & Webster (2000).

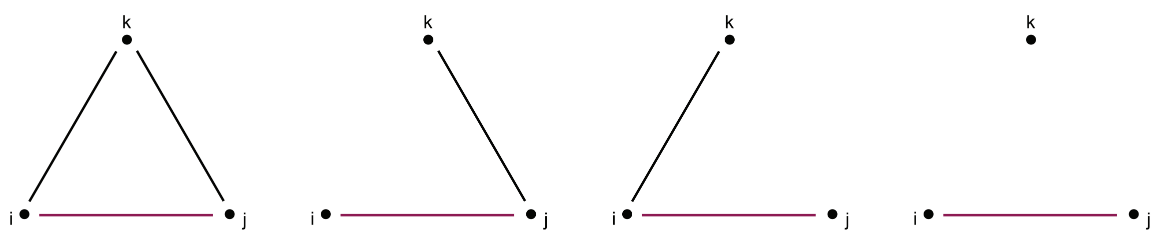
Definition of tie-embeddedness

To systematically introduce negative error into the data in the described manner, an operationalization of tie-embeddedness is required. Here, tie-embeddedness is defined as the proportion of closed triangles involving the edge in cell x_{ij}, compared to the total number of two-paths involving the edge in cell x_{ij}, given that $x_{ij} = 1$. To simplify the concept, the directionality of the ties is thus not considered. This means that the measure of tie-embeddedness is symmetric. The value for the edge in cell x_{ij} is the same for the edge in cell x_{ji} (if the edge exists).

Figure 4.5 contains a comprehensible characterization of the definitions of these triangle counts. When the directionality is not considered and we assume there is an edge x_{ij}, there are only four unique configurations that involve this edge. A is a closed triangle (it has an edge on all three sides), B and C are open two-paths (which could be closed by an additional tie), and D is un-embedded (it has no additional ties). The tie-embeddedness for tie x_{ij} is therefore calculated as the number of triplet configurations of type A it is involved in, proportional to the sum of triplet configurations of type A, B and C it is part of.

Mathematically, the tie-embeddedness C_{ij} is defined as the proportion of triangles (T_{ij}) compared to the total of open and closed two-paths (Q_{ij}). Its value can only be calculated for existing edges ($x_{ij} = 1$); it is undefined when no edge exists between actor i and j. Isolated edges, for which there are no potential triangles, get the value 0:

$$C_{ij} = \begin{cases} \frac{T_{ij}}{Q_{ij}} & \text{if } x_{ij} = 1 \\ 0 & \text{if } Q_{ij} = 0 \\ \perp & \text{if } x_{ij} = 0. \end{cases} \tag{4.1}$$



A: Triangle/closed two-path **B:** Open two-path **C:** Open two-path **D:** Not embedded

Figure 4.5: Potential triplet configurations for a tie $x_{ij} = 1$.

Here, the number of triangles (T_{ij}) is equal to the number of nodes that both actor i and j have a tie to:

$$T_{ij} = \sum_k x_{ik} \cdot x_{jk} \tag{4.2}$$

The total of closed and open two-paths (Q_{ij}) is equal to the total sum of unique neighbours of actors i and j, excluding actors i and j themselves. Every unique neighbour has a tie from at least one of the actors, and thus has the potential to form a triangle with one added tie, if the added tie does not exist yet. Mathematically:

$$Q_{ij} = \sum_{k \neq j} x_{ik} + \sum_{k \neq i} x_{jk} - \sum_k x_{ik} \cdot x_{ij} \tag{4.3}$$

As introduced earlier, it is expected that tie-embeddedness and relationship strength are intertwined. Stronger ties tend to be more strongly embedded in the network structure than weaker ties (Granovetter, 1973, 1983; Friedkin, 1980; Burt, 2000). The Glasgow data contain classifications of relationships types, and can thus be used to assess this. For each friend, students indicated whether it concerned a "best friend", or "just a friend". The tie-embeddedness is higher for best friends ($\overline{C}_{ij}$ = .343, SD = 0.226), than for friends that are considered "just a friend" ($\overline{C}_{ij}$ = .264, SD = 0.220). For ties in Wave 2 and 3 of the data A non-parametric permutation test shows that this result is statistically significant ($\overline{C}_{ij}^{\text{perm}}$ = .079, p < .001). The constructed tie-embeddedness measure is thus indeed related to relationship strength, which contributes to the validity of the operationalization.

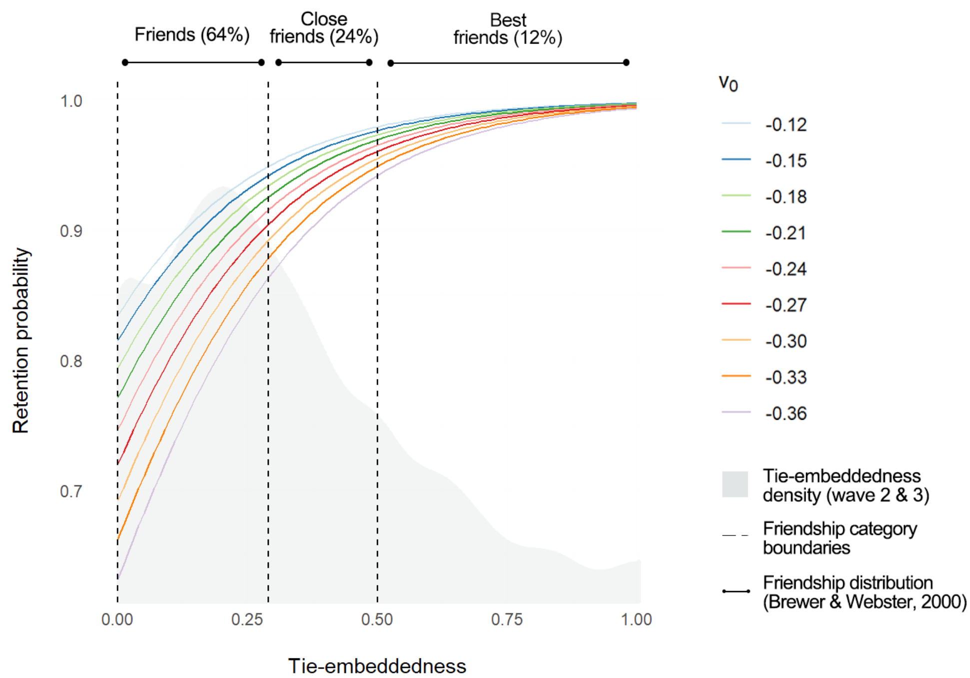


Figure 4.6: Sigmoid functions with varying midway points v_0 that model tie-embeddedness and tie retention probability p_r. The gray area shows the shape of the density of tie-embeddedness values in wave 2 and 3 of the Glasgow study.

Tie-embeddedness and Negative Error Probability

The tie-embeddedness values are related to a corresponding negative error probability through the parameterized sigmoid function

$$p_r(C_{ij}) = \frac{1}{1 + e^{-4.5(C_{ij} - v_0)}}. \tag{4.4}$$

The function models the opposite of the negative error probability: the retention probability (p_r = 1 - p_n). Sigmoid functions allow for a smooth, nonlinear transition from lower to higher retention probabilities. They can be modelled to grow with tie-embeddedness, but with diminishing increments at higher values of tie-embeddedness, due to the sigmoid's natural s-shape. In addition, its output range (0 to 1) matches the intended probability scale. The parameterized version with midpoint v_0 and steepness parameter l gives control over the exact shape of the relation.

The version used in the simulation study sets the steepness parameter l = 4.5, and varies the midway point v_0 in 9 steps from -0.12 to -0.36 to create various error levels. The curves corresponding to these values are displayed in Figure 4.6. The parameter values are informed by the results of Brewer & Webster (2000). Their findings contain information on the distribution of friendship ties over various friendship strength categories, and the corresponding recall probabilities. They are approximated by the sigmoid function with steepness parameter l = 4.5 and v_0 = -0.24. In the other functions, this curve is moved left and right to investigate varying amounts of error. This leads to baseline negative error probabilities ranging from 16.5% ($v_0 = -0.36$) to 36.8% ($v_0 = -0.12$), which form a range around the estimation by Brewer & Webster (2000) of 26%. The baseline negative error refers to the introduced error where the tie-embeddedness value is 0. The steepness parameter is kept constant at l = 4.5, so the retention probability (and thus negative error probability) increases similarly with the tie-embeddedness values in every function.

To align the friendship categories to the tie-embeddedness values, the distribution of ties across strength categories from Brewer & Webster (2000) is mapped onto the tie-embeddedness values distribution observed in the Wave 2 and 3 Glasgow networks. By applying the percentages of ties within each strength category as quantiles to the tie-embeddedness distribution, each friendship category is assigned corresponding lower and upper bounds in terms of tie-embeddedness. The observed recall probability for this friendship category is assigned to the lower tie-embeddedness bound. For example, the category with regular friends contains 64% of the observed friendships, of which 74% is recalled. In the tie-embeddedness distribution, this friendship category corresponds to a lower bound of .00 (quantile = .00), and an upper bound of .29 (quantile = .64). The recall probability of .74 is connected to the lower bound. The sigmoid function with v_0 = -0.24 gives a retention probability of .746 for a tie-embeddedness value of 0. The values for all categories can be found in Table 4.4.

Figure 4.6 also contains the shape of the tie-embeddedness density from Wave 2 and 3 of the Glasgow study. It gives an idea of how many ties fall into which friendship strength categories, and what the overall retention probability is. The distribution is left-skewed: most ties have relatively low tie-embeddness values, and therefore lower retention probabilities.

Error Introduction

The error introduction procedure follows the steps below. The steps are repeated for the start and end network in all 1,000 generated network trajectories, for each combination of baseline negative error rates and positive error rates. For every cell x_{ij} in adjacency matrix x, representing a tie from actor i to actor j where $i \neq j$, positive error rate p_p and sigmoid midway point parameter v_0:

1. If cell x_{ij} contains the value 1, there is an observed relationship between actor i and j. Calculate the probability to replace it with 0 using the following steps:
   (a) Calculate the tie-embeddedness C_{ij} of the tie using Equation 4.1;
   (b) Calculate the probability to retain the tie p_r using Equation 4.4 using midway point parameter v_0;
   (c) Set the negative error probability $p_n = 1 - p_r$.
2. Replace the value in cell x_ij with 0, with probability p_n;
3. If the value of cell x_{ij} is 0, there is no observed relationship between actor i and j. The value is replaced by value 1, representing a relationship, with probability p_p.

Negative Baseline Error

Since the negative error in this scenario is not constant but dependent on tie-embeddedness, there is not one negative error rate that identifies the complete error introduction process. However, to identify the effect of the amount of error on the outcomes of SAOMs, a measure is needed that is closely related to the overall level of error introduction. For this, the *negative baseline error* that was introduced before is used. This is quantified as the value of each sigmoid curve at the tie-embeddedness value 0, and can be interpreted as the probability of removing a tie that is not embedded in the network at all. Since the sigmoid curves used for error introduction do not vary in steepness but only in midway point, the negative baseline error is an identifying feature of each unique sigmoid curve. It is clear and intuitive, and closely related to the overall error introduction. Its partial correlation to the total error rate in the network (the proportion of tie status changes in a simulated dataset that is unequal to the tie status changes in the observed data) is large (ρ = 0.973, p <0.001). This correlation value is controlled for the positie error rate. The total error rate in the network in itself is not used as an identifying value, since it is also affected by the positive error rate and likely differs for start and end networks.

While the baseline negative error rate is an intuitive and practical way to identify the amount of error, it should be noted that they are not directly comparable to the random error rates in the random scenario. To make comparisons between the two type of error patterns and their effects on SAOM outcomes, negative (baseline) rates that lead to start networks with comparable density are used. Comparable densities show that the subsequent total error rates are similar, and indicate that the corresponding error combinations can be used to compare the effect of random error to systematic error.

Table 4.4: Overview of the observed properties of friendship strength categories by Brewer & Webster (2000), the defined tie-embeddedness boundaries of the friendship categories, and the modelled retention probabilities using Equation 4.4 with v_0 = -0.24.

Friendship category (i)	Observed % of friendships (i)	Cumulative proportion (i)	Tie-embeddedness (Lower bound) (ii)	Observed recall (i)	Modelled retention probability (Lower bound)
Friend	.64	.64	.00	.740	.746
Close Friend	.24	.88	.29	.910	.916
Best Friend	.12	1.00	.50	.970	.965

Notes: (i) Columns refer to the results from Brewer & Webster (2000). (ii) Calculated by applying the cumulative proportions as quantiles to the tie-embeddedness distribution of the observed Wave 2 & 3 networks from the Glasgow data.

4.4 Re-estimation and Convergence

After error is introduced into the simulated network data, as described in the error scenarios, a new SAOM model is estimated for each network trajectory. The re-estimated SAOM models estimate the same effects and use the same settings as the original, true model. For each error combination, this leads to 1000 new estimated models with SAOM parameters and convergence statistics. Here, an error combination refers to the unique combination of a (baseline) negative error rate and positive error rate within an error scenario. The error introduction and re-estimation of the SAOM models are performed on the Hábrók supercomputer of the University of Groningen, due to the high computational demands.

The results of these 1000 new analyses are assessed for problems with convergence, since results from non-converged solutions can be misleading. A run does not converge when the average values of the sensitive statistics of the simulated networks are too far from the observed values. For each run, convergence is assessed according to the convergence requirements outlined in section 2.3. Based on these convergence criteria, the overall proportion of non-converging runs was rather high; it fluctuated around 35% across error combinations. Subsequently, all runs that did not converge the first time were run one more time. In these reruns, the results of the initial, non-converged run were used as a starting position. About 87% of these reruns reached convergence. This lead to a total number of converged runs ranging between 930 and 960, in both the random and systematic scenarios. In total, the random scenario has 13,289 converged runs, and the systematic scenario has 18,981 converged runs. The latter number is higher due to the larger number of unique error combinations. In further analyses of network statistics and SAOM parameters, the final non-converged runs are discarded. In Appendix B, it is assessed whether the amount and type of introduced error substantially influenced the probability of both initial and rerun convergence.

4.5 Analysis

To determine the effect of measurement errors on the SAOM results, the converged runs are inspected further. The effects on both the descriptive statistics of the network trajectories and the SAOM parameters are assessed.

4.5.1 Network Trajectory Comparison

The comparison of the error-induced network trajectories and error-free network trajectories is performed by comparing the descriptive statistics of the start and end networks. This includes the density, number of components, transitivity index, reciprocity index, and median distance between actors in the largest component. The change between the start and end networks is also assessed with and without error, using the Jaccard index and Hamming distance.

4.5.2 SAOM Parameters

For each error combination, the SAOM parameters are compared to the true parameters in the original model. The purpose of the analysis is to assess if the parameters estimated from error-induced networks deviate from the real parameters in a meaningful way. Deviations are meaningfull and practically relevant when the difference is large enough to seriously influence conclusions that researchers would draw based on the SAOM parameters.

SAOM parameters in itself are diffcult to interpret in a meaningfull way. They are unstandardized coefficients on a logit scale, and are thus not linearly related to probabilities of network states. Marginal effect-interpretations are currently not possible for Stochastic Actor-Oriented Models. As described by Indlekofer & Brandes (2013), drawing general conclusions on a probability- or odds-ratio-level is problematic. Probabilities are strongly dependent on the local network structure, and are dependent on the set of alternatives that is considered. To meaningfully interpret effect sizes, the full probability distribution needs to be considered. Results based on odds-ratios can be problematic due to correlations of effects, and different scales of SAOM effects. Because of this, researchers often limit conclusions to the sign of the effect, and its significance, without considering effect sizes.

The complexity of SAOM parameter interpretations also influences the results and interpretation of the analysis of this simulation study. To determine whether network error affects conclusions, the first step is to assess the effect of various error amounts and types on the power of SAOM parameters. The power statistic assesses whether the introduced error affects the probability of a type II error: it is the proportion of runs in which a given SAOM effect was found to be significant (given that the null hypothesis is true - which is the case for all effects in the true model). It is also relevant to assess if the introduction of error leads to sign changes, and thus reverses the directionality of the effect. As long as there are no accessible options to calculate marginal effects for SAOMs, these two results together are sufficient to determine whether measurement error affects conclusions about network dynamics for a substantial part of SAOM research.

Although there is no straightforward relation between SAOM parameters and the importance of their effects in network dynamics, the size of SAOM parameters remains of interest. This study therefore also aims to quantify how much bias measurement error causes in SAOM parameters. However, due to the complexity of the parameters, the reported results remain *on a logit scale*. Differences on a logit scale can translate in various ways to a probability scale - small logit differences can become very large or very small on a probability scale and vice versa. Although bias directions will be informative, the bias is not sufficient to draw fully reliable conclusions about the magnitudítude of the effect of measurement error on parameters. In addition, it has to be considered that the scales of SAOM statistics are different. A parameter bias for an effect with a very small scale will be less pronounced than the same bias in an effect with a larger scale. To aid interpretations, an idea of these scales is given by the *mean SAOM counts* reported in Table 4.5. These are the total subgraph counts in the observed end network (the observed sensitive statistics) averaged over the number of actors. Unfortunately, this does not give a full range or distribution of subgraph counts per parameter, but it does give an idea of the magnitude of the scale. As shown in the table, the subgraph counts for indegree-related parameters tend to be higher, while those for the 3-Cycles effect are lower.

A couple of statistics are used to numerically assess the parameter estimates, and to study how they differ from the true parameter. Because the SAOM parameters are unstandardized, the magnitude of the bias is assessed using the relative bias statistic: the bias as a percentage of the true parameter value. Relative bias under 5% are considered small, values between and 5 and 15% are considered moderate, and values exceeding 15% indicate a large bias. In addition, the coverage statistic is used to gives more detailed information on theaccuracy of conclusions drawn from the sample data. It is the proportion of sample 95%-confidence intervals that include the true parameters. Lastly, the standardized variance gives insight into the effect of errors on the variability in estimates. It is the variance of the parameter estimates across simulation runs. The values are standardized per parameter to make the values comparable across parameters.

Linear regression models are used to determine the effect of negative and positive error and their

Table 4.5: The mean SAOM effect counts for the observed end network of the Glasgow student data per SAOM parameter. They are the target sensitive statistics in the real SAOM model averaged over the actors.

	Mean SAOM effect count
Density	3.647
Reciprocity	2.316
Reciprocated triplets	3.925
3-cycles	1.571
Transitivity (gwespFF)	3.780
Indegree popularity	16.368
Indegree activity	14.902
Homophily-Sex	3.278

interaction on the relative bias for each parameter. Since this entails a total of 72 significance tests with the same null hypothesis, the p-values are adjusted for multiple testing using a Holm correction. It should be noted that the sample sizes used for these analyses are extremely large (around 13,000 and 19,000), meaning that even the smallest effects become significant at $\alpha = 0.5$. Statistical significance of the effects of the effects is reported, but it should be emphasized that this statistical signifance does not equate practical significance. Although relative bias on a logit scale cannot fully describe the effect sizes on a probability level, it remains the main criterium to identify relevant effects.

Based on the data, attempts are made to explain the biases found for the various parameters. Due to the inherent complexity of SAOM parameters it was infeasable to set up concrete hypotheses regarding these effects beforehand. The results give more insight into this complexity, and can therefore guide some possible explanations. These explanations can serve as input for hypotheses for further research into the topic of measurement error in SAOMs. However, it should be noted that these explanations are based on results on a logit scale, and may therefore partly be misguided. The interpretation of the results is aided using triad census plots (see Appendix E), which show the average, standardized triad counts are calculated for each error combination - both for the start and end network. These plots give insight into the dependence of various triplet configurations on error rates, for both the start and end network. Triad counts do not exactly match the subgraph counts used in SAOM effects, but they can help understand differences in subgraph counts across error combinations.

Chapter 5

Results

The results of the simulation studies are outlined below for each error scenario. The discussed results include the effect of error on the overall network statistics, and on the parameters of the Stochastic Actor-Oriented Models.

5.1 Random Errors

The first scenario in this simulation study concerns the introduction of random errors. Both negative and positive measurement errors were included in varying amounts, and all errors were introduced independently into both the start and end network of the SAOM trajectory. The negative error rates that were considered range from 0 to .30, with steps of .05. These negative error rates were applied in two different scenarios: without positive error, and with positive error. When positive error is added, non-existing ties are turned into ties with probability .001. The effects of the errors on one of the simulated end networks are visualized in Appendix C.

5.1.1 Network Statistics

Table 5.1 shows the basic network statistics of the start and end networks of simulated SAOM trajectories for two different error scenarios. To highlight the differences caused by the introduction of errors, the scenario without errors is also displayed. The statistics are averaged over all converged runs in the corresponding error scenario. The trends of the statistics across all error combinations can also be viewed in graphs in Appendix E.

The introduction of a substantial amount of random negative error leads to a decrease in density. This is a direct result of the negative error: removing ties simply leads to fewer ties. In addition, more errors lead to increased disconnectivity in the networks. The original components are split up due to the random removal of ties, leading to a larger number of components. It also leads to larger median geodesic distances in the largest network component: actors need more steps to reach other actors. Furthermore, the number of transitive clusters and reciprocated ties decreases due to the introduction of negative error.

The change between the start and end network is also affected. The Jaccard index decreases with the introduction of negative errors, which indicates that the networks are on average less stable over time. The start and end states of the network have fewer common ties. At the same time, the Hamming distance decreases, which indicates that fewer tie changes are needed to transfer from the start to

Table 5.1: Average network statistics for a selection of random error scenarios. Both the mean and SD (between brackets) for the statistics across simulation runs are shown. Statistics related to change across networks are only displayed for the end network.

	Density	Components	Transitivity index	Reciprocity index	Median distance (ii)	Hamming distance	Jaccard index	N
Start network								
No error (i)	.026	24.0	.308	.556	5.0			
Pos = .000, Neg = .200	.021 (.001)	33.9 (3.3)	.246 (.016)	.445 (.023)	5.9 (0.7)			954
Pos = .001, Neg = .200	.022 (.001)	28.9 (3.7)	.225 (.015)	.425 (.023)	5.6 (0.5)			946
End network								
No error	.028 (.001)	13.4 (4.1)	.425 (.028)	.635 (.028)	5.9 (0.5)	464.0 (0) (iii)	.341 (.008)	953
Pos = .000, Neg = .200	.022 (.001)	24.4 (6.3)	.341 (.029)	.508 (.032)	6.8 (0.8)	447.7 (11.6)	.256 (.014)	954
Pos = .001, Neg = .200	.023 (.001)	19.8 (5.4)	.313 (.027)	.488 (.031)	6.1 (0.5)	481.2 (13.1)	.243 (.014)	946

Notes: (i) The start network is the same for every simulated run with no error. Therefore, there are no standard deviations. (ii) This variable refers to the median distance between actors in the largest component of the network. (iii) When simulating the new end networks based on the true model, simulations are terminated when the observed Hamming distance is reached. The Hamming distance is therefore always the same for simulated end networks with no error, leading to a SD of 0.

the end state of the network. This is because the Jaccard index of the original network trajectory is below 50%: fewer than 50% of the ties are stable across both network observations. This means that a uniformly applied random negative error rate affects a larger number of "non-stable" ties, which are only observed in one of the networks. These are now observed as never existing, decreasing the overall change across the network.

The introduction of a few false positive edges mitigates some of the effects caused by the negative error. The density decreases less strongly when some extra ties are randomly introduced in the networks. It also makes the increasing disconnectedness in the networks less severe. The number of components and the median distance between actors increase less quickly with the amount of negative error, when positive error is also introduced. This is a display of the shortcut effect of randomly introduced ties, introduced by Watts & Strogatz (1998). Randomly added edges create "shortcuts" between distant parts of a network, which can strongly decrease the distance between actors. This is also commonly known as the *small-world phenomenon*.

However, the introduction of false positive random error does not mitigate the effect of the negative error on the reciprocity and transitivity indices. It does the opposite - the introduction of positive error makes the decreases in these indices stronger. The random introduction of edges does not necessarily build up the specific network structures that were broken down by the random removal of edges. In addition, it creates more "opportunities" for those network structures that are not fulfilled. For example, a newly introduced edge creates the option for another reciprocated tie, but it is unlikely that it will be reciprocated with another false positive edge. After all, the probability for any non-tie to become a tie is only .001. The fraction of the remaining instances of the network structures are therefore divided by a larger number of opportunities for these structures, leading to a stronger decrease of reciprocity and transitivity indices.

The change between networks is also affected by positive error introduction. The inclusion of extra edges introduces more unstructured change in the network, leading to an increase in Hamming distance. At the same time, extra random edges further disrupt the network stability. The extra "noise" leads to fewer common edges in start and end networks, and thus lower Jaccard indices.

5.1.2 SAOM Parameters

The interpretation of the error effects on SAOM parameters is twofold. First, the effect of error on conclusions purely based on p-values and signs are assessed. The results show that no amount of positive and/or negative random error seriously affects the statistical power of the effects. Almost all power statistics, for all error combinations and all parameters, are 100%, with some dropping to 99%. The lowest power values are 91% and 93% for the 3-cycles effect, for the error combinations without negative error. Even the lowest power values remain high. This indicates that the errors do not have an effect on the probability of finding a significant effect, given that this effect actually exists in reality. In addition, no amount or kind of random error leads to reversals of directions of SAOM parameters. For each combination of error rates, there are 0 runs where the sign of the estimated parameter is reversed compared to the true value. Parameters are overestimated (coefficients tend away from 0) or are underestimated (coefficients tend towards 0), but the effects are never reversed. Therefore, as long as researchers *only* determine the effect of SAOM parameters by their sign and direction, and do not consider the numerical value of the parameter, the considered measurement error will have no effect at all on conclusions.

However, many parameter sizes are significantly and substantially affected by positive and negative random error. In the remainder of this paragraph, the effects of measurement error on the parameter sizes of each SAOM efect are discussed. In Table 5.2, the effects of negative error, positive error, and their interaction on the relative bias of each SAOM parameter are assessed using linear regression models. These models are used to assess the effects of errors in the paragraphs below. Due to the inclusion of interaction effects and the centering of negative error, positive error effects are assessed for average negative error values (p_n = 0.150). The observed effects of errors on the relative bias, coverage, and standardized variance of each SAOM parameter are also visualized in Figure 5.1. Appendix F contains additional plots as further illustrations of the results. This includes plots with the percentile rank of the true parameter value in the estimate distributions by the amount of introduced negative error. It also contains density plots of the estimates of all effects for different negative error rates, to further illustrate the bias. The reader is reminded that all results presented below are on a logit scale. Small deviations on this scale could translate to large deviations on a probability scale, and vice versa.

Overall, the results show that for most parameters, negative error leads to overestimations of parameter coefficients, while a small amount of positive error leads to underestimations - although there are some exceptions. The exact effects and their sizes are parameter-specific, and may be influenced by the exact network properties and parameter selection in this particular model. Almost all parameters are significantly biased by negative error, positive error, and sometimes their interaction. In addition, it is a common pattern across parameters that the variance in the estimated parameters in the simulated data increases with negative error. The only exceptions for this are the rate and density effects. This can also be seen in Figures 5.1e and 5.1f. Below, the effects of errors on each parameter are outlined in more detail.

Rate Parameter

The rate parameter is an estimation of how many opportunities actors have, on average, to make a change to their social relationships between the observed network states. In the "true" model, the estimated parameter is 9.228. The negative (β = -0.397, p < .001) and positive (β = 0.095, p < .001) random error both have a significant effect on the estimated values of the rate parameter. With every 10p.p. (percentage point) increase in negative error rate, the parameter is underestimated with about 4p.p. - given that there is no positive error. This means that actors tend to have fewer opportunities to make network changes when there is more negative measurement error. The underestimation remains small up to 10% error, but becomes moderate in size with more error. This linear effect of negative error can also be seen in Figures 5.1a and 5.1b. As can be seen in Figure 5.1c, the coverage remains around 95% up to 15% negative error. However, with more negative errors this decreases substantially, with only 65.6% of runs having a 95%-confidence interval covering the true value at 30% negative error.

At an average amount of negative error, the introduction of a small amount of positive error (0.1%) leads to a moderate overestimation of the rate parameter of 9.5%: actors have on average more opportunities to make changes to their network. The positive and negative errors thus have opposite effects on parameter bias. With both positive and negative error in the network, around 20-25% negative error cancels out the positive error. Because of these counteracting effects, the average estimates remain closer to the true parameter value. The true value is now more often covered in the 95%-confidence intervals: coverage values all remain above 95%. There is no evidence for an interaction effect between the positive and negative error.

Larger amounts of negative error may thus lead to underestimations of the rate parameter. This is a directly related to the effects of error on the Hamming distance - the sensitive statistic that is used to estimate the rate parameter. As described in Section 5.1.1, the Hamming decreased by negative error, and increased by positive error. This directly translates to bias in the rate parameter.

Density Effect

The density effect reflects the tendency of actors to have ties. In the true model, actors are generally averse to this, with a true parameter value of -1.847. Both random negative and positive error significantly affect the density parameter (β = 0.606, p < .001; β = -0.063, p < .001). In a scenario with no positive error, the introduction of more negative errors leads to an overestimation of this statistic. With a 10p.p. increase in negative error, the density parameter is overestimated with an extra 6p.p on a logit scale, given that there is no positive error. This indicates that the effect of random negative error on the density parameter is small up to about 10% error, but becomes moderate in size for larger error rates. It ranges up to a 16% overestimation at 30% error.

This bias is large enough that the true value falls more often outside the bounds of 95%-confidence intervals than would be expected. Up until 10% negative error, the coverage remains above 95%. With more error, the coverage decreases further, with 75.6% of 95%-confidence intervals covering the true value at 30% error. The introduction of some positive error (0.1%) leads to a similar effect as seen for the rate parameter - it counteracts the bias caused by negative error (β = 0.063, p < .001). With an average amount of negative error, the random introduction of extra ties leads to an underestimation of 6.3% of the density parameter: actors are estimated to be less averse to having ties, than in the true model. The positive effect cancels out the negative error around 15% negative error. Due to the counteracting effects, the coverage remains higher than in the scenario with no positive error. Up until 15% negative error, the coverage remains above 95%, and at 30% negative error the coverage is 87.3%. Furthermore, there is no indication of an interaction effect between positive and negative error.

The absolute value of the true SAOM parameter is one of the larger ones in the model. The mean subgraph count of this parameter of 3.6 is average for the parameters. Together, these two aspects could lead to a larger bias in the objective function than described here on a logit scale.

Since the density effect is directly related to the number of ties in the network, and thus to the random removal and addition of ties, it is not surprising the introduction of negative error causes bias. The absolute number of ties in both networks is lower, strengthening the aversion of actors to have ties. Secondly, the density parameter serves as a type of "intercept" in the SAOM model, setting a baseline tendency to have ties, given the tendencies created by the other effects. Since other effects, such as reciprocity and the degree-related effects, are not centered, their magnitude affects the intercept. A couple of other effects, discussed later on, are overestimated due to the introduction of negative random error. Since tendencies to have ties in the direction of these structures are strengthened, the density effect "balances" these overestimations out with an underestimation of the general tendency to have ties. This way, disproportionate increases in network density, above the level of the end network, are avoided. Strong biases in the other included SAOM parameters, caused by negative measurement error, therefore contributed to the overestimation of the density parameter.

This "intercept-effect" of the density effect can also explain why positive random error leads to the underestimation of the density effect, and thus counteracts the effect of negative error. Positive error reintroduces random noise around the network structures that are overestimated due to negative error. This decreases the focus on ties that form specific network structures, and leads to less overestimation of the corersponding SAOM parameters. To keep the density at the appropriate level, the density effect accounts for a larger number of "random" ties, and is therefore underestimated.

Reciprocity Effect

The reciprocity parameter indicates to what extent actors tend to reciprocate ties that other actors send to them. The true parameter is 2.620, which indicates that actors are inclined to reciprocate ties. Both random negative error (β = -0.141, p $<$ .001) and positive error (β = -0.073, p $<$ .001) significantly affect the bias in this parameter. When there is no positive error, a 10p.p. increase in negative error leads to an underestimation of 1.4p.p. in the reciprocity parameter on a logit scale. The largest underestimation is 3.3% at 30% error, indicating the bias is small. The coverage also remains around 95%, no matter the amount of negative error. The SAOM reciprocity parameter seems to be relatively resistant to random negative error.

The introduction of positive error has a substantially larger influence on the estimations of the reciprocity effect. With average negative error and 0.1% positive error, the positive error causes an underestimation of 7.3% on the logit scale; an underestimation moderate in size. It is also likely that the true value will often not be captured by 95%-confidence intervals when positive error is present. With positive error, the coverage is consistently below 95%. With no negative error, the coverage it is 85%, and with 30% negative error this decreases to 69.6%.

In the objective function, the reciprocity parameter is the largest in absolute size. This is partly counterbalanced by the smaller scale of the subgraph count. The average subgraph count for reciprocity in the observed end network is 2.316. Taking these two together, reciprocity may have a slightly larger effect on the objective function than was found on the logit scale.

It is a noteworthy result that the reciprocity parameter remains relatively stable under random negative errors. The reciprocity index decreases for both networks with the introduction of negative measurement errors. However, it seems that the tendency towards reciprocity from the start to the end networks remains relatively stable. At the same time, random positive error leads to a moderate underestimation of the parameter. The addition of random edges is likely to increase the total number of unreciprocated ties. Due to the low density of the network, the probability of connecting two previously unconnected actors is much higher than reciprocating an existing tie. This additional noise makes reciprocated ties relatively less important in the objective function, leading to an underestimation of the effect.

Triadic Effects

The model includes three effects related to triplets: the transitivity effect (gwespFF), the 3-cycle effect, and the reciprocated triplets effect. All parameters focus on capturing patterns related to triadic closure in the network, but they each highlight a different aspect.

The transitivity parameter, in particular the GWESP variation, indicates to what extent actors tend to become friends with the friends of their friends. The gwespFF variant is geometrically weighted. This refers to the aspect that the extra effect of an additional transitive subgraph diminishes with the number of shared friends. The true value of this parameter is 2.394, which means that actors do tend to become friends with the friends of their friends. The estimated value is significantly influenced by negative ($\beta = 0.507$, $p < .001$) and positive ($\beta = -0.073$, $p < .001$) error. In the case without positive error, more negative error leads to an overestimation of the gwespFF parameter. With each 10p.p. negative error, the parameter is overestimated with about 5p.p. on a logit scale. This amounts to a moderate overestimation of 15.2% at 30% random error, while the bias remains small in size up to 10% error. The bias also affects the coverage: when there is more than 5% negative error, the coverage drops below 95%. With 30% error, only 64.94% of the simulated runs include the true parameter.

The introduction of a small amount of positive error also leads to a moderate bias in the transitivity parameter. At average negative error, the positive error leads to an underestimation of 9.3% on the logit scale. This is again a positive error-bias that works in the opposite direction of the negative-error bias. Around 20% negative error balances the positive error out. The coverage for this scenario increases with the amount of negative error introduced: with no negative error the coverage is 78.5%, with 20% error this increases to 93.3%, but at 30% error this decreases back to 91.8%.

The two other triadic effects are also overestimated when negative errors are introduced, but both the 3-cycle and the reciprocated triplets effects respond much more strongly on a logit scale. The 3-cycle effect reflects the tendency of actors to create relations in circles; if actor *a* has a relationship with actor *b*, and actor *b* with actor *c*, actor *c* also tends to create a relationship with actor *a*. In the true model, the 3-cycles parameter has a value of 0.514, indicating that actors tend to form these patterns. The reciprocated triplets effect corresponds to the interaction between reciprocity and transitivity. It reflects the tendency of actors to form reciprocated ties within transitive triplets. The value of the true parameter is -0.542, which means that actors tend to be averse to reciprocating ties within transitive triplets.

The introduction of negative error leads to significant (β = 4.810, p < .001; β = 5.189, p < .001) and very substantial overestimations of both the 3-cycles and reciprocated triplets effects. Each 10p.p. increase in random negative error leads to parameter overestimations of 48.1p.p. for the 3-cycles effect, and 51.9p.p for the reciprocated triplets effect. At 30% negative error, this amounts to parameter overestimations of 149.3% and 157.3%. This is also reflected in the coverage statistic. Even with as little as 5% negative error, the true value is covered in only 81% and 74% of the 95%-confidence intervals. This is already substantially lower than the 95% one would expect. With negative error rates of 10% and larger, the coverage drops below 50% for both parameters, and they tend to 0% as the negative error rate increases. With the presence of random negative error, it is thus therefore very difficult to correctly draw conclusions about the size of the 3-cycle and the reciprocated triplets efects. They are very likely to be overestimated.

The introduction of random positive error also significantly (β = -0.131, p < .001; β = -0.224, p

< .001) influences the 3-cycle and reciprocated triplets effects. With average negative error, this error leads to underestimations of respectively 13.1% and 22.4% of the true parameter on a logit scale. These effects are moderate and large in size, and have the opposite effect compared to the negative error. However, the overestimations caused by negative error are far larger than the underestimations caused by positive error, meaning that the positive error does not have much of a compensating effect in itself. The positive error with no negative error leads to coverage values of 93.7% and 87.0%, but this quickly decreases and approaches 0 with more negative error.

Random positive error does also significantly alter the effect of negative error on relative bias for both parameters (β = -0.372, p < .001; β = -0.424, p < .001). It not only counteracts, but also diminishes the magnitude of the bias caused by negative error. When positive error is added, the effect of negative error on the SAOM parameter is diminished with 3.72p.p. per 10p.p. negative error for the 3-cycle effect, and 4.24p.p. per 10p.p. negative error for the reciprocated triplets effect. At 20-30% negative error, this interaction effect is of moderate size, and therefore a noteworthy finding.

It should be noted that the absolute coefficient sizes as well as the parameter scales differ across the triadic parameters, and thus likely differ in their effect on the objective function. The transitivity effect (gwespFF) has a large coefficient and also has a moderately large scale - the mean subgraph count in the observed end network is 3.780. These two factors mean that this parameter, as well as as its bias, are likely to have a relatively large impact on the objective function. The bias that is here measured on a logit scale may therefore be translated to an even larger effect within the objective function, and potentially (but not certainly) on the resulting probabilities of network states. On the other hand, the large overestimation of the 3-cycle effect may have weaker implications in the objective function, since both its scale (mean SAOM subgraph count of 1.571) and its parameter are relatively small. The reciprocated triplets effect resides somewhere inbetween. It has a larger scale (mean SAOM subgarph count of 3.925), but a small parameter.

A potential explanation for the overestimation of these three triadic effects is that the remaining instances of each triadic effect become relatively more important in the objective function. The introduction of random negative error leads to a decrease in the total number of ties. Although the removed ties may also break up some of the relevant triadic structures, it also makes the remaining triadic structures stand out more. This may lead to an overestimation of their effects on network development. The triad census patterns in Appendix E are in line with this. With more negative error, the number of 030C (cyclic) and 030T (transitive) triplets increase in absolute sense in both the start and end networks. The reciprocated triplets structure (120C) only shows a strong increase in the end networks. At the same time, most other triad counts decrease, making these specific triadic structures stand out.

This can also explain why the transitivity (gwespFF) effect responds much less strongly to the introduced error than the other two triadic effects on a logit scale. The gwespFF effect is geometrically weighted: additional closed triplets have a smaller influence on the effect as the total number of closed triplets increases. In a sparser network, the contribution of the remaining transitive structures to the objective function is moderated by this geometric weighting, which may prevent this effect from standing out as much as the other two triadic effects.

The introduction of random positive error mitigates the overestimation of the triadic effects, which could be because it increases the density of the networks slightly. In particular, it makes less complex triadic structures (e.g. 012 and 021C) involving only one or two ties more prevalent. This noise introduction decreases the relative importance of the remaining triadic structures, and therefore limits the overestimation of the parameters.

Indegree-related Effects

The two indegree-related effects, indegree popularity and indegree activity, are relatively robust to the introduction of positive and negative error. The indegree popularity parameter shows to what extent actors tend to form relationships with actors that already have a large indegree. The true parameter is -0.223, which indicates actors are generally averse to forming relationships with popular actors. The indegree activity parameter reflects to what extent act actors with higher indegrees tend to form more relationships with others. In the true model, this parameter has a value of -0.300, which indicates that popular actors tend to form fewer new relationships.

There is a significant effect of the negative error rate on the relative bias of both parameters (β = 0.253, $p < .001$; β = 0.514, $p < .001$). with no positive error, each 10p.p. increase in negative error leads to parameter overestimations of 2.5p.p and 5.1p.p. on a logit scale. Although the size of the bias is small up to 10% error, it increases to moderate and large overestimations of 10.2% and 17.1% at 30% error. The coverage fluctuates around 95% for both parameters and all error rates, which indicates that most estimation procedures would calculate 95%-confidence intervals that include the true values.

Random positive error is also found to significantly affect the relative bias of the two parameters (β = -0.118, $p < .001$; β = -0.106, $p < .001$). At average negative error, positive error leads to an underestimation of about 10% for both effects. Positive error also interacts with the amount of negative error: when positive error is introduced, negative error leads to less parameter overestimation (β = -0.104, p = .044; β = -0.144, p = .006). However, the practical relevance of these interaction effects is small: positive error reduces the overestimation of negative error with only 1 to 1.5p.p. on a logit scale. With positive error, the coverage values always remain above 90%, with fluctuations around 92% for indegree popularity, and 95% for indegree activity.

It should be noted that the scale for indegree-related effects is larger than for the other effects considered in this model. The mean SAOM subgraph counts in the observed end network are about 15 for the indegree activity, and 16 for the indegree popularity. However, these larger scales are counterbalanced by smaller absolute coefficient. There are no clear indications that the bias found on the logit scale would have a much smaller or larger weight in the objective function.

It is noteworthy that the coverage of these statistics is relatively resistant to negative error, while the error does lead to relative bias values of moderate size. This is because the variance in the simulated parameters is relatively large for these coefficients. The true value therefore remains more central in the parameter distributions based on error-induced data. This can also be seen in the percentile rank plots in Appendix F: the percentile ranks of the true values of the indegree-related effects remain much more close to the 50th percentile than parameters with similar relative bias values (e.g. transitivity). So, although the point estimates of these parameters become moderately biased as a consequence of measurement errors, 95%-confidence intervals tend to remain reliable.

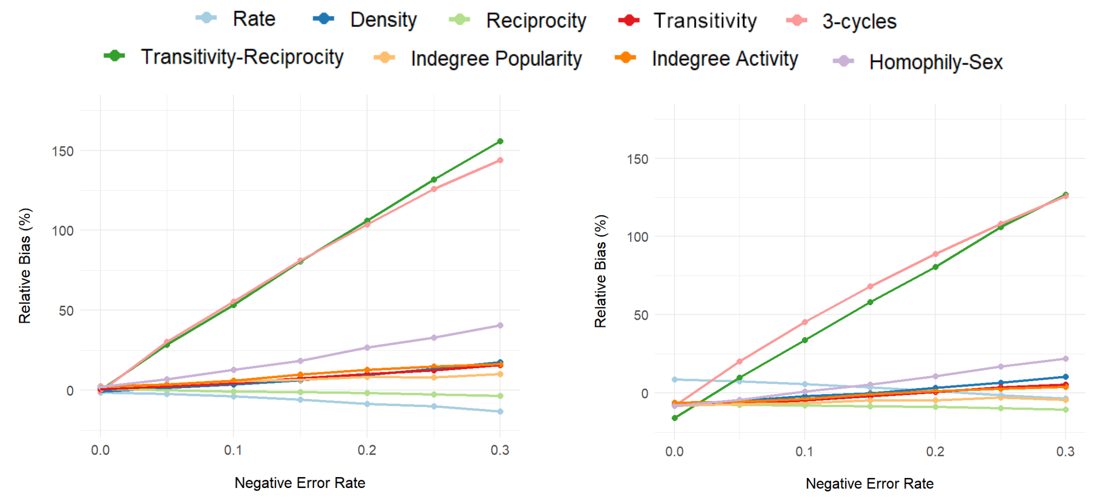
Sex-based Homophily Effect

The homophily effect in this model reflects to what extent actors form connections to other actors of the same sex. The true parameter value is 0.581, which shows actors have a tendency to form relationships with actors of the same sex. Its estimation is significantly influenced by the introduction of negative (β = 1.293, $p < .001$) and positive (β = -0.139, $p < .001$) errors, as well as their interaction (β = -0.269 $p < .001$). With no positive error, the introduction of negative error leads to overestimations of the homophily effect of about 12.9p.p. with every 10p.p. increase in negative error. This overestimation remains moderate in size up until about 15% negative error. Between 15 and 30% error the overstimation of the homophily effect is large, with the largest overestimation at 39.5%. This also affects the coverage, which remains around or above 95% up until 10% error, and then further decreases towards 79.1% at 30% error.

Positive error again has the opposite effect, and leads to an underestimation of the parameter of 13.9% at average negative error on a logit scale. The overestimation caused by negative errror and understimation caused by postive error balance each other out around 15% negative error. The counteracting effects make that the SAOM parameters remain closer to the true value: the coverage remains above 95% up to 20% negative error, after which it decreases to 87%. The interaction between positive and negative error is small to moderate in size: when positive error is introduced, the overestimating effect is diminished by around 2.7p.p. on a logit scale. This can translate to a total reduction of 8.07p.p. in negative error effect, at 30% negative error.

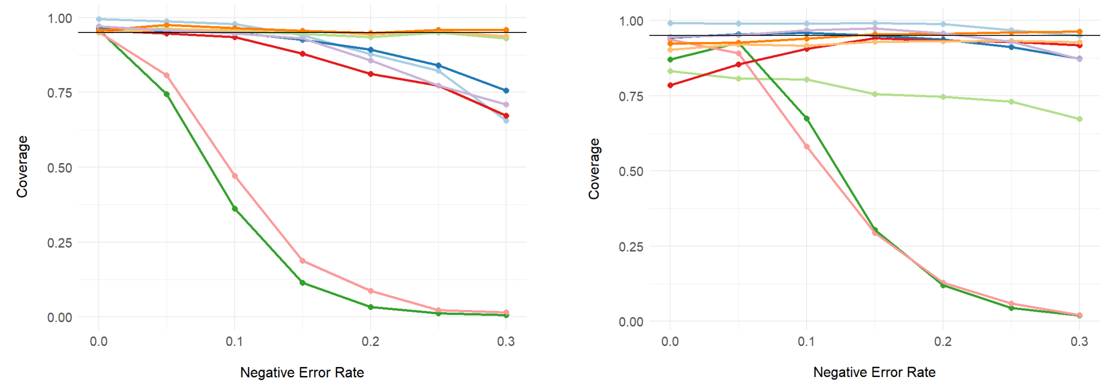
The scale of the homophily effect is quite average compared ot the other effects: the mean subgraph count in the observed end network is 3.278. Combined with a relatively small parameter, the bias found here on the logit scale could translate to a relatively smaller bias within the objective function.

This overestimation of the homophily effect can be explained in way similar to the overestimation of the triadic effects. The homophily in the original networks without error is very large: around 90% of the ties concern a same-sex friendship. By decreasing the density of the network, the remaining homophily-pattern becomes relatively more important in driving the dynamics of tie creation and dissolution. Although the negative error will remove some same-sex ties, this is not be enough to diminish the effect of the general pattern. Even with 30% negative error, 90% of the ties remain between actors of the same sex. When positive error is introduced, this effect is mitigated because of two reasons. First, the density is slightly increased by the introduction of random ties, making the homophily-effect relatively less important. Second, the random added ties are likely to create additional ties between actors of a different sex, which decreases the homophily effect. In networks with positive error, around 88% of the ties are to a same-sex actor.



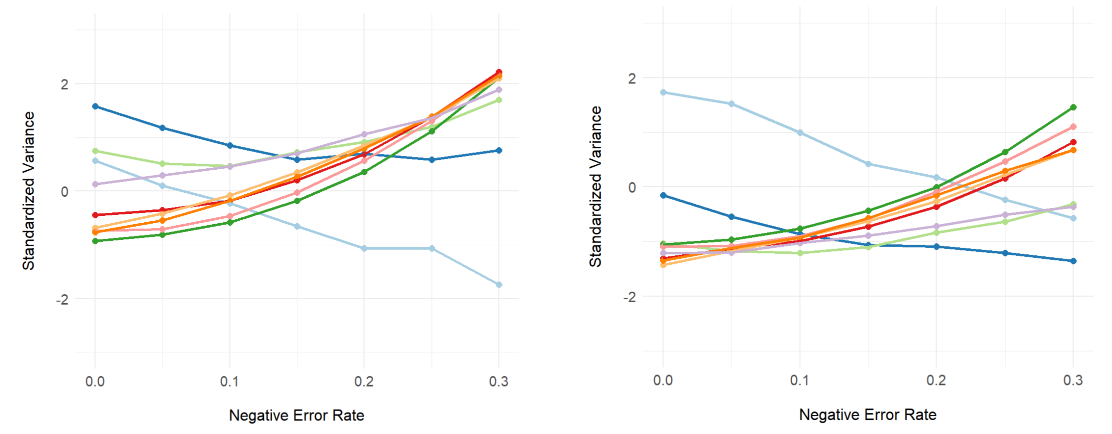
(a) Relative bias of SAOM parameters by negative error rate - No positive error.

(b) Relative bias of SAOM parameters by negative error rate - With positive error.



(c) Coverage of SAOM parameters by negative error rate - No positive error.

(d) Coverage of SAOM parameters by negative error rate - With positive error.



(e) Standardized variance of SAOM parameters by negative error rate - No positive error.

(f) Standardized variance of SAOM parameters by negative error rate - With positive error

Figure 5.1: The coverage, relative bias, and standardized variance of the simulated SAOM parameters by negative error rate, for the random error scenario. Left figures show situations without positive error, and right figures show situations with positive error.

Table 5.2: Linear regression models to assess the relation between the amount of introduced positive and random negative error, and their interaction, and the relative bias in proportions in SAOM parameters in each individual run: Rate, Density, Reciprocity, Transitivity (GWESP), 3-Cycles, Reciprocated Triplets, Indegree Popularity, Indegree Activity, and Homophily based on Sex. Independent simulation runs are the unit of analysis. N = 13289.

	1: Rate		**2: Density**		**3: Reciprocity**		**4: Transitivity**		**5: 3-Cycles**		**6: Rec. Triplets**		**7: Indeg. Pop.**		**8: Indeg. Act.**		**9: Homophily - Sex**	
	β (SE)	p	β (SE)	p	β (SE)	p	β (SE)	p	β (SE)	p	β (SE)	p	β (SE)	p	β (SE)	p	β (SE)	p
Intercept	-0.064 (0.001)	<.001	0.073 (0.002)	<.001	-0.012 (0.001)	<.001	0.077 (0.001)	<.001	0.773 (0.003)	<.001	0.797 (0.003)	<.001	0.064 (0.002)	<.001	0.094 (0.003)	<.001	0.201 (0.003)	<.001
Negative Error (Centered)	-0.397 (0.008)	<.001	0.606 (0.017)	<.001	-0.141 (0.009)	<.001	0.507 (0.010)	<.001	4.810 (0.035)	<.001	5.189 (0.034)	<.001	0.253 (0.025)	<.001	0.514 (0.029)	<.001	1.293 (0.026)	<.001
Positive Error (0 = 0, 0.001 = 1)	0.095 (0.001)	<.001	-0.063 (0.002)	<.001	-0.073 (0.001)	<.001	-0.093 (0.001)	<.001	-0.131 (0.005)	<.001	-0.224 (0.005)	<.001	-0.118 (0.004)	<.001	-0.106 0.004	<.001	-0.139 (0.004)	<.001
Interaction Neg x Pos	-0.019 (0.012)	.587	-0.024 (0.024)	.587	0.017 (0.013)	.587	-0.035 (0.015)	.157	-0.372 (0.049)	<.001	-0.424 (0.015)	<.001	-0.104 (0.035)	.031	-0.144 (0.041)	.005	-0.269 (0.036)	<.001
R^2_{adj}	.450		.196		.222		.387		.733		.778		.086		.080		.294	
F	3622	<.001	1080	<.001	1264	<.001	2796	<.001	12207	<.001	15659	<.001	420	<.001	387	<.001	1849	<.001

Notes: (i) The negative error predictor is centered to avoid multicollinearity. The positive error predictor is treated as a dummy variable with 0 = error rate of 0, 1 = error rate of .001. (ii) All parameter p-values in this table, as well as those in Table 5.4, are corrected for multiple testing using the Holm method.

5.2 Systematic Errors

The second scenario in this simulation study concerns the introduction of systematic errors, where the negative error probability is dependent on the tie-embeddedness of a tie. As outlined before, the error is introduced using sigmoid curves. The degree of negative error is varied to assess various error rates, but the shape of the relation between tie-embeddedness and negative error probabilities remains constant. The various sigmoid curves that are used to introduce error relative to tie-embeddedness have baseline negative error rates varying from .165 to .368. In all graphs and tables in this section, each sigmoid curves is indicated by their corresponding baseline negative error. These negative error rates were applied in two different scenarios: without positive error, and with random positive error. In the positive error scenario, non-existing ties are turned into ties with probability .001. Again, the effects of the errors on one of the simulated end networks are visualized in Appendix C.

5.2.1 Network Statistics

Table 5.3 shows the basic network statistics of the start and end networks of the simulated SAOM trajectories for two different systematic error scenarios. They contain the results of the sigmoid curve that is based on the results of Brewer & Webster (2000) ($v_0 = -0.24$), with and without positive error. The corresponding baseline negative error rate is .254. The statistics are averaged over all converged runs. The trends across all error combinations can also be viewed in Appendix E.

The results are relatively similar to the patterns in the random error scenario. Negative error decreases the density, and disconnects the network: the number components and median distance between actors increase. The median distance increases more quickly with negative error than in the random scenario with similar density measures ($p_n = .100$). The systematic error pattern favours clusters and is more likely to break up weak ties. Usually, weak, less-embedded ties are the ones that create "shortcuts" between different clusters in a network (Granovetter, 1973). These ties are now more likely to be removed. Reciprocity also decreases with negative error. It decreases more quickly than in the random scenario with the error rate than in the random error scenario. This is likely because stronger (and thus more embedded) ties tend to have a higher probability to be reciprocated (Vaquera & Kao, 2008), and more embedded ties have a higher probability to be removed. Lastly, the systematic negative error pattern leads to an increase in the transitivity index. By favouring stronger, more triangularly embedded relations, triangular patterns in the networks are more likely to be retained than other ties.

The change between the start and end network is more strongly affected by the systematic error than by the random error, although the directionality of the effect is the same. The systematic removal of ties leads to less stability in the ties over time (lower Jaccard index), and fewer tie changes between networks (lower Hamming distance). The effects are stronger than in the random error scenario.

The positive error in this scenario follows the same pattern as in the random scenario, and has the same effects on the network statistics. It increases the density and connectedness of the network. It adds noise to the networks, which obscures existing network structures and therefore leads to decreases in transitivity and reciprocity indeces. The noise also contributes to the dissimilarity of the start and end network, increasing the Hamming distance and decreasing the Jaccard index.

Table 5.3: Average network statistics for a selection of systematic error scenarios. Both the mean and SD (between brackets) for the statistics across simulation runs are shown. Statistics related to change across networks are only included for the end network.

	Density	Compo-nents	Transitivity index	Reciprocity index	Median distance (ii)	Hamming distance (iii)	Jaccard index	N
Start network								
No error (i) (iii)	.026	24.0	.308	.556	5.0			
Pos = .000, Neg = .254	.023 (3.9×10^{-4})	31.0 (2.7)	.313 (.011)	.515 (.017)	5.9 (0.6)			950
Pos = .001, Neg = .254	.024 (4.6×10^{-4})	26.9 (3.2)	.290 (.011)	.493 (.011)	5.5 (0.5)			947
End network								
No error (iii)	.028 (.001)	13.4 (4.1)	.425 (.028)	.635 (.028)	5.9 (0.5)	464.0 (0)	.341 (0.008)	953
Pos = .000, Neg = .254	.025 (.001)	23.3 (6.1)	.440 (.032)	.606 (.031)	6.6 (0.8)	433.9 (9.9)	.315 (.012)	950
Pos = .001, Neg = .254	.026 (.001)	19.2 (5.3)	.408 (.031)	.583 (.031)	5.9 (0.5)	467.5 (11.7)	.299 (.012)	947

Notes: (i) The start network is the same for every simulated run with no error. Therefore, there are no standard deviations. (ii) This variable refers to the median distance between actors in the largest component of the network. (iii) The systematic simulation set-up itself does not contain a separate "no error" scenario. Values from the random scenario are used for comparison. (iii) When simulating the new end networks based on the true model, simulations are terminated when the observed Hamming distance is reached. The Hamming distance is therefore always the same for simulated end networks with no error, leading to a SD of 0.

5.2.2 SAOM Parameters

Again, the analysis of the effect of systematic measurement error on SAOM parameters is first assessed for conclusions purely based on p-values and signs. Similar to the random scenario, results show that no amount of positive and/or negative random error seriously affects the power. The lowest power value was 98.9% for the 3-cycles effect, which means that measurement error is unlikely to cause non-rejection of null hypotheses, when effects actually exist. In addition, the sign of the SAOM parameters is not affected by any kind or amount of measurement error in this scenario. For each combination of error rates, there are no runs where the sign of the estimated parameter is opposite to the true values sign. Conclusions solely based on parameter significance and sign, without considerations of the parameter size, will therefore also not be affected by systematic measurement error.

However, similarly to the random error scenario, the introduced errors do affect parameter sizes. In the next sections, the effects of measurement error on the parameter sizes are considered. Table 5.4 contains linear regression models that estimate the effects of systematic negative error (quantified as baseline negative error), positive error, and their interaction on the relative bias of each parameter. These are used to assess the effects of errors on parameter bias below. For the positive error effect, interpretations are given for average baseline negative error rates (22.8% error). Note that baseline negative error is not directly comparable to the random error rates, since it is only the error rate at tie-embeddedness value 0. Additionally, Figure 5.2 visualizes the effects of various amounts of errors on the relative bias, coverage and standardized variance of the parameters. Again, visualizations that further illustrate the percentile rank of the true parameter and the densities of parameter estimates of the effects can be found in Appendix F.

Generally, negative error again leads to parameter overestimations, while positive error leads to parameter underestimations; effects that are similar to the random scenario. However, the some of the sizes of these effects, and especially the negative error effects, tend to differ from the random error scenario. The differences come from the different error patterns: whereas random negative error leads to a uniform thinning of the network, the systematic negative error selectively preserves network structures. When assessing differences in results between the random and systematic scenario, a comparison is made for error combinations that have start networks with similar densities after error introduction. These reflect similar total amounts of error, and are thus most comparable. The systematic scenario with a baseline negative error rate of .368 is comparable to the random scenario with a negative error rate of .200. The reader is referred to Appendix G for a table that compares these results in a structured way.

Rate Parameter

The introduction of more systematic negative error leads to an increasing and significant underestimation of the rate parameter (β = -0.249, p < .001); on average, actors have fewer opportunities to make changes to their relationships than in the true model when more ties are removed. With every 10p.p. increase in negative baseline error rate, the rate parameter is underestimated with 2.5p.p. The bias is moderate in size for the considered negative baseline error rates, with the relative bias underestimation ranging between about 5% and 10% when there is no positive error. For baseline negative error rates above 20%, the coverage drops below 95%, with a coverage of 83.0% at its lowest point.

The underestimation of the rate parameter is slightly *stronger* than in the random error scenario when comparing error scenarios. This is likely because systematic error keeps relations relatively more stable between the start and end network than random error, since it tends to preserve network structures. Therefore, fewer changes are needed to transfer from the start to end network state. However, the difference between the two scenarios is not very large: an underestimation of 8.35% in the random scenario (p_n = 0.200). corresponds to an underestimation of 10.47% in the systematic scenario (p_n = 0.368). The negative error in itself is more important in explaining the underestimation than the pattern of the negative error. The effect of positive error is similar to its effect in the random error scenario.

Density Effect

The true density effect indicates an aversion of actors to have ties. The introduction of systematic negative error again leads to an overestimation of this aversion. With more negative error, strength of the negative effect increases significantly (β = 0.599, p < .001). Given there is no positive error, each 10p.p increase in baseline negative error leads to an overestimation of the density effect of about 6p.p, on a logit scale. The overestimation is moderate to large in size: it ranges from 9.7% at 16.5% baseline negative error, to 20.7% at 36.8% baseline error. Any amount of studied systematic error decreases the coverage of this parameter below 95%, ranging from a coverage of 90.6% at 16.5% error, to 69.7% at 36.5% baseline error. In the presence of systematic measurement error, it is thus unlikely to capture the true parameter in a 95%-confidence interval with 95% confidence.

The explanation of the overestimation caused by systematic negative error is the same as in the explanation in the random error scenario. The simple decrease in number of ties and network density make that actors are less driven to create ties, and the "intercept-effect" of the density effect causes the parameter to adjust according to biases in other SAOM parameters. It is noteworthy that the overestimation of the density effect is stronger for systematic errors than for random errors. The relative bias is around 10 percentage points larger in the systematic scenario. This is likely because the intercept-effect is larger in this scenario. Systematic error selectively preserves certain types of network structures while removing noise, leading to overestimations in the corresponding SAOM parameters. As further explored later, this leads to stronger overestimations of homophily and transitivity, which is balanced out by the intercept-effect.

The positive error has a similar effect as in the random error scenario. Positive error by itself leads to an underestimation of the density effect by simply adding more ties. For this scenario, it also significantly influences the impact of the amount of negative error on the bias of this parameter (β = -0.093, p < 0.001). With the presence of positive error, the systematic negative error biases the density effect to a lesser extent. Its effect becomes about 1p.p. smaller for every 10p.p. increase in error, so the difference is not very large. The interaction effect is related to the systematic nature of the negative error. As explained before, the intercept-effect is an important part of the explanation of the bias caused by this type of error. The positive error decreases this intercept-effect. The added random ties do not only counteract the simple removal of ties, as in the random error scenario. They also decrease the effect of negative error on the density parameter by obscuring its selective preservation of network structures, leading to lower overestimations of the corresponding SAOM parameters. There is therefore less need for a density effect overestimation that counteracts those overestimations.

Reciprocity Effect

The reciprocity effect is not significantly affected by systematic negative error (β = 0.013, p = 0.587). Given there is no positive error, each 10p.p. increase in negative baseline error leads to an overestimation of the SAOM parameter of 0.1p.p. on a logit scale - a bias that is also negligible in size and not of practical importance. The coverage also fluctuates steadily around 95%, indicating that 95%-confidence intervals for this parameter can be interpreted as usual - even with large amounts of systematic negative error. Similar to the random error scenario, the reciprocity effect is also robust against systematic negative error. The biasing effect of positive error, as well as its lack of interaction with the baseline negative error effect, are similar to the random scenario.

Triadic Effects

The three included triadic effects, the transitivity (gwespFF), 3-cycle and reciprocated triplets effects, are most interesting to study. They relate most directly to the patterns introduced by the systematic negative error, since the pattern favours ties that are embedded in triangles.

The original transitivity parameter was 2.394, which shows that actors tend to form ties to friends of their friends. This effect is significantly overestimated with more negative error (β = 0.383, p < .001). Without positive error, an increase of 10p.p. in baseline negative error corresponds to a parameter overestimation of 3.8p.p. on a logit scale. In the considered range of 16.5% up to 36.8% negative baseline error, this therefore translates to an overestimation that is moderate in size (ranging from 6.4% to 14.1% relative bias). This overestimation is stronger than in the random error scenario, with a difference of about 4 percentage points. The bias does lead to low coverage: none of the baseline error rates have a coverage above 95%. At 36.8% baseline error, the coverage decreases to 68.4%.

The 3-cycle and reciprocated triplets effects have true values of 0.514 and -0.542 respectively. Actors tend to cyclic structures, but are averse to recreating reciprocating ties within triads. Both effects are strongly overestimated with more systematic negative error (β = 1.734, p < .001; β = 1.531, p < .001). Given there is no positive error, an increase of 10p.p in negative baseline error leads to overestimations of respectively 17.3p.p. and 15.3p.p on a logit scale. These are still very large overestimations, and lead to respective maximal overstimations of 60.6% and 54.6% at 36.8% negative baseline error. These remain substantial overestimations that could largely influence conclusions, but they are much smaller than in the random error scenario. In the scenario comparable to 36.8% negative baseline error, the overestimations are respectively 101% and 105%. The overestimations again lead to very low coverage values (44.0% and 31.2% at their lowest).

Similar to the random scenario, the introduction of random positive error significantly affects the relative bias in the transitivity, 3-cycles and reciprocated triplet parameters (β = -0.098, p < .001; β = -0.099, p < .001, β = -0.187, p < .001). Compared to the random scenario, the effect for the transitivity effect remains similar (an underestimation of about 10p.p in both scenarios), whereas the underestimations are about 5p.p. smaller for the 3-cycles and reciprocated triplet effects in the systematic scenario (underestimations of respectively 11% and 20.8%). In addition, all interaction effects in the systematic scenario for triadic parameters have become small enough to be of little interest. If positive error is introduced, the effect of negative error decreases with at most 0.5p.p. to 1.5p.p, depending on the parameter.

The overestimations of the 3-cycle and reciprocated triplets effects is thus much less severe than in the random scenario, and the interaction effects between positive and negative error have become much smaller. These results are likely related to the pattern of the introduced error. In the random scenario, the introduction of error led to a "uniform thinning" of the network. More complex structures were broken down into patterns that fit the triadic effects, making these configurations relatively important in a sparse context. This resulted in strong overestimations. While the 030T, 030C and 120C triplets still gain relative importance with the introduction of negative error, they stand out less. In the systematic scenario, the error changes the networks in such a way that network structures are selectively preserved. Cohesive, densely clustered network structures are likely to remain (largely) intact. This is also visible in the triad count: fully, or almost fully, connected triads (210 and 300) remain more important - especially in the end networks. This is captured by the larger overestimation of the transitivity effect. The effect specifically focuses on edgewise shared partners, with (almost) fully connected triads making a significant contribution to its value.

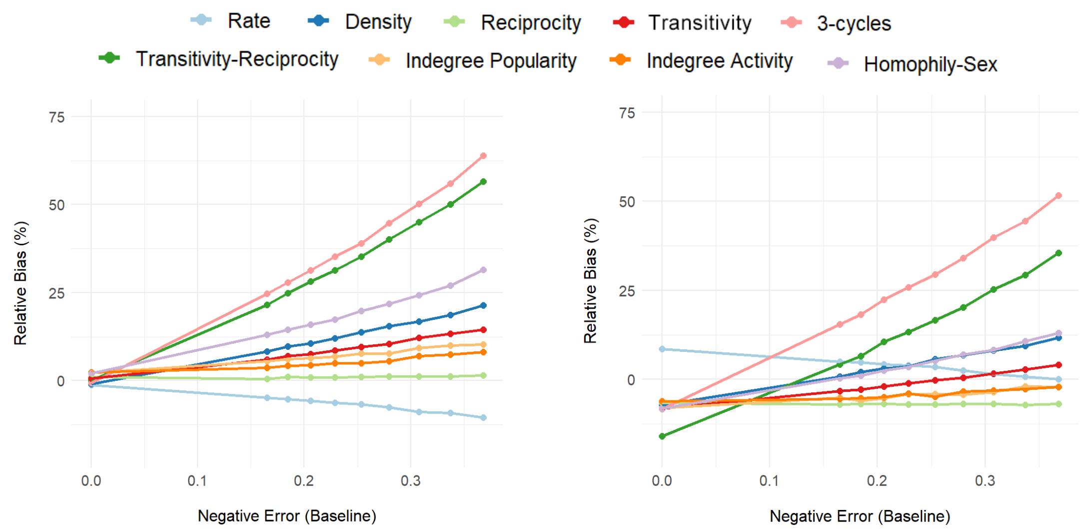
Indegree-related Effects

Indegree popularity and indegree activity refer to popular actors respectively receiving and creating more ties over time. The true parameters for these effects are -0.223 and -0.300, which implies that popular actors are averse to receiving and creating more ties. Both effects are significantly overestimated when more systematic, negative error is included (β = 0.226, p < .001; β = 0.161, p < .001). With each 10p.p. increase in baseline negative error, the parameters are overestimated by respectively 2.26 and 1.61p.p. on a logit scale. These overestimations remain small up until a baseline negative error of about 20%, but become of moderate size for more error - up to 10.2% and 7.4% at 36.8% baseline negative error. These overestimations are comparable to the ones in the random scenario. Similar to the random scenario, the coverage is not affected: the coverage values remain around 95%. Positive error (β = -0.117, p < .001; β = -0.095, p < .001) has a similar effect as in the random scenario, and leads to underestimations of parameters of about 10%. There are again no relevant interactions between positive and negative error.

Sex-based Homophily Effect

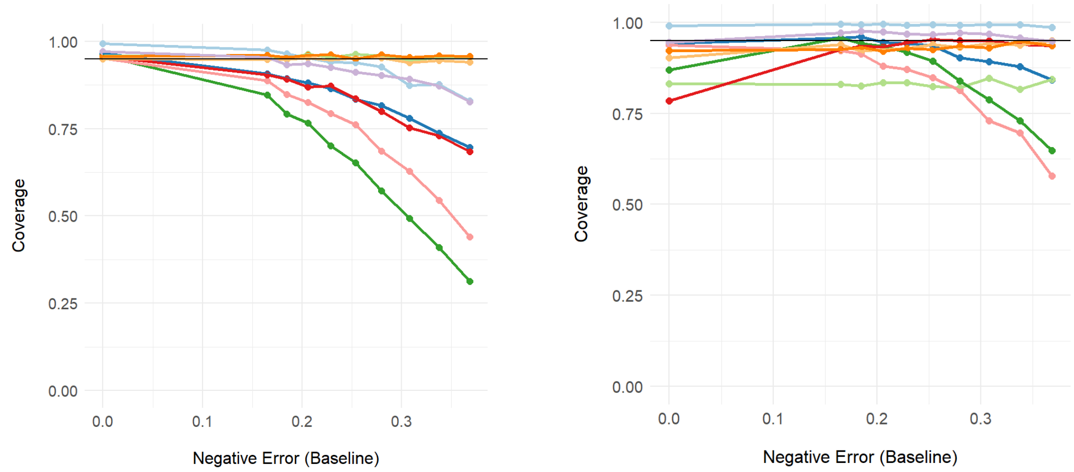
The homophily effect, reflecting the tendency of actors to prefer forming ties to others of the same sex, has a true parameter of 0.581. More systematic negative error leads to a significant overestimation of this parameter ($\beta = 0.779$, p $<$ .001). At no positive error, a 10p.p. increase in baseline negative error, leads to an overestimation of 7.79p.p. The relative bias increases up to 29.2% at 36.8% baseline error, and is therefore overall large in size. The bias remains moderate up until about 20% baseline negative error. The coverage does drop below 95% for all scenarios with systematic negative error. This decreases from 91.2% at 25.4% baseline error, to 82.7% at 36.8% baseline error. When it comes to positive error and its interaction with negative error, there are no substantial differences with the random scenario.

The overestimation is slightly stronger than in the random error scenario; by about 3 percentage points, so the difference is not very large. The explanation again stems from the structured pattern in the error. In the random error scenario the homophily effect was overestimated because it was a network structure that remained strong in a sparse context, where many other stuctures were thinned out. In this case, the overestimation is driven by the systematic pattern. As is also shown by the network visualizations in Appendix C, ties between actors of the same sex are often ties embedded in a cluster of actors of the same sex. On the other hand, the few ties that are part between actors of the opposite sex, often form bridges between male and female clusters. These bridges are much less embedded in the network structure, and are therefore more likely to be removed by the negative error pattern. This increases the proportion of same-sex ties compared to opposite-sex even more, which strengthens the homophily effect.



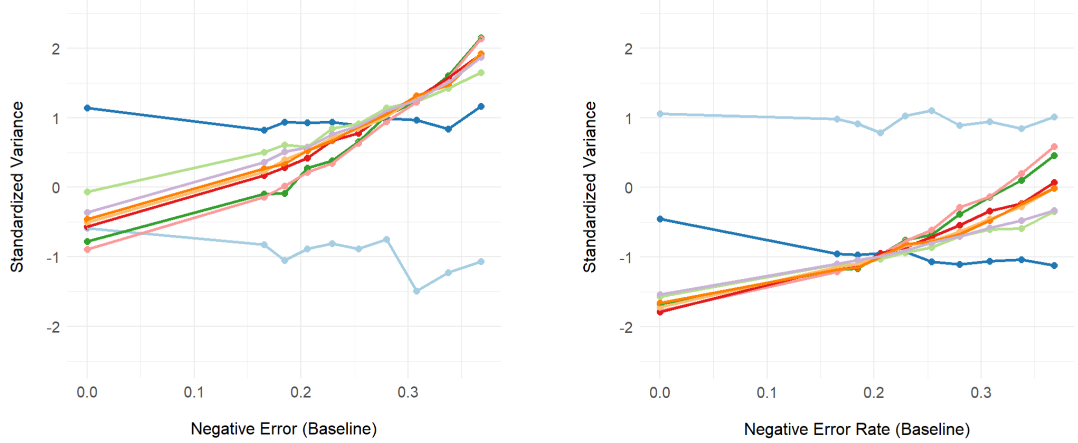
(a) Relative bias of SAOM parameters by negative error rate - No positive error.

(b) Relative bias of SAOM parameters by negative error rate - With positive error.



(c) Coverage of SAOM parameters by negative error rate - No positive error.

(d) Coverage of SAOM parameters by negative error rate - With positive error.



(e) Standardized variance of SAOM parameters by negative error rate - No positive error.

(f) Standardized variance of SAOM parameters by negative error rate - With positive error

Figure 5.2: The coverage, relative bias, and standardized variance of the simulated SAOM parameters by baseline negative error rate for the systematic error scenario. Left figures show situations without positive error, and right figures show situations with positive error.

Table 5.4: Linear regression models to assess the relation between the amount of introduced positive and systematic baseline negative error, and their interaction, and the relative bias in proportions in the SAOM parameters: Rate, Density, Reciprocity, Transitivity (GWESP), 3-Cycles, Reciprocated Triplets, Indegree Popularity, Indegree Activity, Homophily based on Sex (N = 18981).

	1: Rate		**2: Density**		**3: Reciprocity**		**4: Transitivity**		**5: 3-Cycles**		**6: Rec. Triplets**		**7: Indeg. Pop.**		**8: Indeg. Act.**		**9: Homophily - Sex**	
	β (SE)	p	β (SE)	p	β (SE)	p	β (SE)	p	β (SE)	p	β (SE)	p	β (SE)	p	β (SE)	p	β (SE)	p
Intercept	-0.067 (0.001)	<.001	0.126 (0.001)	<.001	0.010 (0.001)	<.001	0.089 (0.001)	<.001	0.372 (0.003)	<.001	0.333 (0.002)	<.001	0.072 (0.002)	<.001	0.052 (0.002)	<.001	0.187 (0.002)	<.001
Baseline Negative Error (Centered)	-0.249 (0.007)	<.001	0.599 (0.015)	<.001	0.013 (0.008)	.587	0.383 (0.009)	<.001	1.734 (0.026)	<.001	1.531 (0.021)	<.001	0.226 (0.020)	<.001	0.161 (0.023)	<.001	0.779 (0.023)	<.001
Positive Error (0 = 0, 0.001 = 1)	0.101 (0.001)	<.001	-0.080 (0.001)	<.001	-0.080 (0.001)	<.001	-0.098 (0.001)	<.001	-0.099 (0.004)	<.001	-0.187 (0.003)	<.001	-0.117 (0.002)	<.001	-0.095 0.003	<.001	-0.144 (0.003)	<.001
Interaction Neg x Pos	0.016 (0.010)	.587	-0.093 (0.021)	<.001	-0.023 (0.011)	.255	-0.050 (0.012)	<.001	-0.117 (0.037)	.019	-0.144 (0.030)	.001	-0.062 (0.028)	.227	-0.044 (0.032)	.584	-0.205 (0.032)	<.001
R^2_{adj}	.393		.138		.212		.347		.315		.408		.092		.047		.165	
F	4100	<.001	914	<.001	1703	<.001	3362	<.001	2910	<.001	4358	<.001	640	<.001	315	<.001	1250	<.001

Notes: (i) The baseline negative error predictor is centered to avoid multicollinearity. The positive error predictor is treated as a dummy variable with 0 = error rate of 0, 1 = error rate of .001. (ii) All parameter p-values in this table, as well as those in Table 5.2, are corrected for multiple testing using the Holm method.

Chapter 6

Conclusion & Discussion

The primary goal of this thesis was to contribute to the field of statistical analysis of social networks by exploring the effects of measurement errors on the results of SAOMs. Effects of various patterns of mismeasured ties were assessed, with a special focus on forgetting ties (negative error). Various amounts of tie-forgetting were considered, where error introduction weas either random, or dependent on tie-embeddedness; which is a systematic error pattern informed by social and cognitive sciences. In addition, small amounts of random positive error were considered.

6.1 Conclusions

Due to the difficult interpretations of SAOM parameters, conclusions about SAOM effects are often based on parameter significance and direction of the effects. Such conclusions are not affected by any of the types or amounts of measurement error considered in this study. There are no differences in power for any of the coefficients, and measurement error does not lead to sign reversals of coefficients.

However, measurement errors do affect the sizes of SAOM coefficients. Hence, conclusions based on the parameter sizes in SAOMs can be biased by measurement error. The findings presented in this thesis regarding effect sizes are not directly interpretable: the presented biases are on a logit scale, for unstandardized coefficients. Due to their nonlinear relationship, small bias on a logit scale can translate to large bias on a probability scale, and vice versa. The exact biases on a logit scale differ per parameter, but the following observations tend to hold for most considered SAOM effects:

I. Negative error leads to overestimation, while positive error leads to underestimation. Parameters tend to increase in a linear fashion with negative error, both random and systematic. On the contrary, a small amount of random, positive error tends to underestimate parameters, counteracting the effect of negative error. When the two error types are combined, parameter estimates therefore stay closer to the true parameter value. In the parameters with larger biases (especially the 3-cycles, reciprocated triplets and homophily effects), there are also interaction effects where positive error reduces the effect of negative error on the parameters.

The opposing effects of positive and negative error suggest that (part of) the bias is related to network density: a decrease in ties (due to negative error) leads to a sparser network where the remaining effects become relatively more important and are overestimated. On the other hand, positive error reintroduces random noise and increases the density. Effects become relatively less prominent, and are underestimated.

There are two exceptions to this general observation: the rate parameter, and reciprocity effect. The first, not part of the SAOM objective function, is related to the Hamming distance instead of subgraph counts of effects. The reciprocity parameter, while affected by positive error as described, is not meaningfully biased by negative error in both scenarios. Reciprocity is likely not an effect that stands out as much as the other effects with a lower network density, since its subgraph counts are largely impacted by negative error: with more than 50% of the ties being reciprocal, and one tie removal leading to two fewer reciprocal ties, it decreases quickly with negative error. Hence, it has less potential than other effects to be relatively important in a sparser network.

II. Overestimations caused by negative error tend to be small to moderate on a logit scale. For most studied parameters, overestimations caused by negative random error tend to be small op until 10%. With little negative error, conclusions about these effects may remain relatively reliable. With negative random error between 10 and 30%, these biases become moderate in size, and have to be interpreted with more care. Bias sizes tend to be similar in the systematic scenario.

Only the 3-cycles and reciprocated triplets effect, and the homophily effect are much more strongly overestimated. For the first two this could be because they are triadic effects that are not geometrically weighted. Their importance in less dense networks is therefore emphasized much more severely. The homophily effect is also strongly overestimated – although less severely than the triadic effects. Its larger overestimation could be due to the magnitude of the effect in the network: 90% of the ties are between same-sex actors. Even if 30% of these ties disappear, it remains a very prominent subgraph structure.

III. Underestimations caused by positive error tend to be moderate on a logit scale. The effect of a small, set amount of random positive error is moderate in size across most parameters, in both the random and systematic scenario. Relative bias fluctuates between 6 and 15%, with density and reciprocity effects on the lower end, and homophily and 3-cycles effects on the higher end. The only exception to this is the reciprocated triplets effect, where positive error leads to a large underestimation. Given the small amount of positive error that was introduced, the biases are quite substantial; if larger amounts of positive error were to occur, biases may become much larger.

IV. Differences between the random and systematic scenarios concentrate in the effects that are related to the systematic error-pattern. Most effects in the random and systematic scenario are comparable in direction and size, with most biases being tied to network density. The largest discrepancies are found in the 3-cycle, reciprocated triplets, transitivity, and homophily parameters (all correlated with the systematic error pattern), as well as the density parameter. The first two parameters are severely overestimated in the random scenario due to their relative importance in networks that are "uniformly thinned" by negative error, exacerbated by their lack of geomatic weighting. While this overestimation remains very large in the systematic scenario, it decreases substantially. In the systematic scenario networks also decrease in density with negative error, but it selectively preserves network structures – and particularly triadic structures. The transitivity and homophily effects, which are related to the error pattern, are therefore also more strongly overestimated (although the differences are not large on a logit scale). These overestimations partly overshadow the 3-cycle and reciprocated triplets effects, making their overestimations smaller. Lastly, the density effect is more strongly overestimated in the systematic scenario due to its stronger intercept effect. Due to the lack of centered parameters in the SAOM model, the density effect has to counterbalance overestimations in other parameters to accurately calibrate overall network density.

V. Biases likely depend on data and model characteristics. Due to the complexity of SAOMs and its simulation-based estimation, interpreting the biases is challenging. However, two factors return in most of the potential bias explanations. First, the biases depend on characteristics of these network data, and the strengths of effects in the data in particular. For example, it is unlikely that a homophily-related effect is so strongly overestimated in networks without homophily. Second, the interplay between parameters is a relevant factor in bias direction and magnitude, because parameters are not centered. This is apparent in the density parameter, which is overestimated to counteract the overestimations of other model parameters. When other parameters are less biased, the density effect will also be less biased. Other parameters can also affect each other, due to correlations between network structures.

6.2 Expectations & Relation of Conclusions to Existing Literature

Due to the complexity of the SAOM model, no concrete hypotheses were formulated about the effects of errors. Instead, broader, less specific expectations were outlined based on other measurement error-research in social networks. Firstly, studies on measurement errors affecting descriptive statistics of social networks showed dependencies of results on network characteristics (Wang et al., 2012; Frantz et al., 2009), which could also apply to SAOMs. As described in the previous section, there are signs that this is indeed the case. The explanations of bias for both the reciprocity and the homophily effect rely on the strengths of their effects in the network. This means that it may be quite difficult for research to assess how much they can expect their SAOM parameters to be biased: this is likely to differ across data sets.

In addition, it was expected that randomly introduced errors could lead to bias of effect parameters towards 0. Kim et al. (2016) found similar results for ERGMs. Since ERGMs and SAOMs both use subgraph counts as statistics, results could be similar. However, this study finds that SAOM parameters tend to be overestimated instead of underestimated by random negative error – the opposite effect. Differences could be attributed to differences in model specifications and ratios of positive and negative errors, but also to differences between the models. A more extensive comparison is needed to properly explain the difference, but that is beyond the scope of this thesis.

Lastly, it was expected that complex network effects are more affected by negative random errors, due to their dependence on specific tie-configurations. This expectation is also not supported by the (interpretation of) the results. Both complex effects involving a specific tie configuration, such as triadic effects, and less complex effects involving only one tie, such as homophily, are strongly overstimated.

6.3 Strengths & Limitations

The presented simulation study has a couple of strengths. The study is the first exploration of the sensitivity of SAOMs to measurement errors. It gives a first insight into the way various aspects of SAOM results respond to the introduction of error. The study is also not limited to random errors, as most other studies relating measurement error and social network analysis results are (Borgatti et al., 2006; Frantz et al., 2009; Wang et al., 2012; Kim et al., 2016). Systematic negative errors are considered, where the probability of tie-forgetting depends on the embeddedness of each tie. This systematic type of error is informed by social and cognitive theory (Brashears & Quintane, 2015; Brewer, 2000; Roth et al., 2021; Marin, 2004), and therefore likely closer to error in real research projects. Actual error is likely somewhere inbetween the random and systematic scenarios, so the considered scenarios give a broad overview of the potential biases researchers can encounter. Lastly, the study is based on a model fit to a real friendship network, and therefore reflects elements of real-life networks. This means that the results reflect effects of measurement errors in networks that are close to those in (a type of) social reality, and can thus provide insights into potential biases in research projects that analyze friendship networks using SAOMs.

On the other hand, some limitations of the study must be noted. While using a "true" model that reflects a type of social reality has an advantage, it also means that results are based on the specific characteristics of this model and data. Parameter biases are likely (in part) dependent on the interplay between the selected model parameters as well as characteristics of the network data. Results are therefore only applicable to (friendship) networks with similar characteristics, and this selection of model parameters. The used model is also not a fully accurate reflection of a type of social reality. The data on which the true model was based, used an upper bound for friendship nominations per participant of 6 friends. Using a capped nominations design decreases the average number of nominated friends (Neal, 2024), which makes the considered "true" model a less accurate depiction of social reality. Another limitation to this study is that both the random and systematic scenarios considered do not fully capture actual error patterns. While the systematic scenario likely comes closer than the random scenario, it is unlikely to be a complete reflection of real measurement error. This is in general difficult to accomplish, and requires more research on specific error patterns that occur.

All in all, the set-up and limitations of this study make that the results should in the first place be seen of a first exploration, a type of case study, on the effects of measurement errors on SAOMs. It gives valuable insights into the potential biases various types of measurement errors can create during research involving SAOMs. However, results are not necessarily predictive of biases in other studies due to their dependence on (constrained) network data, this specific model, and the lack of a fully reliable specification of error patterns.

6.4 Future Research Directions

To achieve a more accurate picture of the overall effects of measurement error on SAOM results, more research is needed. Future studies should assess the effect of measurement errors for different network structures and/or effect strengths, as well as vary the selection of parameters included in the model. In particular, the effect of error geometrically weighted triadic effects could be assessed, to determine if the use of geometrically weighted versions of for example 3-cycles and reciprocated triplets effects can mitigate their strong bias. Attempts can also be made to more accurately define and introduce real measurement errors patterns by integrating social and cognitive theory with research on reliability of peer nomination data. An example of this would be to include temporal dependence of measurement errors between data observation wavese. Other potential areas of interest are systematic error related to preferential attachment in positive error (actors forming false positive ties to popular actors), or dependence of errors on exogenous variables such as actor sex.

A second potential research area could make an extension of this research to co-evolution models, which analyse network dynamics over time together with a behavioural variable. By co-analysing these two concepts, conclusions can be drawn about dynamics where actors influence each other's behaviour, or select friends based on behavioural similarity. The behavioural variable in this model can also contain measurement error, which can affect SAOM results in itself, and/or interact with measurement errors in network data. Preliminary findings by Sorjonen et al. (2023) show that measurement errors in the behaviour variable could distort behavioural influence effects.

Lastly, it should be noted that the results of this study are limited to the bias in SAOM coefficients on a logit scale. When statistical techniques become available to assess marginal effects in SAOMs, it would be interesting to assess the effects of random and systematic measurement error on a probability scale, or another scale that is easier to interpret. A starting point could be to assess the effects of errors on the relative importance measure of Indlekofer & Brandes (2013).

6.5 Take-Home Message

In summary, conclusions about SAOMS that are solely based on parameter significance and direction can be reliably used to answer research questions. However, researchers that attempt to tie conclusions to parameter sizes should be aware that parameters are likely to be moderately to largely overestimated. Especially the homophily, 3-cycles and reciprocated triplet effects are likely to be strongly biased. Researchers should exercise care when interpreting their parameter sizes. Positive error may partly diminish the bias due to the counteracting effects of positive and negative error, but this cannot be reliably expected due to a lack of information on the actual error rates in the used dataset. [1]

The findings highlight the need for researchers to carefully assess their methods of social network data collection, and their interpretation of SAOM results. Especially when measurement errors are anticipated, the potential for substantial parameter biases should be addressed and acknowledged. This can contextualize results and robustness of conclusions, and contribute to a more nuanced understanding of social networks.

[1] It is therefore also not suggested to randomly add positive error to a dataset to diminish bias. This is impossible to accurately calibrate without knowledge on the amount of error present in the data. In addition, positive error can reduce accuracy for parameters that are not or less impacted by negative error, such as reciprocity

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Appendix A

Code & Data

The R code and simulated data used to execute the simulation study can be found on this Github page.

Appendix B

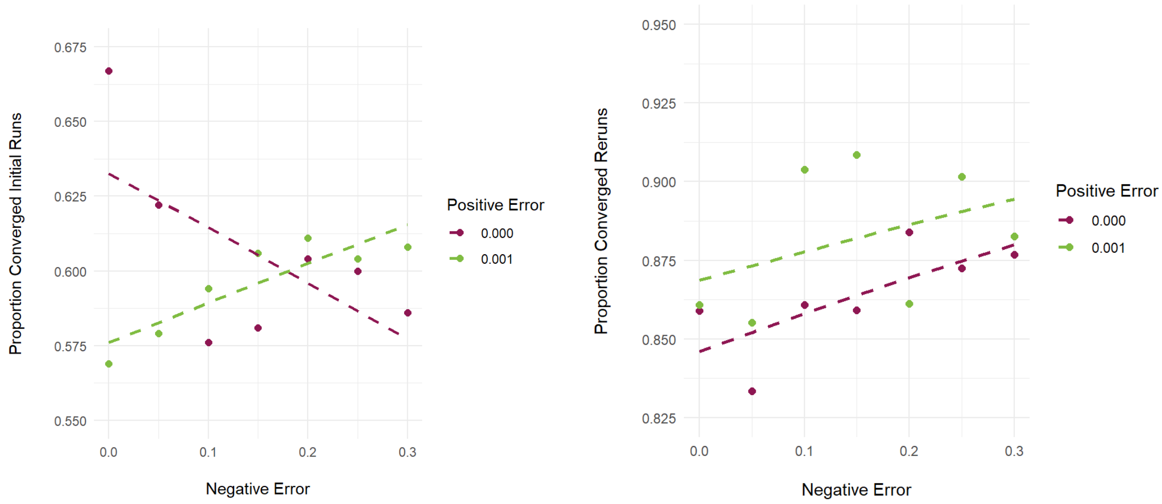
SAOM Convergence under Measurement Error

In this Appendix, the effect of the positive and negative error rates on the ability of the estimation algorithm to converge is assessed. A SAOM simulation run converges when the expectations of sensitive statistics in the simulated networks approach the values of the simulated statistics in the observed networks. As discussed in Section 2.3, convergence is achieved when the absolute t-ratio for each single parameter is below .1, and the maximum convergence ratio for any linear combination of parameters is below .25.

The introduction of error into network data may deconstruct or obscure patterns in the data, and could therefore make it more difficult to detect and simulate the effects in the objective function in the SAOM analysis. It may therefore affect the probability that a run can converge under the default RSiena settings. Below, the convergence probability is assessed under the default settings of the RSiena simulation algorithm. However, deviations from these settings may improve convergence, and can lead to different results than presented here. For further information on the default SAOM settings and the potential adjustments to the algorithm settings, the reader is referred to Ripley et al. (2024, p. 76).

Below, the convergence probability of the SAOMs under default settings is assessed for both the random and embeddedness-based error scenarios. Results are explored using visualizations and logistic regression models. This is done separately for the initial convergence and rerun convergence. Initial convergence is convergence on the first attempt. Rerun convergence refers to the second attempt: each of the initially non-converged runs had a second convergence attempt, with the solution of the previous run as a starting point.

It is important to note that the statistical tests from the logistic models presented below have little power. Since the model outcome is binary (convergence versus no convergence), the analysis is performed on the level of error combinations. For example, the error scenario with no positive error and a negative error rate of 0.1 is taken as one observation, with the proportion of converged runs as its value. Therefore, the effective sample size is very low: 14 in the random error scenario, and 20 in the systematic error scenario. This is very little data to draw conclusions from. This also means that the results are very easily influenced by outliers. Drawing robust conclusions from these analyses is not possible.



(a) Proportion of converged initial runs

(b) Proportion of converged reruns

Figure B.1: The proportion of (a) initial convergence of simulation runs under default SAOM settings and of (b) successful reruns under default SAOM settings after non-convergence, by the amount of introduced positive and negative error. Dots indicate observed proportions, and dotted lines show the fitted values based on the GLM results in Table B.1.

B.1 Random Errors

All logistic regression results for initial and rerun convergence under the random error scenario can be found in Table B.1. The results are visualized in Figure B.1.

B.1.1 Initial Convergence

In Figure B.1, the observed proportions of convergence are shown for both initial runs and reruns, based on the amount of introduced error. The proportions of initial convergence fluctuate between .569 and .667 - the highest being for the scenario without any error. Overall, the proportion of initial convergence is significantly higher for the no-error scenario (.667) than for scenarios with error (.595) ($\chi^2(1) = 19.554, p < .001$). This difference is moderate in size: negative error makes convergence under default settings more difficult to achieve.

The relations between the amount of error, as well as their interaction, and the proportion of initial convergence, are assessed using a logistic regression model. The results are shown in Model 1 of Table B.1. Results show that the amount of negative error (β = -0.775, p = .002) significantly influences the probability of convergence. More error is related to a lower probability of convergence when there is no positive error: the break-down of network structures leads to relatively more noise, making it more difficult for the SAOM algorithm to identify structural patterns. However, this effect is mostly driven by the larger convergence probability of the scenario with no error. Without this scenario, the strength of the effect decreases and becomes non-significant.

Positive error (β = -0.039, p = .258) is by itself not significantly related to the probability of initial convergence. However, there is a significant interaction effect between the two types of errors (β = 3.747, p < .001): when positive error is introduced, more negative error increases, instead of decreases, the initial convergence probability. This effect decreases in size, but remains significant when outliers are removed. The interaction likely reflects the balance between additional random noise caused by positive error and increased sparsity caused by negative error. Little negative error together with

Table B.1: Logistic regression models to assess the relation between the amount of introduced positive and negative error, and their interaction, and the convergence rate under default SAOM settings of the initial run and rerun. The unit of analysis is the error-combination: for each unique combination of errors, the probability of convergence is assessed.

	1: Initial Convergence			**2: Rerun Convergence**		
	β (SE)	OR	p	β (SE)	OR	p
Intercept	0.428 (0.024)		<.001	1.847 (0.056)		<.001
Negative Error (Centered)	-0.775 (0.245)	0.461	.002	0.965 (0.567)	2.624	.089
Positive Error (0 = 0, 0.001 = 1)	-0.039 (0.035)	0.962	.258	0.165 (0.081)	1.180	.040
Interaction Neg x Pos	1.321 (0.345)	3.747	<.001	-0.140 (0.811)	0.869	.863
χ^2	16.311		<.001	8.844		.032
df	10			10		

positive error leads to a lot of extra noise on top of the network structure, while negative error has not yet led to substantial network sparsity. This makes it more difficult for the algorithm to distinguish network patterns from noise. When negative error increases and makes networks more sparser, this effect of positive error is mitigated. However, because negative errors also break down network structures, the SAOM convergence does not come close to the convergence in the scenario with no errors.

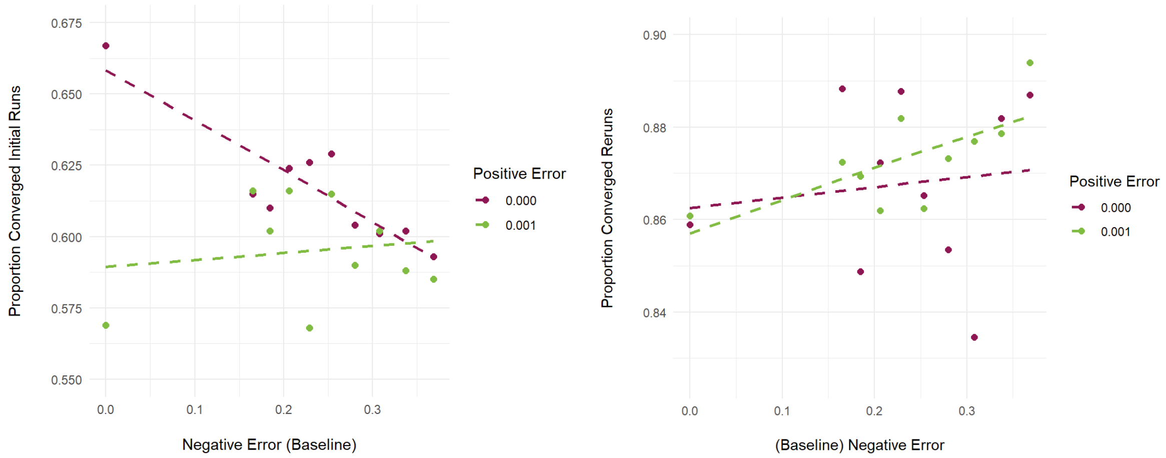
B.1.2 Rerun Convergence

Runs that did not converge on the first attempt, had the opportunity to reach convergence one more time in a second run. This rerun had the results of the previous run as the starting state. Since the starting state is likely to be closer to the solution, the probability of convergence was higher in the second run. The lowest proportion was .833, while the highest was .910 - substantially higher than the initial convergence rates.

When looking at the convergence of reruns, the effect of the (amount of) error on the convergence probability seems to disappear. The no-error scenario does not have a significantly higher convergence probability than all other error scenarios together ($\chi^2(1) = 0.450, p = .450$). Figure B.1b does indicate that rerun convergence increases slightly with negative error, but this is not significant (β = 0.965, p = .089). This is a noteworthy difference compared to the initial convergence analysis, where negative error has a negative effect on convergence. In addition, positive error significantly increases the probability of rerun convergence (β = 0.165, p = .040) - an effect that was not significant for initial convergence. However, the introduction of positive error only leads to an increase in the convergence probability of around 0.025, which is not a very large effect. In addition, no significant interaction effect is found (β = 0.869, p = .863). In the reruns, convergence probabilities increase slightly (and insignificantly) with negative error, regardless of the presence of positive error.

B.2 Embeddedness-based Errors

The analysis presented above is repeated for the scenario with embeddedness-based errors. Logistic regression analyses assessing the influence of the error rates can be found in Table B.2, and are



(a) Proportion of converged initial runs

(b) Proportion of converged reruns

Figure B.2: The proportion of (a) initial convergence of simulation runs under default SAOM settings and of (b) successful reruns under default SAOM settings after non-convergence, by the amount of introduced positive and systematic (baseline) negative error. Dots indicate observed proportions, and dotted lines show the fitted values based on the logistic regression results in Table B.2.

visualized in Figure B.2. The regression analyses use the negative baseline error rates as an indicator of the various sigmoid curves introducing negative error, to determine the effects of various amounts of systematic negative errors.

B.2.1 Initial Convergence

The initial convergence results for the systematic scenario are almost the same as for the random scenario. The convergence of SAOM models on the first run fluctuates around 60%. The runs with systematic negative error have, on average, a significantly ($\chi^2(1) = 15.13, p < .001$) lower convergence ratio (0.605) compared to the scenario with no error (.667). Furthermore, the logistic regression analysis in Model 1 of Table B.2 shows that the convergence probability significantly decreases with more negative error , and is significantly lower when positive error is also introduced . Again, there is a significant interaction between positive and negative error : when there is positive error, the convergence probability slightly increases with extra negative error. However, an important note is that the scenario with no (negative) error strongly influences the results. It is a strong outlier on a dataset with an effective sample size of 20 - especially since the systematic scenario has no data between a baseline negative error rate of 0.000 and 0.165. When removing the two observations that have no negative error, all coefficients strongly decrease in size and become non-significant.

B.2.2 Rerun Convergence

The rerun convergence ratios are again higher than the initial convergence ratios, since the starting positions of the algorithm are closer to the solution than a random starting position. They fluctuate around 87.5%, which is consistent with the random error scenario. Both the graph in Figure B.2b and the results in Model 2 of Table B.2 suggest an absence of a clear effect of both types of error on rerun convergence ratio (β = 0.014, p = .985; β = 0.055, p = .441). This is in contrast with the random error scenario, where positive error leads to a small but significant increase of the rerun convergence ratio.

Table B.2: Logistic regression models to assess the relation between the amount of introduced positive and systematic (baseline) negative error and their interaction, and the convergence rate of the initial run and rerun under default SAOM settings. The unit of analysis is the error-combination: for each unique combination of errors, the probability of convergence is assessed.

	1: Initial Convergence			**2: Rerun Convergence**		
	β (SE)	OR	p	β (SE)	OR	p
Intercept	0.478 (0.021)		<.001	1.881 (0.048)		<.001
Baseline Negative Error (Centered)	-0.764 (0.209)	0.465	<.001	0.194 (0.489)	1.214	.692
Positive Error (0 = 0, 0.001 = 1)	-0.093 (0.029)	0.911	.001	0.051 (0.067)	1.052	.448
Interaction Neg x Pos	0.867 (0.292)	2.380	.003	0.414 (0.669)	1.513	.536
χ^2	29.93		<.001	2.429		.488
df	16			16		

B.3 Conclusion

Although no definitive conclusions can be drawn from the analyses presented above, the results suggest that more negative error leads to a lower probability of a SAOM converging in one attempt under the default SAOM settings. However, this effect is mostly driven by the scenario with no negative error: when there are no errors in the data, convergence seems to be much easier. This means that data sets with either random or systematic negative measurement error may require non-standard settings of the estimation algorithm to converge more quickly.

On the other hand, random or systematic negative error does not decrease convergence probability in a second run, when a more informative starting solution is used. Convergence is slightly (but not significantly) higher for datasets with more negative error. A potential explanation for this is that datasets with negative measurement error tend to need a longer time to reach convergence: although they are less likely to converge initially, non-converged runs tend to still converge after more simulations and parameter updates. This suggests the maximum number of iterations should be increased in the initial SAOM estimation, so that the model can converge in one go.

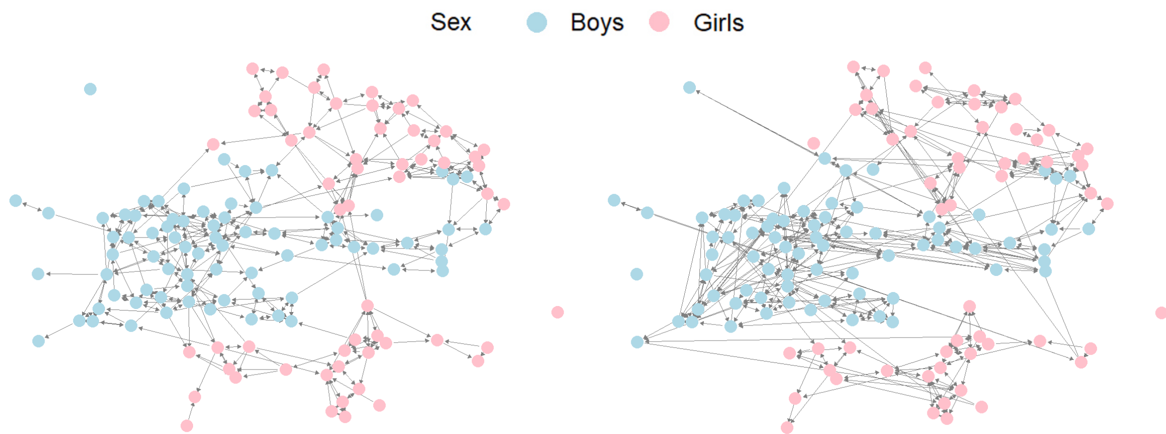
Positive error in itself also seems to decrease the initial convergence probability: in the scenario with no negative error, the positive error substantially decreases the initial convergence rate by adding aditional noise. This may therefore also require non-standard algorithm settings, with potentially more iterations, to reach convergence without needing a rerun.

In presence of both positive and negative error, the results are less consistent across models and seem to be more complex. The data presented here are insufficient to properly identify these patterns.

Appendix C

Network Visualizations

C.1 Observed Networks

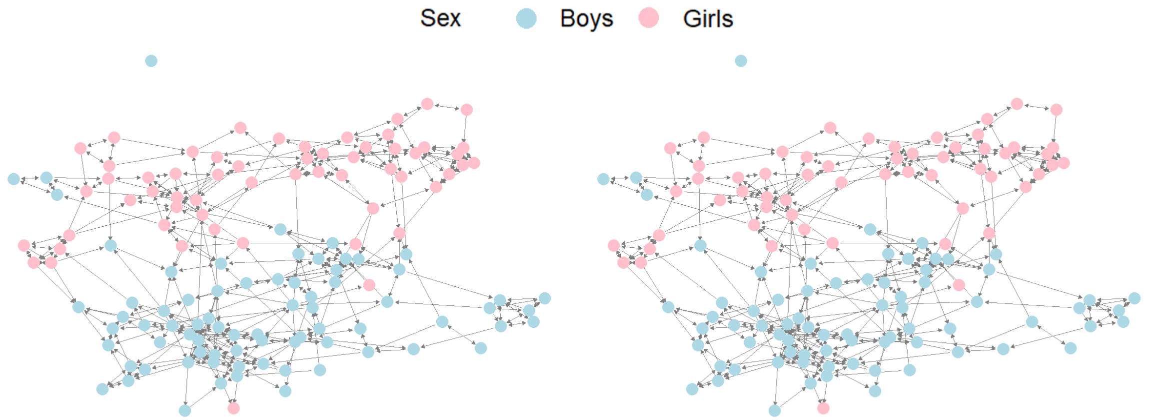


(a) Wave 2 (Start Network)

(b) Wave 3 (End Network)

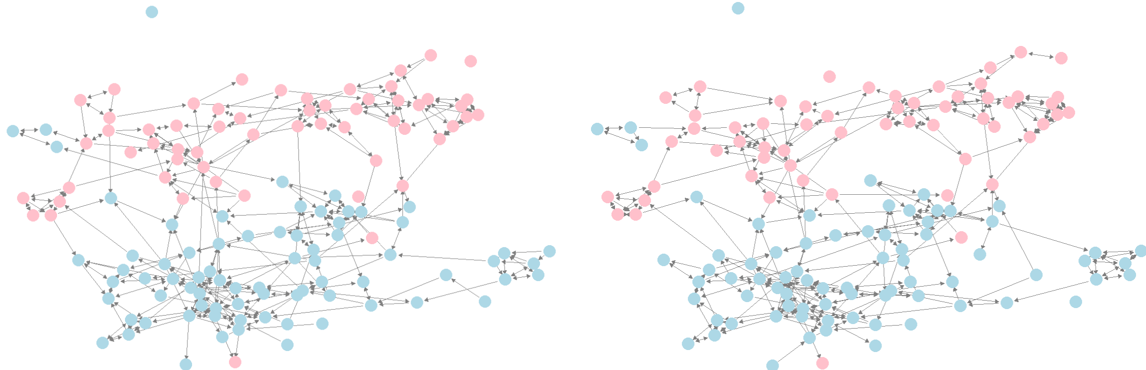
Figure C.1: Visualizations of the observed Glasgow data networks, with nodes coloured by student sex. Nodes are positioned to clearly show network changes between Wave 2 and 3.

C.2 Error-Induced Networks



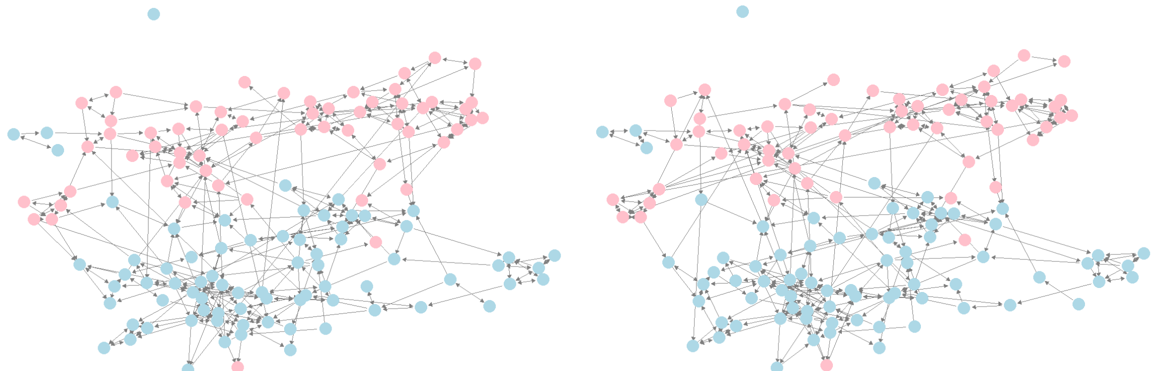
(a) No error

(b) No error



(c) 0.20 Random negative error without positive error

(d) 0.36 Systematic baseline negative error without positive error



(e) 0.20 Random negative error with positive error

(f) 0.36 Systematic baseline negative error with positive error

Figure C.2: Visualizations of a simulated end network with and without measurement error. Figures on the left show the random error scenario, and figures on the right show the systematic error scenario. Figures a and b are equal, and are both displayed for ease of comparison.

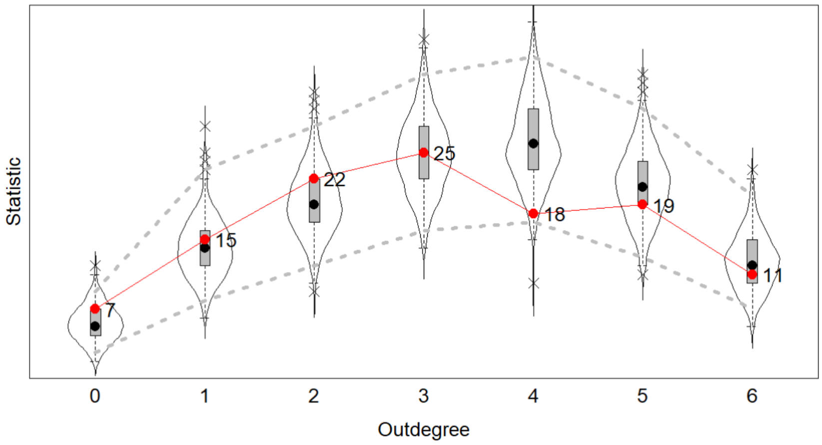
Appendix D

SAOM Fit to Empirical Data

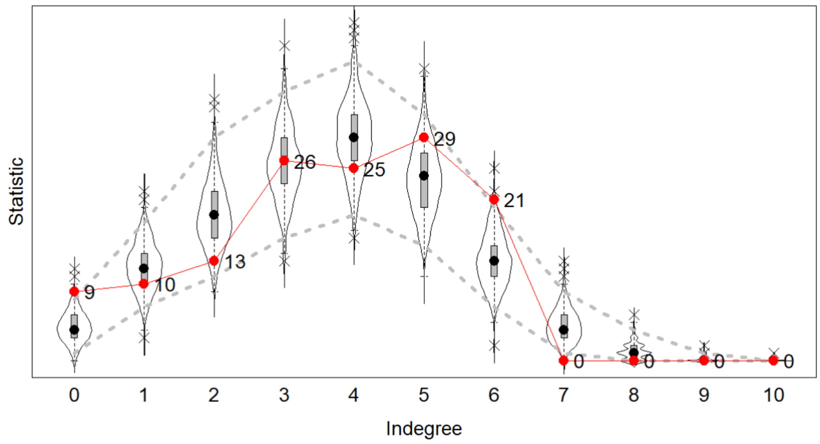
One of the objectives of this study was to assess the effects of measurement error in a scenario that mimics an actual research project. Therefore, the aim was to use a "true" model that has features that occur in real social networks. To achieve this, a model was calibrated to the Glasgow study data (Michell & West, 1995): a study that addressess the evolution of friendship networks among pupils over time. This model was then used to simulate network trajectories that have similar network dynamics to the Glasgow study data, and thus have elements of true friendship data such as tendencies towards homophily and reciprocity.

The true model was calibrated to fit the Glasgow data well. In Figure D.1, various plots show how well the observed network attributes in the Glasgow data (the red line) compare to the attributes of the networks that are simulated by the estimated model. Goodness of fit tests were also executed, to determine if the observed indegree, outdegree and triad census distributions deviate strongly from the distributions in the simulated networks.

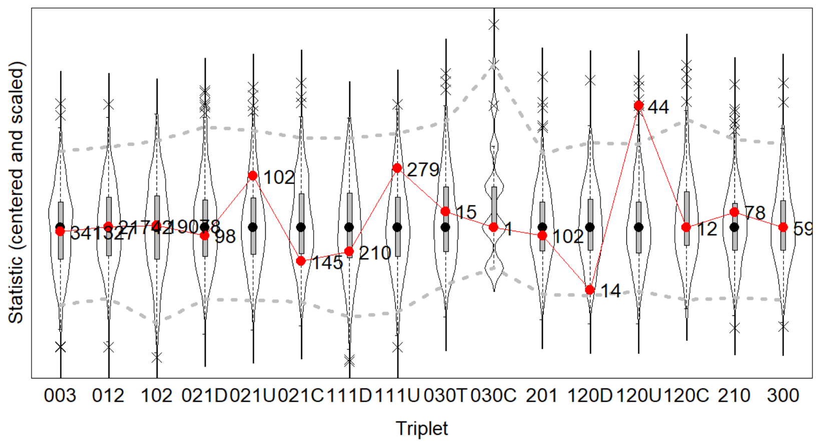
The model fits the outdegree distribution of the observed networks relatively well. The distribution of actor outdegree values in simulated networks does not significantly deviate from the observed networks ($p = 0.376$). The indegree fits less well: the simulated networks have fewer actors with indegree 5 or 6, and a few more with 7 or higher. The observed data does not contain actors with an indegree value of 7 or more. The deviation of the indegree distributions is not strong enough to be statistically significant ($p = 0.052$), but the distributions vary enough to yield a relatively low p-value. The variation in indegree is therefore slightly smaller in the studied "true model" than in a real friendship network. Lastly, the distribution of the triad census counts in simulated networks also matches the observed data relatively well: there is no significant difference ($p = 0.182$). Notable differences are found in the 120U and 120D triplets, which respectively occur more and less often in the observed data.



(a) Outdegree Goodness of Fit



(b) Indegree Goodness of Fit



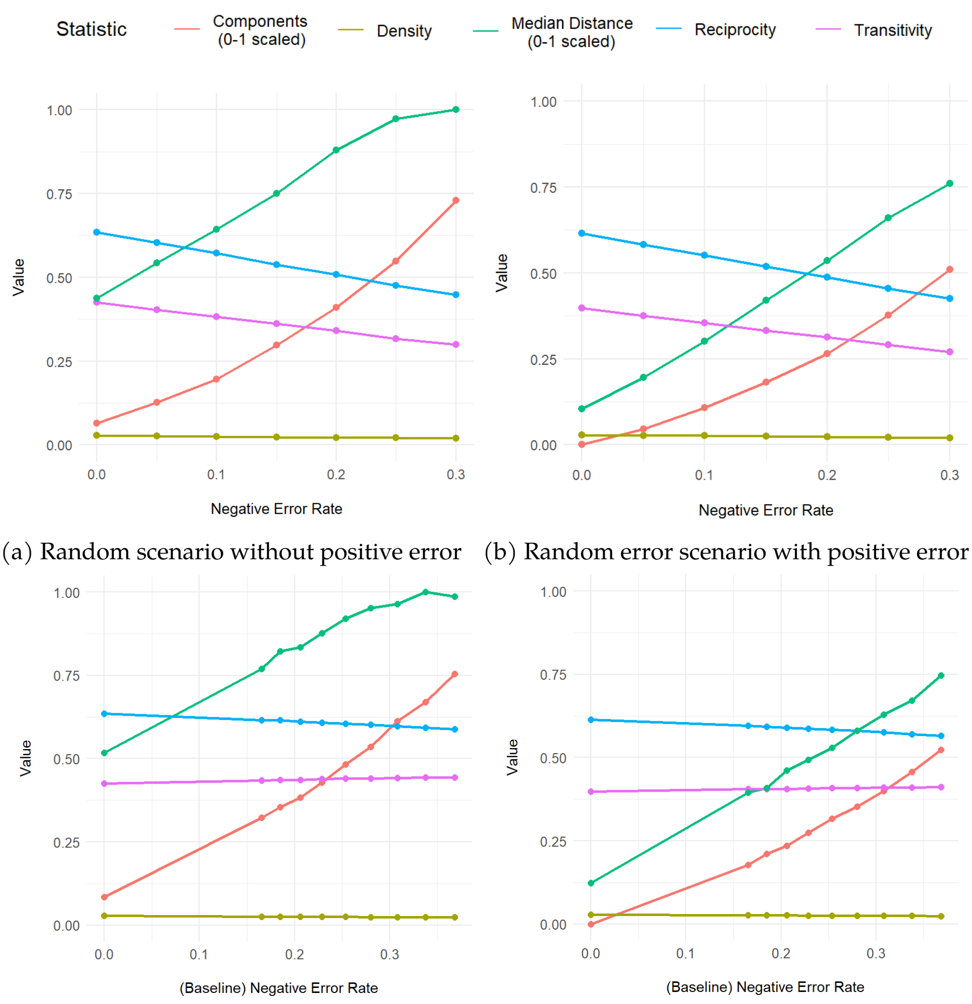
(c) Triad Census Goodness of Fit

Figure D.1: Goodness of fit plots for the "true" model on Wave 2 and Wave 3 of the Glasgow data. The red line indicates the statistics from the observed networks. The distributions show the statistics in networks that are simulated from the estimated true model (without error).

Appendix E

Network Statistics

E.1 Descriptive Network Statistics End Networks



(a) Random scenario without positive error

(b) Random error scenario with positive error

(c) Systematic error scenario without positive error

(d) Systematic error scenario with positive error

Figure E.1: Network statistics by (baseline) negative error rate in the random and systematic scenarios, with and without positive error, in the error-induced end networks. Number of components and median distance are brought to a 0-1 scale using min-max normalization.

E.2 Triad Census Plots

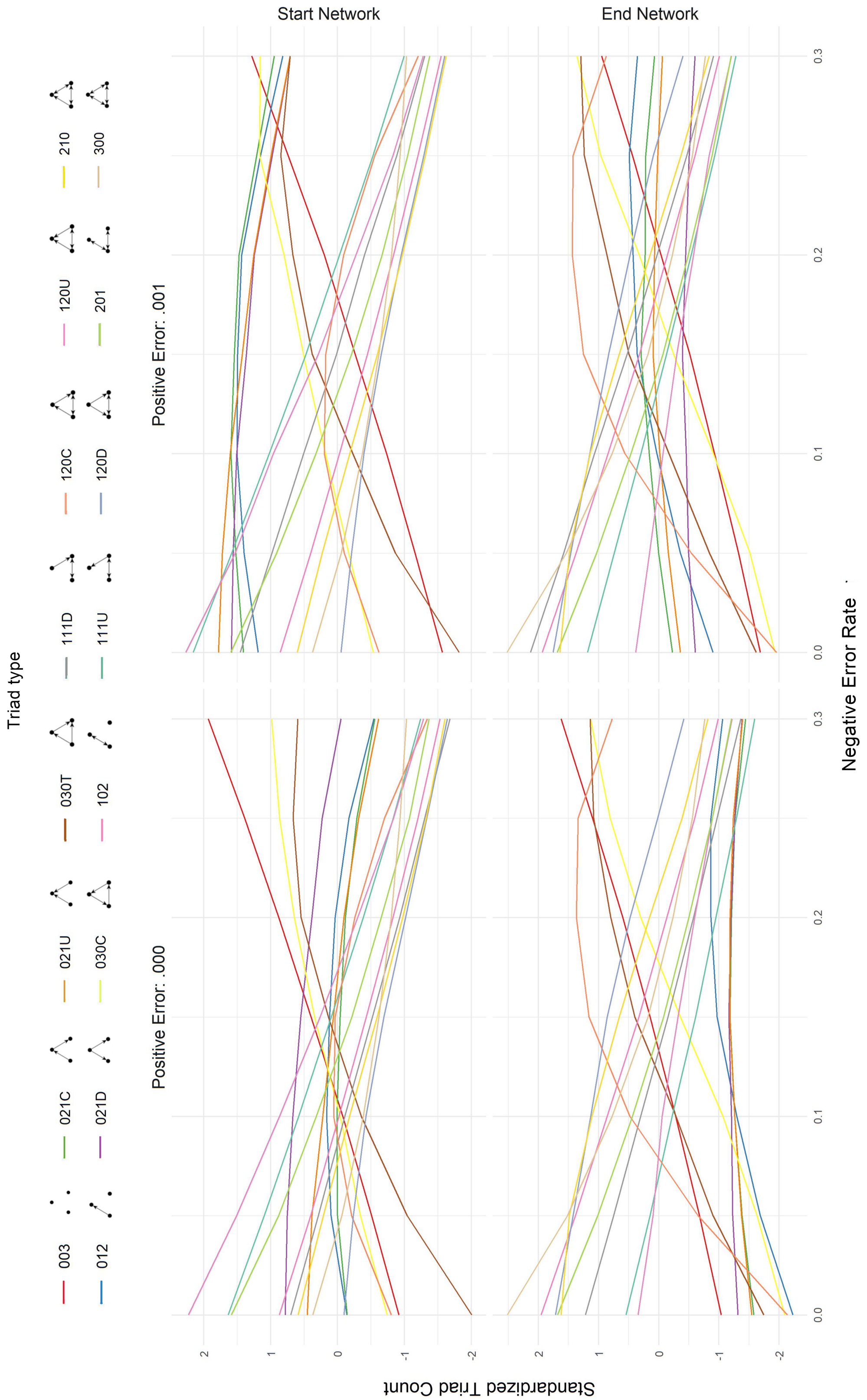


Figure E.2: Average standardized triad census counts by negative error rates for the random error scenario. Counts are standardized per triad type, across all simulation runs and error rates in the random scenario. The counts are displayed for both the end and the start network, with and without positive error.

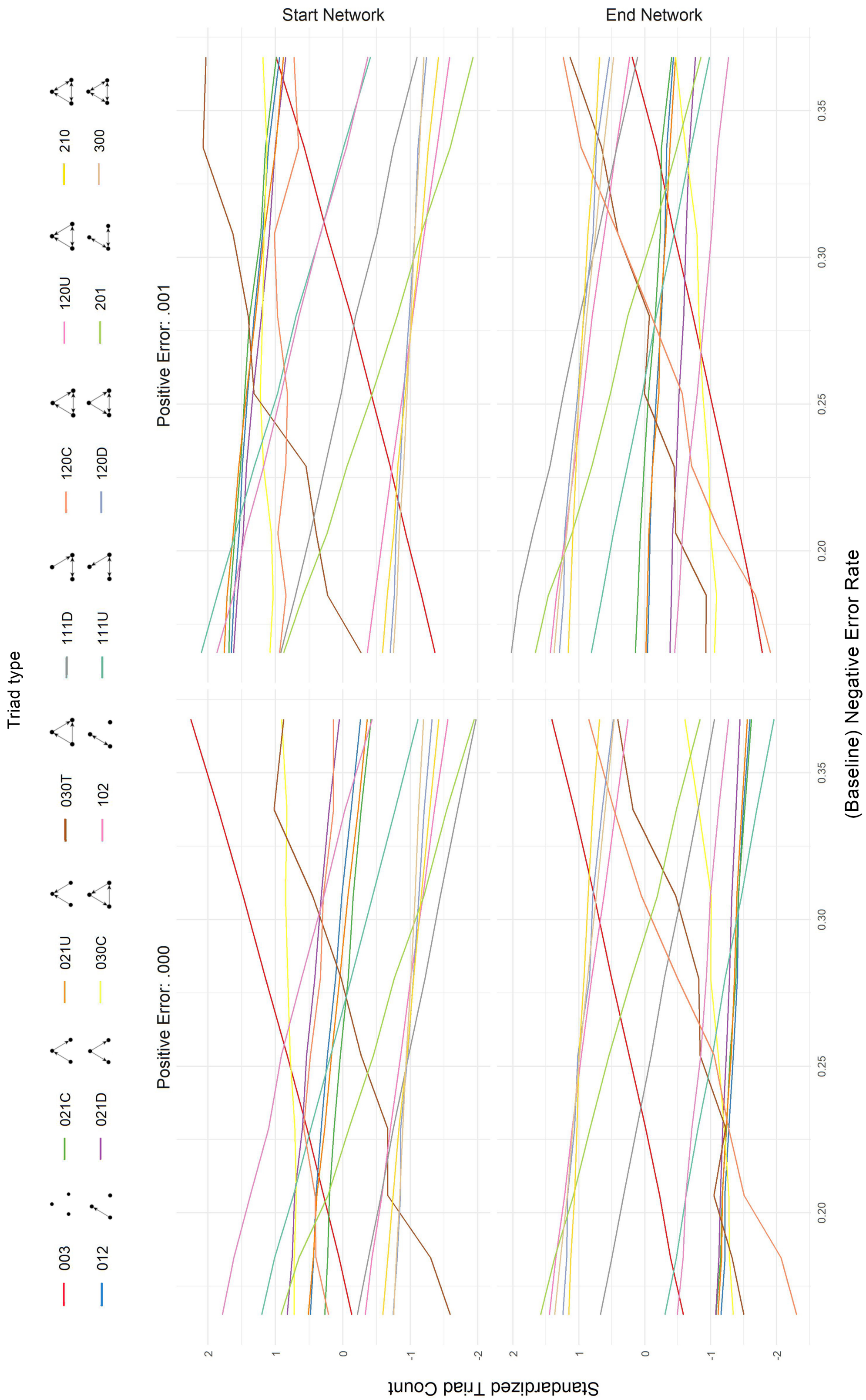
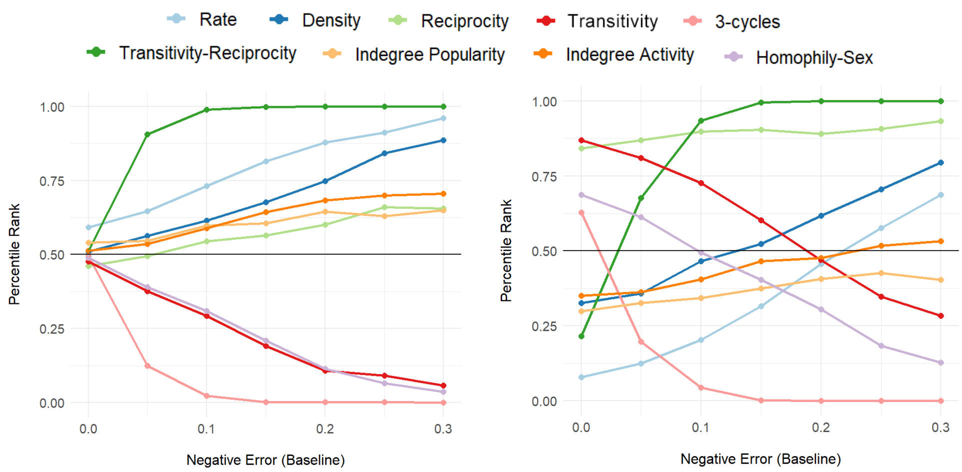


Figure E.3: Average standardized triad census counts by baseline negative error rates for the embeddedness-based systematic error scenario. Counts are standardized per triad type, across all simulation runs and error rates in the systematic scenario. The counts are displayed for both the end and the start network, with and without positive error.

Appendix F

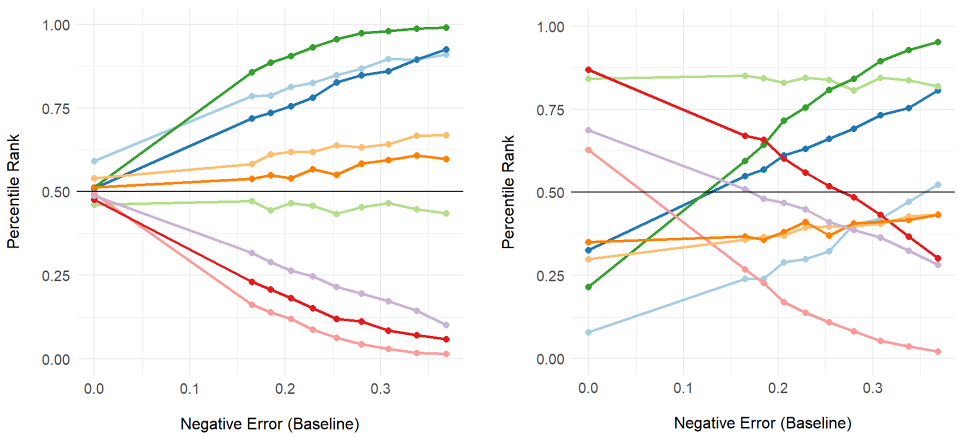
Additional Parameter Visualizations

F.1 Percentile Rank Plots



(a) Random error scenario, without positive error

(b) Random error scenario, with positive error



(c) Systematic error scenario, without positive error

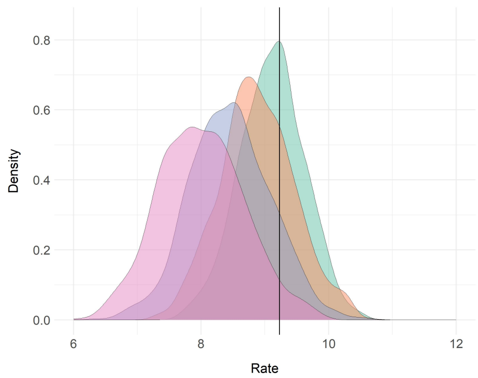
(d) Systematic error scenario, with positive error

Figure F.1: The average percentile of the true parameter value in the distribution of estimated parameter values in the simulated data, by negative error rate. Plots are shown for the random and systematic error scenario, with and without positive error.

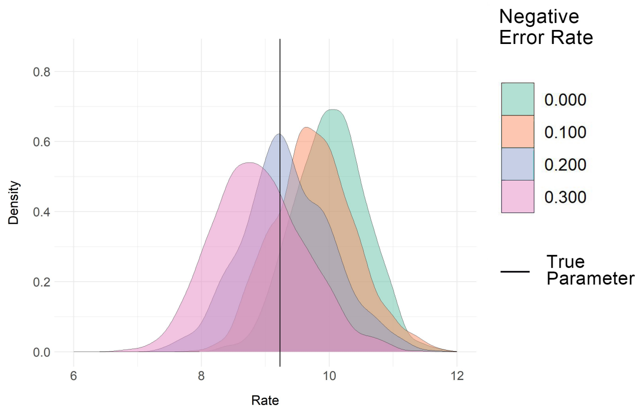
F.2 Density Plots of Parameter Estimates by Error

In the plots below, the density plots of the parameter estimates are shown by error scenario and combination of error rates to further illustrate the effects of measurement error on the estimates. Note that the scenarios are not directly comparable. The negative error rates shown for the systematic scenario refer to the *baseline* negative error rate.

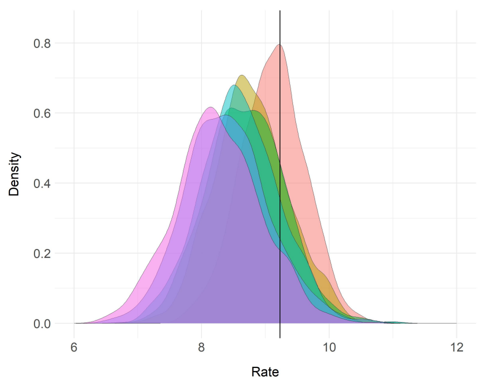
Rate Parameter



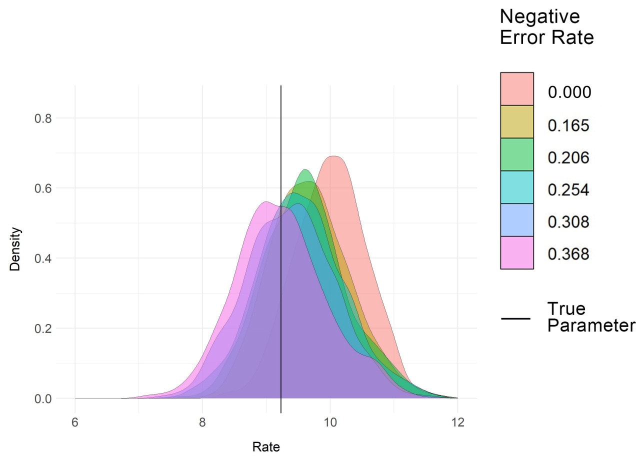
(a) Random error scenario, without positive error



(b) Random error scenario, with positive error



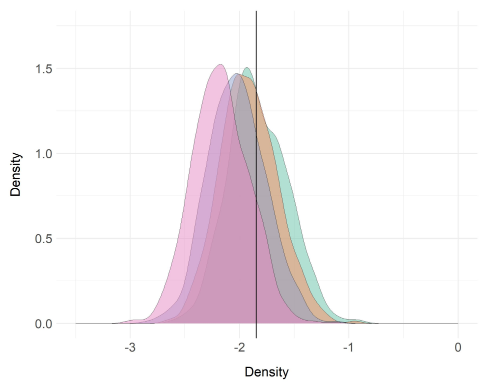
(c) Systematic error scenario, without positive error



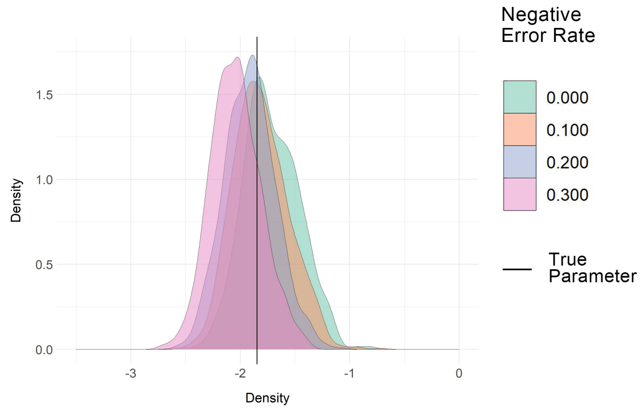
(d) Systematic error scenario, with positive error

Figure F.2: Density plots of estimates of the rate parameter in the simulated data by negative error rate. Plots are displayed for the random and systematic scenario, with and without positive error.

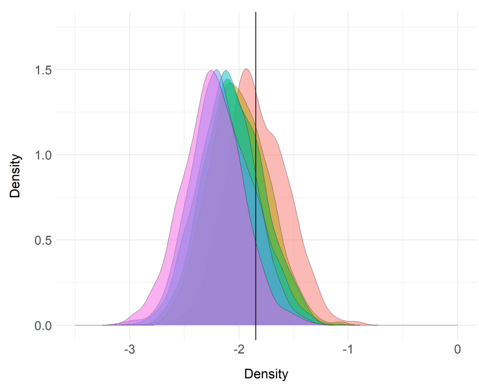
Density Effect



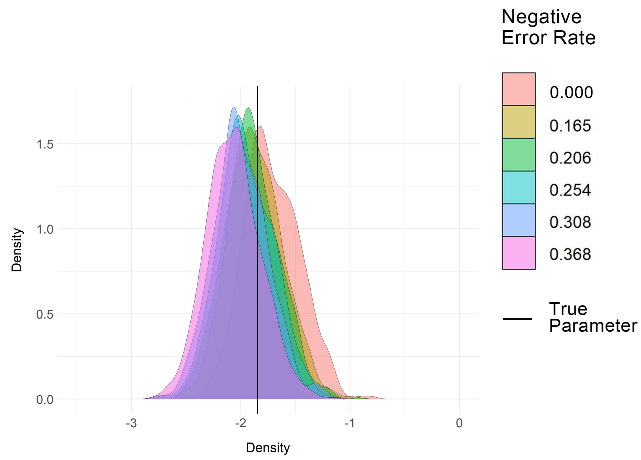
(a) Random error scenario, without positive error



(b) Random error scenario, with positive error



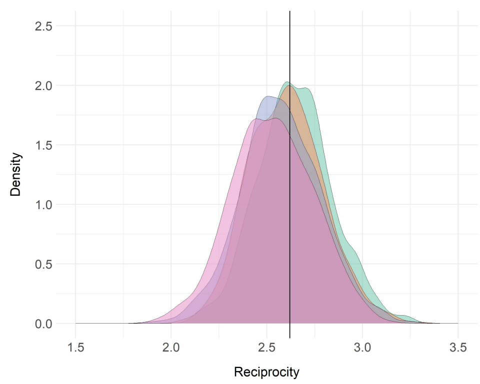
(c) Systematic error scenario, without positive error



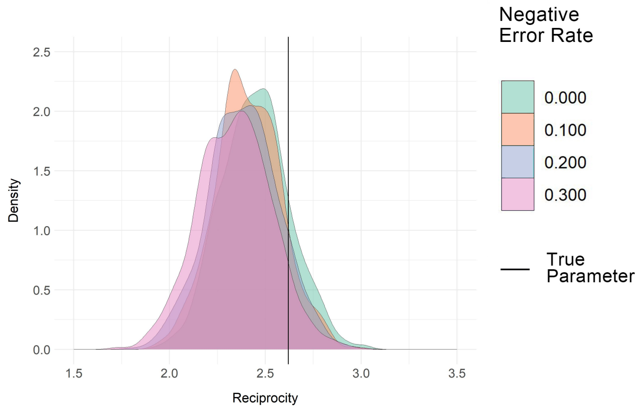
(d) Systematic error scenario, with positive error

Figure F.3: Density plots of parameter estimates of the density effect in the simulated data by negative error rate. Plots are displayed for the random and systematic scenario, with and without positive error.

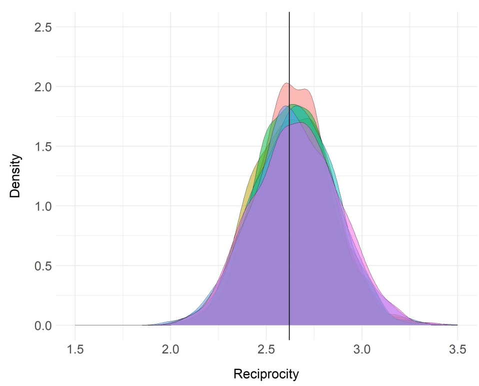
Reciprocity Effect



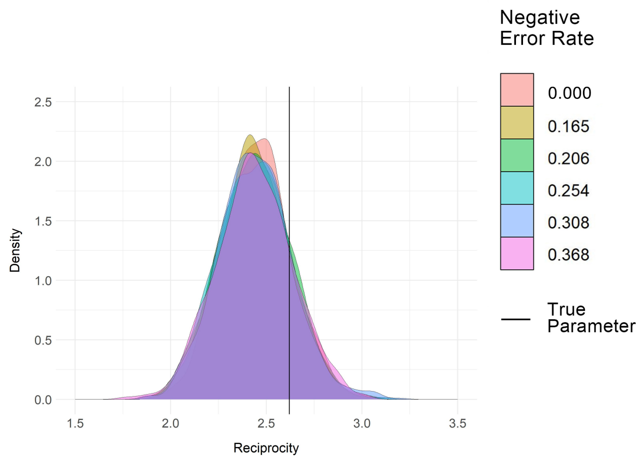
(a) Random error scenario, without positive error



(b) Random error scenario, with positive error



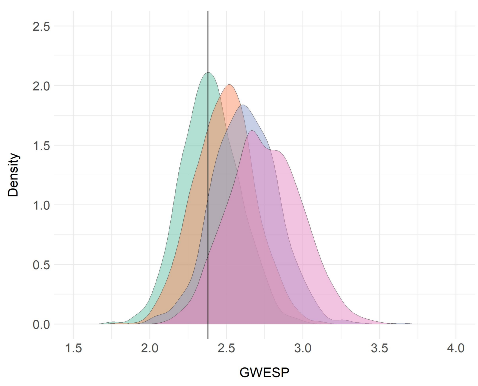
(c) Systematic error scenario, without positive error



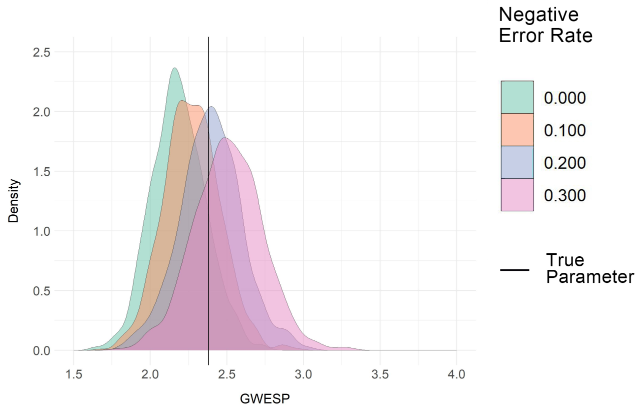
(d) Systematic error scenario, with positive error

Figure F.4: Density plots of parameter estimates of the reciprocity effect in the simulated data by negative error rate. Plots are displayed for the random and systematic scenario, with and without positive error.

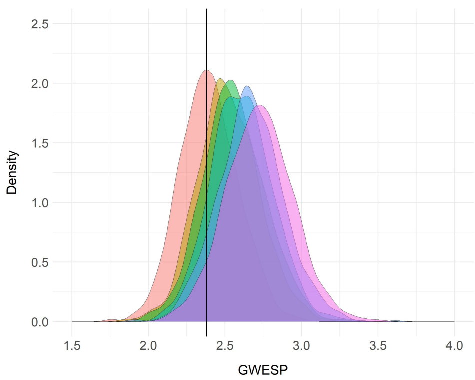
Transitivity (gwespFF) Effect



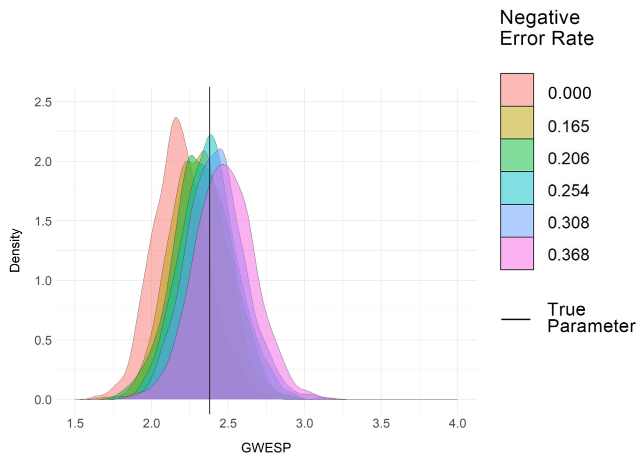
(a) Random error scenario, without positive error



(b) Random error scenario, with positive error



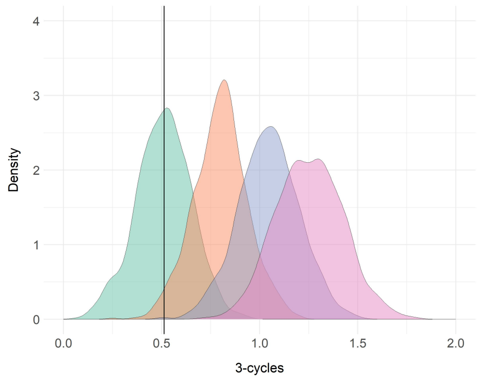
(c) Systematic error scenario, without positive error



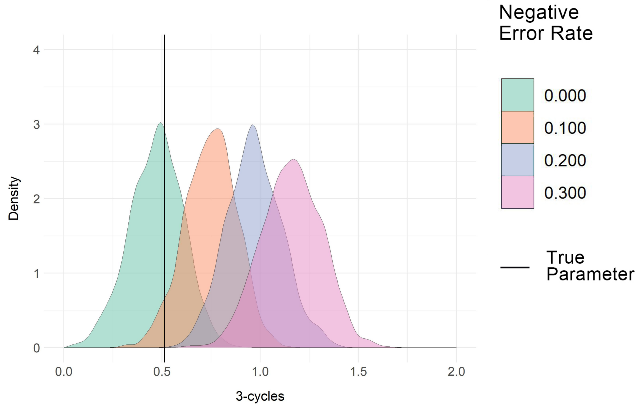
(d) Systematic error scenario, with positive error

Figure F.5: Density plots of parameter estimates of the Transitivity effect (gwespFF) in the simulated data by negative error rate. Plots are displayed for the random and systematic scenario, with and without positive error.

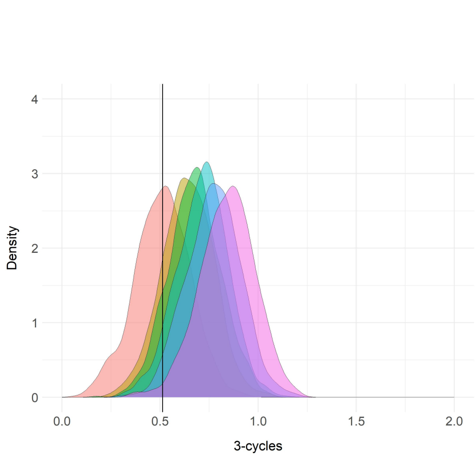
3-Cycles Effect



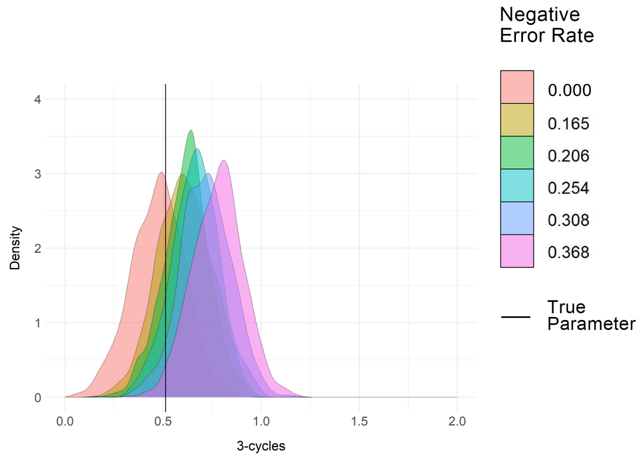
(a) Random error scenario, without positive error



(b) Random error scenario, with positive error



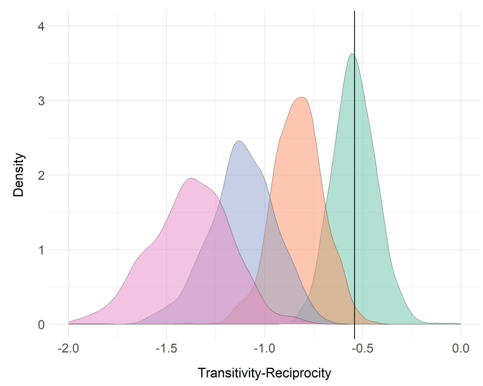
(c) Systematic error scenario, without positive error



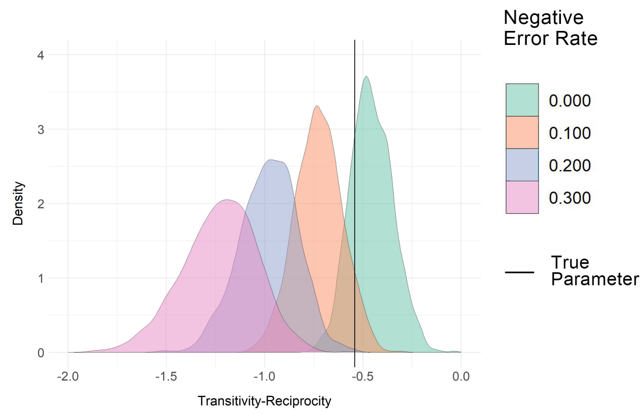
(d) Systematic error scenario, with positive error

Figure F.6: Density plots of parameter estimates in simulated data of the 3-cycles effect by negative error rate. Plots are displayed for the random and systematic scenario, with and without positive error.

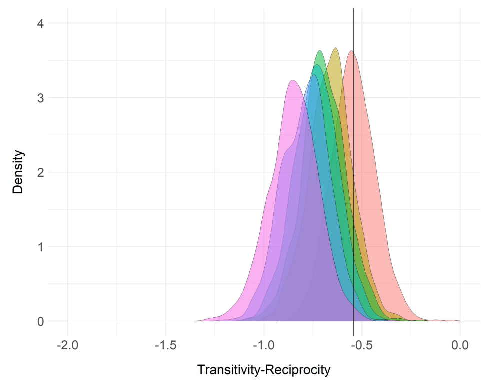
Transitive Reciprocated Triplets Effect



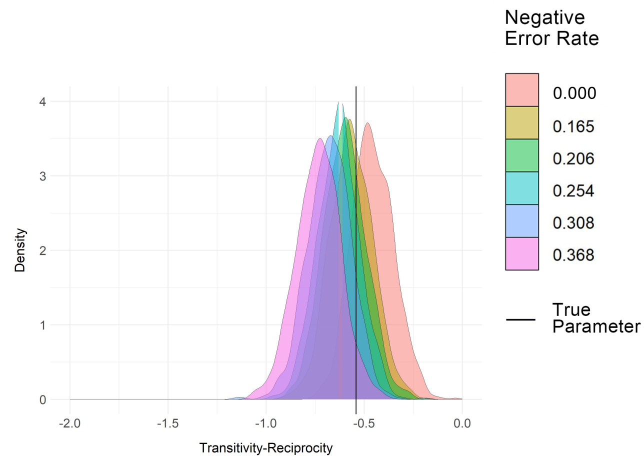
(a) Random error scenario, without positive error



(b) Random error scenario, with positive error



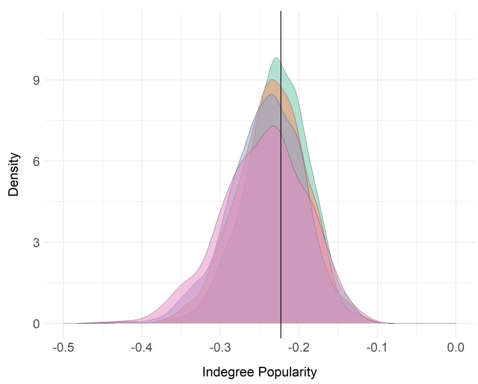
(c) Systematic error scenario, without positive error



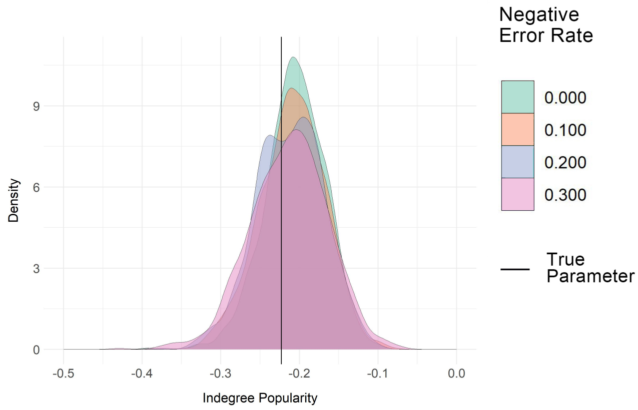
(d) Systematic error scenario, with positive error

Figure F.7: Density plots of parameter estimates in simulated data of the reciprocated triplets effect by negative error rate. Plots are displayed for the random and systematic scenario, with and without positive error.

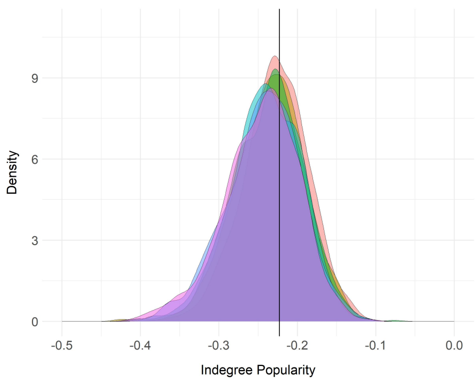
Indegree Popularity Effect



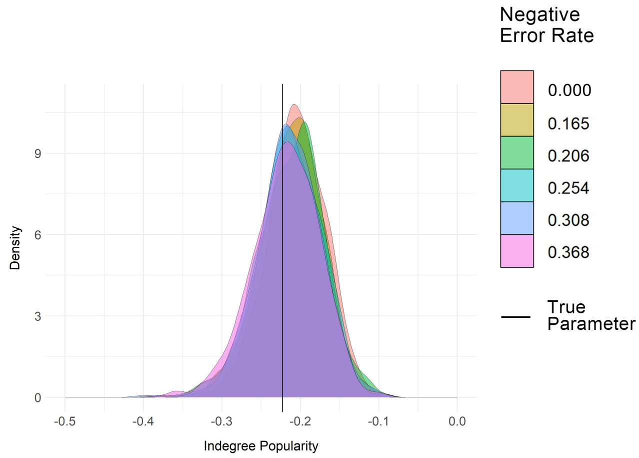
(a) Random error scenario, without positive error



(b) Random error scenario, with positive error



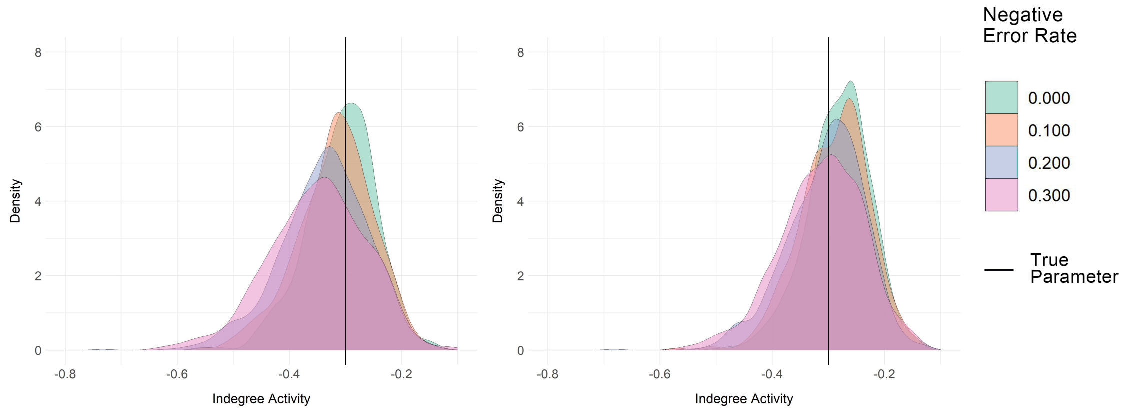
(c) Systematic error scenario, without positive error



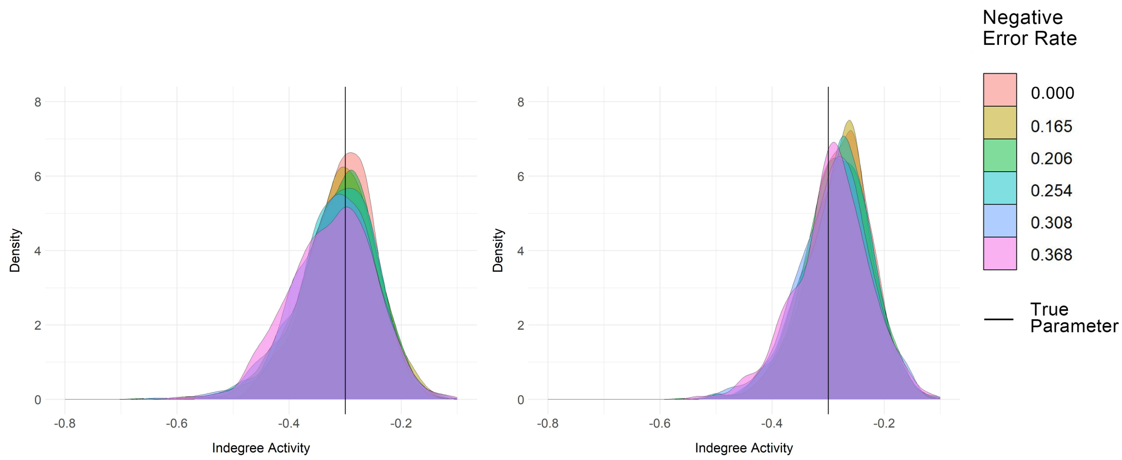
(d) Systematic error scenario, with positive error

Figure F.8: Density plots of parameter estimates in simulated data of the indegree popularity effect by negative error rate. Plots are displayed for the random and systematic scenario, with and without positive error.

Indegree Activity Effect



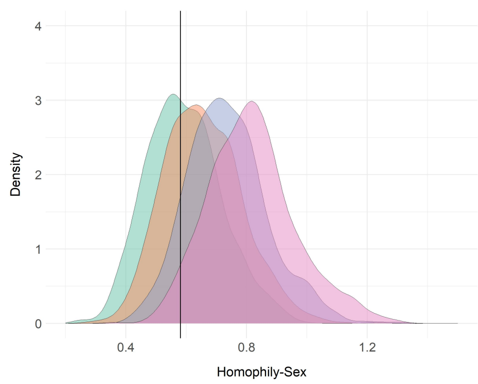
(a) Random error scenario, without positive error (b) Random error scenario, with positive error



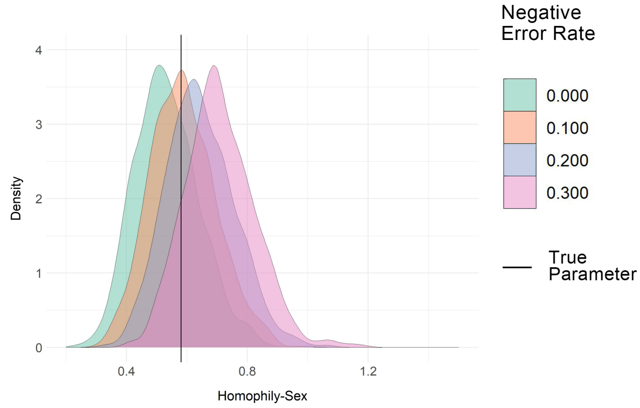
(c) Systematic error scenario, without positive error (d) Systematic error scenario, with positive error

Figure F.9: Density plots of parameter estimates in simulated data of the indegree activity effect by negative error rate. Plots are displayed for the random and systematic scenario, with and without positive error.

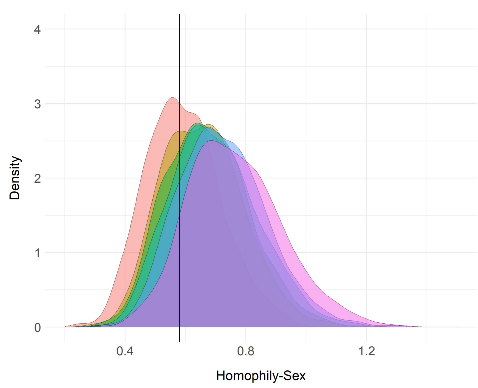
Sex-based Homophily Effect



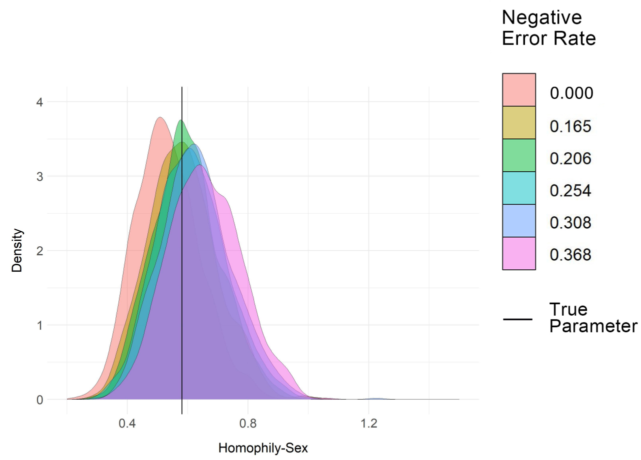
(a) Random error scenario, without positive error



(b) Random error scenario, with positive error



(c) Systematic error scenario, without positive error



(d) Systematic error scenario, with positive error

Figure F.10: Density plots of parameter estimates in simulated data of the sex-based homophily effect by negative error rate. Plots are displayed for the random and systematic scenario, with and without positive error.

Appendix G

Scenario Comparison

Table G.1: Comparison of the real parameter to the estimates in the random error scenario (negative error rate = 0.200) and systematic error scenario (baseline negative error rate = 0.368), without positive error. Observed relative bias and coverage are given for both scenarios. The two scenarios have starting networks with comparable densities, and can therefore be compared.

	Real Parameter	**Random Error**			**Systematic Error**		
		Estimate	**Relative Bias**	**Coverage**	**Estimate**	**Relative Bias**	**Coverage**
Rate	9.228	8.457	-8.35%	87.73%	8.262	-10.47%	83.02%
Density	-1.846	-2.022	9.50%	89.30%	-2.243	21.44%	69.71%
Reciprocity	2.620	2.576	-1.69%	93.50%	2.658	1.46%	95.39%
Transitivity	2.380	2.625	10.31%	81.23%	2.725	14.51%	68.45%
3-Cycle	0.514	1.046	103.59%	8.70%	0.842	63.89%	44.03%
Rec. Triplets	-0.542	-1.116	106.02%	3.35%	-0.848	56.59%	31.24%
Indegree Pop.	-0.223	-0.242	8.51%	94.34%	-0.246	10.34%	94.03%
Indegree Act.	-0.300	-0.338	12.83%	94.76%	-0.324	8.12%	95.70%
Homophily (Sex)	0.581	0.736	26.62%	85.64%	0.764	31.46%	82.70%