Review of the Diamond-Dybvig Model for Bank Runs, including its Extensions and Applications

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1 Introduction

Banks play a vital role in the stability and economy of the financial world. While these entities are often highly regulated, they are still subject to a phenomenon known as a bank run, even in the modern world. These runs can occur when a large number of customers who have deposited their money at a bank all make a simultaneous attempt to withdraw their financial assets. Historically, these runs are most likely to happen when there is a loss of faith in the bank either due to internal issues such as bad investments or external issues such as an economic downturn. They can even still happen in today's modern technological world, such as the Silicon Valley Bank crisis in March of 2023, proving that there is still a place for bank run models.

The Diamond-Dybvig Model is an economic model that simulates the environment required for a run and explores how banks function as intermediaries that create liquid claims against illiquid assets [1]. The main idea behind the model involves banks providing services to both depositors, who prefer accounts with liquid assets for easy access to funds, and debtors or loan-takers who prefer long-maturity, low-liquidity loans. Debtors can be both businesses and single individuals who wish to make a big purchase, such as a house. This paper aims to give an overview of the original model from Diamond and Dybvig, then explore some criticisms and extensions of the model from Ross and Cooper while synthesizing additional sources. Finally, the paper will look into the collapse of Silicon Valley Bank and attempt to answer if the Diamond-Dybvig model can explain what went wrong.

1.1 3 Main Ingredients

As stated by Wallace (1988), there are three main ideas or ingredients to the Diamond and Dybvig model. The first is that individual customers are uncertain about when they will need to spend money. This leads to a demand for "liquid assets", or assets which can be easily converted into cash within a short period of time. The second is known as the *sequential service constraint*, which is the idea that different people spend money in succession. For example, the withdrawal demands of separate customers must be dealt with separately and not all at the same time. In other words, the bank's reserve cash management problem becomes an inventory problem, but the exact definition will be later discussed in further detail. Finally, the third ingredient is that investment projects in real life are very costly to restart if interrupted. For instance, one can think of assets that must be held to maturity and will result in a loss if they are sold earlier.

Wallace makes the point that these ingredients by themselves are not new. However, Diamond and Dybvig use these ingredients to produce a model which allows the mathematical deduction of the consequences from differing policies applied to bank management. Before getting deep into the model, it may be wise to give an analogy to understand the three ingredients in a more familiar setting before entering the financial world. One such analogy describes a simple camping trip [2].

1.2 Overview Using a Camping Analogy from Wallace (1988)

Wallace starts with a group of N people on the last evening of their camping trip. They are planning two meals: one late-night snack and a breakfast the next morning. The breakfast will be the last meal before their return home. In the evening, each camper has y units of food available. This food will grow if stored, and each unit will become R_1 units if held until the late-night snack and will become R_1R_2 units if held until breakfast. R_1 and R_2 are stated as being fixed returns on the investment.

However, any food taken out from storage at night is either wasted or eaten. This allows for the modeling of the cost of interrupting an investment project (one of the main ingredients). Next, all campers are aware that they will awaken sometime during the night and will either be hungry and prefer to eat then and skip breakfast, or will not be hungry and will prefer to wait and eat at breakfast. This allows for uncertainty of expenditures to be included in the model (another main ingredient). An additional important idea to note is that the campers care about both possibilities, namely how much they will be able to eat during the night and at breakfast depending on when they become hungry. Finally, campers have a rough idea about the fraction of the group that will be hungry during the night.

Before introducing the idea of sequential service (the last main ingredient), Wallace first describes the situations of joint action versus autarky. Take α_1 to be the fraction of the group that will awaken hungry during the night, and $\alpha_2 = 1 - \alpha_1$ as the alternative. Wallace also denotes the quantity of the meal per person in the following way: c_1^1 for the late-night snack if the camper is hungry at night, and c_2^2 if not hungry at night. The subscript denotes the time of the meal: 1 for night, 2 for morning. The superscript describes the camper's state during the night: 1 for hungry and 2 for not hungry. If each camper acts alone, or autarkically, each person will have a late-night snack of $R_1 y$ or a breakfast of $R_1 R_2 y$. Thus, a payoff of $c_1^1 = R_1 y$ and $c_2^2 = R_1 R_2 y$ is achieved. In this way, the total amount of food consumed during the night is $C_1 = \alpha_1 N R_1 y$. If instead campers pool their resources together rather than acting alone, they can plan to have either more or less than C_1. This is what Wallace calls joint action.

In order to determine how much more or less, suppose C_2 is the total amount consumed at breakfast. Given C_1, the maximum of C_2 is $R_2(N R_1 y - C_1)$. As one can see, after the late-night snacks C_1 are subtracted, the remaining amount accumulates at a rate of R_2 and is the amount which will be consumed at breakfast. If the group of campers can divide C_1 equally among the $\alpha_1 N$ people who wake up hungry and divide C_2 equally among the $\alpha_2 N$ people who wake up not hungry, then $c_1^1 = C_1/(\alpha_1 N)$ and $c_2^2 = C_2/(\alpha_2 N)$. Taking $C_2 = R_2(N R_1 y - C_1)$ and substituting the expressions with respect to C_1 and C_2 results in:

$$c_2^2 = R_2(R_1 y - \alpha_1 c_1^1)/\alpha_2.$$

Now the possible combinations arising from joint action can be compared with autarky, as seen in Figure 1 below. Here the non-negative combinations (c_1^1, c_2^2) that satisfy the above equation are shown.

According to Wallace, if campers were able to rank all combinations on the line, there is no reason that the autarkic combination would be the favorite, and instead a combination below and to the right would be preferable. The point in question is labeled as $(\tilde{c_1^1}, \tilde{c_2^2})$. In other literature, the reasoning behind this combination being more preferable is due to relative risk aversion assumptions which will be discussed later [3]. Note that the preferred amounts involve campers eating less for breakfast and more for a late-night snack; this is a point involving marginal utility benefit and cost which we will come to later.

Now that the idea of joint action is explained, the third ingredient can be introduced. Suppose that the campers are isolated from each other and cannot meet during the night. Thus, they cannot come together to decide with certainty how many people are hungry during the night and produce an efficient and feasible amount for the late-night snacks and breakfasts. Now, any arrangement must follow the rule that campers wake at a random time and do not interact with others during the night. Therefore, late-night withdrawal demands from the shared resources must be handled sequentially, hence the name *sequential service*. In order for the group to deal with this new situation, suppose that the campers have a sort of cash-machine-like robot which

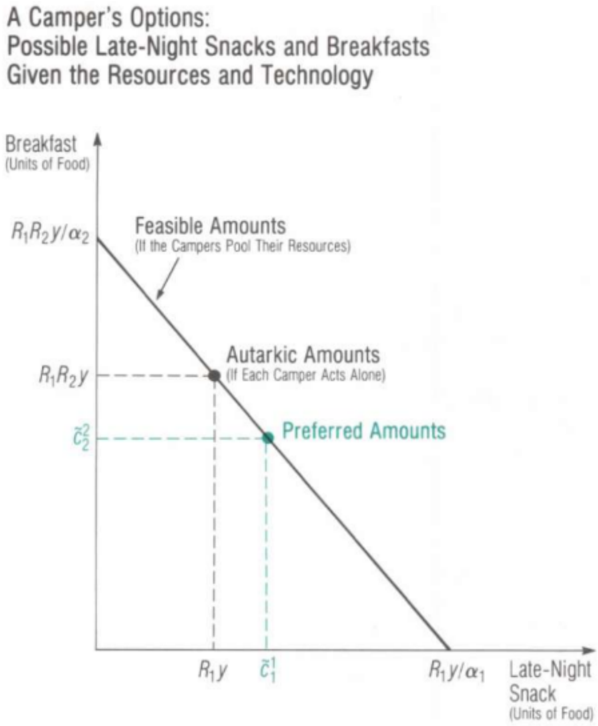

Figure 1: Taken from Wallace (1988)

is stocked with and can dispense food for people. The food inside the robot also grows with the returns R_1 and R_2. While the machine can check identities and prevent withdrawals from a different account, it cannot check if a person is actually hungry or making the night withdrawal for some other reason, an example being their worry that the robot will run out of food and there will be nothing for breakfast. As the robot can be programmed to accumulate withdrawals and make future withdrawals depend on past ones, the problem then becomes how campers should stock and program the robot such that the group obtains their preferred amounts and is the happiest with their meals. Keep in mind that not using the robot at all is also an option in this scenario. While a camping trip is far from the world of banking and finance, the idea behind the Diamond and Dybvig bank run model is quite similar. Using their paper and its extensions, we will take a look in the next sections at how the campers can solve their dilemma (though we will omit the analogy). From here, we can begin to explore the ingredients in further mathematical and financial detail.

2 Diamond and Dybvig Model and Techniques Used

The original paper begins with a model of periods $T = 0, 1, 2$, and payoff $R > 1$ in period 2 for every monetary unit given in period 0. If the unit(s) are withdrawn in period 1, the value is just the initial investment. In reality, R can be thought of as the payoff from a long-term investment, and withdrawing in period 1 is analogous to withdrawing funds before the investment project comes to fruition. The model then specifies between two different types of consumers, or *agents* as they are called. Each consumer learns what type they are in period 1. Type 1 agents care about consumption in period 1 and derive utility only from this period, while type 2 agents are willing to wait until period 2 to receive their utility. This effectively models consumers who need to withdraw their money early versus consumers who can afford to wait and collect interest on their investment. A fraction $t \in (0, 1)$ determines the ratio of type 1 agents to type 2 agents. For example, if $t = \frac{1}{4}$, it infers that one-fourth of the continuum of agents are type 1 and care about period 1 consumption. Here, the aim is to model bank customers who must withdraw early due to impending payments, such as bills or large necessary repairs.

2.1 Agent Utility

The model first utilizes a utility function to determine why agents act in certain ways. Each agent j has a state-dependent utility function which has the form:

$$U(c_1, c_2; \theta) = \begin{cases} u(c_1), & \text{if } j \text{ is of type 1 in state } \theta \\ \rho u(c_1 + c_2), & \text{if } j \text{ is of type 2 in state } \theta \end{cases}$$

where c_T represents goods or money received by an agent at period T. The goods can be either stored or consumed depending on the type of agent. Here, state θ can be thought of as the circumstances surrounding the agent and their outcome which can affect their utility or decision process. For example, if the bank does not have enough liquidity to pay a type 2 agent back in period 2, the agent may decide to withdraw at period 1, thus changing their utility. While θ is not defined in detail nor used within the paper itself, state-dependent utility functions are described in other literature surrounding economic theory [4].

ρ is defined as $R^{-1} < \rho \leq 1$, and $u : \boldsymbol{R}_{++} \rightarrow \boldsymbol{R}$ is twice continuously differentiable, increasing, strictly concave, and satisfies Inada conditions $u'(0) = \infty$ and $u'(\infty) = 0$ in order to make it a viable utility function.

As mentioned by Kang (2020), ρ is used to represent the rate at which type 2 agents discount future utility [5]. In this way, cost of waiting can be measured and balanced against receiving a quicker payoff. Notice that their utility cannot be discounted lower than by a factor of 1/R, as then it's better to be type 1 anyway.

Agents are also assumed to have high risk aversion (the relative risk aversion coefficient is greater than 1), and will run to withdraw their money at the slightest hint of trouble as well as be more willing to engage in risk sharing. This reflects real world conditions where depositors are not risk seeking with the money they place in their checking account. Finally, agents wish to maximize their expected utility: $E[U(c_1, c_2; \theta)]$.

2.2 Assets Held Directly

In order to demonstrate the roles banks can play, Diamond and Dybvig first consider the situation where there is no bank (assets are held by agents directly), and types are revealed privately and observed privately. All agents then invest in period 0, and their type is revealed privately in period 1. Then type 1 agents liquidate their investment in period 1 and consume 1, while type 2 agents liquidate their investment in period 2 and consume R. If c_k^i represents consumption in period k of an agent of type i, then we have $c_1^1 = 1$, $c_2^1 = c_1^2 = 0$, and $c_2^2 = R$. This equilibrium is known as 'autarky' which was also mentioned by Wallace above [6].

2.3 Optimal Consumption

On the other hand, if types would be publicly observed at period 1, then a type of insurance contract could be written. This would allow agents to insure against the probability of becoming a type 1 agent by increasing their period 1 consumption while decreasing their period 2 consumption and would further increase their utility, hence why they would wish to enter into such a contract. This is known as *risk sharing* and is viable due to agents' risk adverseness. According to Diamond and Dybvig, optimal consumption written as $\{c_k^{i*}\}$ satisfies the following constraints:

$$c_1^{2*} = c_2^{1*} = 0 \text{ [If agents can delay consumption, they will do so]} \tag{1}$$

$$u'(c_1^{1*}) = \rho R u'(c_2^{2*}) \text{ [Marginal benefit equal to marginal cost]} \tag{2}$$

$$tc_1^{1*} + [(1-t)c_2^{2*}/R] = 1 \text{ [Resource constraint].} \tag{3}$$

Using the fact that $\rho R > 1$, and the relative risk aversion exceeds 1, Diamond and Dybvig conclude that the optimal consumption levels satisfy $c_1^{1*} > 1$ and $c_2^{2*} < R$. Thus, the competitive outcome where types are revealed privately can be improved. One can also note that $c_2^{2*} > c_1^{1*}$ by equation (2), again using that $\rho R > 1$. In other words, agents accept a lower consumption in period 2 for a reasonable return if they cash out before the investment project reaches maturity. This is where financial intermediaries such as banks step in. Banks are able to pool resources and pay agents what they will respectively consume at the different periods.

2.4 Demand Deposit Contract Model

But how would a bank go about doing this? A demand deposit contract can be thought of as a basic checking account. It is an agreement between a depositor and a bank where the depositor can withdraw their funds as soon as they require. One more assumption Diamond and Dybvig use is that the bank is "mutually owned" and liquidated in period 2 where the proceeds go to the agents withdrawing in period 2. Therefore, the amount each type 2 agent gets is calculated after the withdrawals at period 1. This of course can give type 2 agents incentive to also withdraw in period 1 if they believe that the bank will run out of assets before it can be liquidated. The following equations are used as a basis for such a model, with V_T being the payoff per unit deposit withdrawn in period T:

$$V_1(f_j, r_1) = \begin{cases} r_1, & \text{if } f_j < r_1^{-1} \\ 0, & \text{if } f_j \geq r_1^{-1} \end{cases}$$

$$V_2(f, r_1) = \max\left\{\frac{R(1-r_1 f)}{1-f}, 0\right\}.$$

The contract gives each agent that withdraws in period 1 a payoff of r_1 per unit deposited at $T = 0$, and f_j is the number of withdrawals completed before agent j written as a fraction of total demand deposits. f is then the total number of demand deposits withdrawn in $T = 1$ over the total demand deposits created. Note that $1/r_1$ is used as stopping time since f_j is a fraction.

Additionally, Diamond and Dybvig explain the idea of a sequential service constraint. This is defined as "a bank's payoff to any agent can depend only on the agent's place in line and not on future information about agents behind him in line." Hence V_1 is dependent on the location of a single agent f_j, rather than how many agents there are in total.

This reflects the real world situation where a bank cannot look into the future and see how many more agents will withdraw, but rather can only service the agents *currently* withdrawing. Diamond and Dybvig assume that a bank must obey this constraint when delivering withdrawals. Thus, if $T = 1$, V_T is dependent on an agent's place in line, and if $T = 2$, it is dependent on the total withdrawals at $T = 1$. Breaking down $V_2(f, r_1)$, one sees that it is the investment outcome R, times the amount of resources left in the bank after the first round of withdrawals at $T = 1$ (that is $1 - r_1 f$), and divided among the fraction of agents who did not withdraw at $T = 1$. The maximum with 0 is used in the case that the bank runs out of money before the second round of withdrawals can be paid.

2.5 Suspension of Convertibility Improvement

Now, Diamond and Dybvig attempt to model how a bank can suddenly stop withdrawals in the event of a run. Almost everything is identical to the previous model, besides that here they take a fraction $\hat{f} < r_1^{-1}$ as the "stopping point". If any agent tries to withdraw in the first period after $\hat{f}$ of all deposits have been withdrawn, the agent receives nothing, hence:

$$V_1(f_j, r_1) = \begin{cases} r_1, & \text{if } f_j < \hat{f} \\ 0, & \text{if } f_j \geq \hat{f} \end{cases}$$

$$V_2(f, r_1) = \max \left\{ \frac{R(1 - r_1 f)}{1 - f}, \frac{R(1 - r_1 \hat{f})}{1 - \hat{f}} \right\}.$$

Unfortunately, this only works optimally when the fraction of type 1 agents is known (t). Optimally, consumption is the same for all agents of a certain type, and depends on the realization of t. However, if t is stochastic, denoted as $\tilde{t}$, this system fails due to the incompatibility with the sequential service constraint. This is described in the following proposition from the paper:

Diamond and Dybvig Proposition 1. *Bank contracts (which must obey the sequential service constraint) cannot achieve optimal risk sharing when t is stochastic and has a non-degenerate distribution.*

The proof can be found in the Appendix.

The proof can be found at the end of this thesis, but simply put, Diamond and Dybvig argue that a bank cannot optimally make use of the suspension of convertibility if they do not know the fraction of type 1 agents. The main problem occurs when $\tilde{f}$ is less than the largest possible realization of $\tilde{t}$. In that case, there is an inefficiency as some type 1 agents cannot withdraw. Diamond and Dybvig argue that this is where government deposit insurance shines.

2.6 Government Deposit Insurance Improvement

Finally, Diamond and Dybvig create a system revolving around deposit insurance, where a secure financial institution secures the deposits of agents to ensure that the payoff of a bank run is limited. Firstly, they assume that the government is the best financial institution to provide insurance. This also implies that the government can raise a tax to pay for this insurance if it is needed. The tax is dependent on the number of withdrawals. They denote the tax as a function of f, $\tau : [0,1] \to [0,1]$ such that:

$$\tau(f) = \begin{cases} 1 - \frac{c_1^{1*}(f)}{r_1}, & \text{if } f \leq \bar{t} \\ 1 - \frac{1}{r_1}, & \text{if } f > \bar{t}. \end{cases} \tag{4}$$

Here $\bar{t}$ is the greatest possible realization of $\tilde{t}$. Now they denote $\hat{V}_1(f)$ as the after-tax payoff, which is given by the following equation:

$$\hat{V}_1(f) = \begin{cases} c_1^{1*}(f), & \text{if } f \leq \bar{t} \\ 1, & \text{if } f > \bar{t}. \end{cases} \tag{5}$$

$c_1^{1*}(f)$ is the optimum consumption for a type 1 agent at period 1 dependent on the total fraction of withdrawals over deposits. Then, the net payments to the agents who withdraw at period 1 determine the after-tax value of the withdrawals at period 2. Thus, $\hat{V}_2(f)$ denotes the after-tax proceeds per initial deposit unit of a withdrawal at $T = 2$:

$$\hat{V}_2(f) = \begin{cases} \frac{R(1-[c_1^{1*}(f)f])}{1-f} = c_2^{2*}(f), & \text{if } f \leq \bar{t} \\ \frac{R(1-f)}{1-f} = R, & \text{if } f > \bar{t}. \end{cases} \tag{6}$$

Seeing as how $\hat{V}_1(f) < \hat{V}_2(f)$ for all $f \in [0,1]$, no type 2 agents will want to withdraw in the first period, even if they anticipate a run. Additionally, $\hat{V}_1(f) > 0$ shows that all type 1 agents will wish to withdraw in the first period as they care only about consumption at $T = 1$.

2.7 Findings

The Diamond-Dybvig model demonstrates that there are two Nash equilibria when not considering the autarky solution. The first achieves optimal risk sharing where type 1 agents withdraw at $T = 1$ while type 2 agents wait until $T = 2$ with no issues for any party. The second is that of a bank run, where all agents panic and attempt to withdraw at $T = 1$ as there is less utility in withdrawing later. They also find that any liquidity offering service is subject to runs.

The first improvement they make to the model, suspension of convertibility, shows that if the fraction of type 1 agents t is constant and known, the run equilibrium is avoided and the first equilibrium mentioned above is a dominant strategy equilibrium. However, in the more general case where t is stochastic, suspension of convertibility fails and the bank cannot achieve optimal risk sharing (proposition 1).

The second and final improvement reveals that it never helps the consumers to participate in a bank run as long as government deposit insurance is in place. Diamond and Dybvig demonstrate that demand deposit contracts with this insurance achieve the unconstrained consumer optimum as a unique Nash equilibrium. This is also a dominant strategy equilibrium. They show this with the following proposition:

Diamond and Dybvig Proposition 2. *Demand deposit contracts with government deposit insurance achieve the unconstrained optimum as a unique Nash equilibrium if the government imposes an optimal tax to finance the deposit insurance.*

The proof follows equations from section 2.6 and can be found in the Appendix.

2.8 Conclusions of the Diamond-Dybvig Model

Historically, banks often tried to stop bank runs using "suspension of convertibility," and shut off the ability of the customers to make withdrawals completely [1]. While this method can be effective at blocking a run taking place, it simply does not address the underlying fear that prompted the run in the first place, and can adversely affect consumer trust in the bank. Additionally, it can result in some depositors still not having access to their funds when they need them.

Diamond and Dybvig argue that deposit insurance is a preferable alternative when it comes to managing bank runs, as opposed to suspension of convertibility practices. They conclude that deposit insurance issued by a central bank or federal government can solve this issue while still giving the best utility to all types of agents.

However, there are potential downsides to deposit insurance not discussed in this paper. Banks can often be incentivized through this insurance to make riskier investments than they would otherwise. After all, if the deposits are insured by the federal government, the risk of insolvency is much lower. This is a concept known as *moral hazard* and it is explored in mathematical detail by Cooper and Ross [7].

3 Improvements by Cooper and Ross

Moral hazard is defined as "...an incentive to take unusual risks in a desperate attempt to earn a profit before the contract settles" [8]. An example from the field of banking and investment theory would be an investor who has incentive to take unnecessary risks which may endanger the funds entrusted to them. This was a major problem in the 1980s during what became known in the United States as the savings and loan (S&L) crisis. This event led to taxpayers' having to shoulder an enormous amount of bad debt despite, or perhaps even due to, deposit insurance secured by the government. Cooper and Ross attempt to not only model moral hazard, but also explain the crisis itself in an effort to show how deposit insurance alone will not solve the problem of bank runs. Even though the insurance may avoid bank runs, it can have adverse effects to the incentives of both depositors and bank managers.

3.1 Techniques

The techniques used by Cooper and Ross are at first quite similar to those from Diamond and Dybvig. They begin once again with periods $T = 0, 1, 2$ and the two types of agents. A fraction π[1] learn that they obtain utility from early consumption (period 1), and the rest obtain utility from late consumption (period 2). They assume π is nonstochastic and known to all agents. They describe a utility function $U(c)$ and the consumption levels for period 1 and period 2 consumers, c_E and c_L respectively.

Similar to the original paper, there are two technologies available: an illiquid productive technology, which offers a return of $R > 1$ in the second period, and a storage technology, which gives one unit in period $T + 1$ per unit of period T investment. Again, the realistic example for the illiquid productive technology would be interest gained from a deposit while the storage technology could be a simple checking account.

3.2 Constraints and Contracts

Assuming it is free and possible to verify agent types, an intermediary would offer an optimum contract $\delta^* = (c_E^*, c_L^*)$ which solves the following expression:

$$\max_{c_E, c_L} \pi U(c_E) + (1 - \pi)U(c_L) \tag{7}$$

$$\text{s.t. } 1 = \pi c_E + \frac{(1 - \pi)c_L}{R}.$$

The idea is similar to the optimum consumption from Diamond and Dybvig. The first equation is necessary to maximize the utility for all types of consumers, hence the utility function and fraction π. The second equation is the resource constraint and ensures that consumption by agents is limited.

If consumer tastes are private information, however, multiple equilibria including a "bank run" equilibrium can exist. A bank can respond to this in one of two ways: either they can write a contract that is not vulnerable to bank runs at all, or they can alternatively look at which equilibrium is likely and reduce the impact of runs should they occur [9]. The first method involves adding the additional constraint $c_E \leq 1$; this way there are always enough period 1 resources to pay consumers. In fact, the best runs-preventing contract will require $c_E = 1$ and

[1] π here is analogous to t from Diamond and Dybvig

$c_L = R$, though this is not necessarily optimal consumption for agents, as shown by Diamond and Dybvig.

The alternative method involves the variable q defined as the probability of economy-wide pessimism leading to a bank run. Here, a publicly observable event is what determines the behavior of depositors, which is known as a *sunspot* in the literature. Therefore, the contract now solves:

$$\max_{c_E, c_L} (1-q)[\pi U(c_E) + (1-\pi)U(c_L)] + qU(c_E)(1/c_E)$$

$$\text{s.t. } 1 = \pi c_E + \frac{(1-\pi)c_L}{R}.$$

It is assumed that the early consumption c_E is greater than 1, otherwise this contract is dominated by the best runs-preventing contract where $c_E = 1$ and $c_L = R$. One notes that, in the event of a run, higher early consumption decreases overall utility as the probability of an agent actually receiving c_E is much lower. This q will be included later in the extended model with depositor monitoring, but before that, Cooper and Ross first give depositors a reason to monitor; namely an investment option with a higher paying, but riskier, investment option for the bank manager.

3.3 Extended Model: Richer Investment Option

In order to fully encompass what moral hazard implies, Cooper and Ross create an additional multiperiod technology which yields one unit at $T = 1$ and a return of λR with probability ν and nothing otherwise at $T = 2$. The risky investment should have a higher return upon success, but a lower expected return than the riskless technology. Thus assume that $\lambda > 1$ and $\nu\lambda \leq 1$. Also assume that the government provides complete deposit insurance such that even late consumers receive a complete payment, with notation $I(c_E)$ and $I(c_L)$ for the payments given to early and late consumers respectively. For example, if $I(c_L) = c_L$, the government is paying full deposit insurance to late consumers.

Finally, assume that any funds remaining after the payment to late consumers are retained by the shareholders of the bank. Then, with i indicating the amount of resources per unit of deposit that the bank invests into the risky fund, the following equation represents the money retained by the shareholders with the first-best contract:

$$\max_i [\nu(i\lambda R + (1 - i - \pi c_E^*)R - (1-\pi)c_L^*) + (1-\nu)\max((1 - i - \pi c_E^*)R - (1-\pi)c_L^*, 0)].$$

A very interesting fact to note is that the the best solution to the equation is for the intermediary to place all funds in the risky investment. This is because the bank will earn no profits from the riskless investment as all money goes to the depositors, and if the risky investment fails, the bank also has no return. Therefore there is no incentive for the manager to invest anything in the riskless illiquid technology as there is no payoff. One may ask why the depositors accept the bank taking such risks, but one should remember that the government will fully reimburse them should the bank become insolvent. Thus, the agents also have no reason to oppose this moral hazard.

3.4 Extended Model: Monitoring

The second change from the Diamond and Dybvig model is depositor monitoring. The main idea is that a depositor can "check" the bank's investment at $T = 0$ and force the bank to change their portfolio to what the depositor wants. The depositor incurs a utility loss Γ in doing so. Cooper and Ross state that if moral hazard is present, monitoring will occur if and only if:

$$(1-\pi)(1-\nu)(1-q)[U(c_L) - U(I(c_L))] \geq \Gamma.$$

The inequality above takes into account several conditions:

1. Monitoring is only useful to type 2 agents as the yield in period 1 is the same for both investments, and monitoring happens before the agents learn their types. $[1-\pi]$
2. Without monitoring, the bank will use the risky investment which yields c_L with probability ν. Hence the risky investment's probability of failure cannot be too low otherwise there is no incentive in monitoring. $[1-\nu]$
3. There is no point in monitoring if there is a bank run, since once again both investments yield the same return in period 1. Thus the probability of optimism also cannot be too low. $[1-q]$
4. If the government pays too much deposit insurance, i.e. $I(c_L) \approx c_L$, there is also no incentive to monitor. $[U(c_L) - U(I(c_L))]$

3.5 Extended Model: Capital Requirements

The final extension from Cooper and Ross is the inclusion of capital requirements to the liquidity of a bank. This is done by the government requiring that the shareholders contribute κ units per unit of deposit to the bank. Then the bank's portfolio is determined by:

$$\max_i [\nu(i\lambda R + (\kappa+1-i-\pi c_E)R - (1-\pi)c_L) + (1-\nu)\max((\kappa+1-i-\pi c_E)R - (1-\pi)c_L, 0)]. \quad (8)$$

From the following proposition, Cooper and Ross formally show that if the capital requirement κ is large enough, shareholders will not prefer the risky investment and moral hazard is no longer an issue. Additionally, with complete deposit insurance, bank runs are also no longer a problem, and depositors will not have to lower their utility by monitoring.

Cooper and Ross Proposition 1. *If $I(c_L) = c_L$ for $c_L \leq c_L^*$, $I(c_L) = c_L^*$ for $c_L > c_L^*$, $I(c_E) = c_E$ for $c_E \leq c_E^*$, $I(c_E) = c_E^*$ for $c_E > c_E^*$, and $\kappa \geq \kappa^* \equiv [\nu(\lambda-1)]/[1-\lambda\nu]$, then the first-best allocation of (c_E^*, c_L^*) is achievable without bank runs and without monitoring.*

The proof can be found in the Appendix.

3.6 Findings

An interesting finding of Cooper and Ross involves the parameters q and κ, and their critical levels within the model. The q, representing the probability of economy-wide pessimism, and it's critical point, $q^* \in (0,1)$, show the best type of contract given a particular economic situation. If pessimism in the economy is large and runs are probable, then the best runs-preventing contract dominates the best contract with runs. The opposite is true if there is economic optimism and q is small enough. Formally, this is written as $q > q^*$ for the first scenario, and $q < q^*$ for the second.
On the other hand, the κ representing a capital requirement by the government also has a critical value. As seen in proposition 1, this value is

$$\kappa \geq \kappa^* \equiv \frac{\nu(\lambda-1)}{1-\lambda\nu}.$$

Any κ lower than κ^* will create an incentive to be exposed to a risky investment. One point of interest are the effects of a parameter change on this critical level. If one increases λ and decreases ν in such a way that $\lambda\nu$ remains the same, the critical level will increase. In other words, if a risky asset is less likely to succeed, but the expected return is held constant, higher capital will be required to prevent moral hazard. Additionally, raising only one of the parameters and keeping the other one constant will also raise the minimum capital requirement. If a risky asset is more efficient, one needs to impose tighter requirements to once again avoid moral hazard.

Finally, Cooper and Ross make the statement that deposit insurance does not create the moral hazard problem, as managers would take the same risks for the shareholders with or without the insurance. Deposit insurance instead reduces the incentive of depositors to monitor banks and force changes to their portfolios. Thus, in certain situations deposit insurance can create even more problems than it would solve given the right environment.

3.7 Conclusions from Cooper and Ross

Cooper and Ross make the conclusion that first-best allocation of assets to depositors is possible using a combination of capital requirements and deposit insurance. However, they make the point that in reality, there can be limitations to solutions due to the presence of moral hazard. This can be between bank owners and managers. Additionally, banks and financial institutions may have difficulties in raising and maintaining sufficient equity capital, thus the critical κ may not be feasible. Finally, the level of required capital must also be adjusted by the government in response to changes in the economic environment, which can happen suddenly and frequently.

4 Probabilistic Improvements by Goldstein and Pauzner

Similarly to Cooper and Ross, Goldstein and Pauzner create a modified version of the original Diamond and Dybvig model. They not only explore the maturity mismatch between assets and liabilities leading to bank runs like papers before it, but also seek to address the likelihood of each equilibrium presented by Diamond and Dybvig. Their point being that in one equilibrium, the welfare of the agents is increased and in another, the welfare is decreased. Since Diamond and Dybvig did not give any indication of how likely one equilibrium is to another, the overall utility gained or lost through demand-deposit contracts could not be determined. Therefore, Goldstein and Pauzner modify their model to where the fundamentals of the economy direct the likelihood of a bank run as opposed to agent behavior. In this way, the model has a unique Bayesian equilibrium, where each agent's strategy is optimal given their beliefs about the other agents' private information. Here, a bank run occurs if and only if the economy fundamentals are below a critical value.

4.1 Assumptions

They firstly assume that the fundamentals of the economy are stochastic. Additionally, they assume that agents do not all hold common knowledge of how the economy is doing. They instead observe noisy private signals. This assumption can even be thought of as more realistic as most investors do not share the same information and opinions about how the economy is doing at any given moment. Rather, they obtain their news from separate sources and digest that information with their own private ideas.

Goldstein and Pauzner stress that even if the economy fundamentals uniquely direct a bank run, the runs that occur in their model are driven by bad expectations and are thus still panic-based. The economy does not directly control the agents or their actions, however it serves as a tool which coordinates how they believe that other agents will react. This is both similar and different to Cooper and Ross' q and $q*$. It is similar in that they are both variables to represent the fundamentals of the overall economy and this affects how agents will react. On the other hand, it is different since q is a publicly observable event that determines the behavior of agents, but Goldstein and Pauzner utilize private noisy signals which influence agents but does not control them.

4.2 Basic Framework

4.2.1 The Economy

Goldstein and Pauzner use the same basic framework as both Diamond and Dybvig and Cooper and Ross. Again, type 1 agents consume in period 1 and obtain utility $u(c_1)$ and patient agents can consume in either period with utility $u(c_1 + c_2)$. While not explicitely mentioned in Cooper and Ross, both Diamond and Dybvig and Goldstein and Pauzner define the utility function u as twice continuously differentiable, increasing, and, for any $c \geq 1$, the function has a relative risk-aversion coefficient $-cu''(c)/u'(c) > 1$. Finally, $u(0) = 0$. Again, as previously seen, each unit invested in period 0 gives one unit of output in period 1 if withdrawn. However, a key difference in this model is what happens in period 2. A withdrawal here yields R units of output with probability $p(\theta)$ and 0 units with probability $1 - p(\theta)$. The variable θ is the current state of the economy drawn from a uniform distribution on $[0, 1]$. It is also unknown to agents before period 2. $p(\theta)$ is strictly increasing in θ since the better the state of the economy, the greater the probability that one will receive R. Finally, in order for type 2 agents to obtain greater utility by waiting, $p(\theta)$ also satisfies $\mathbf{E}_\theta[p(\theta)]u(R) > u(1)$.

4.2.2 Risk Sharing and Utility Gain/Loss

Similar to previous definitions of autarky, type 1 (impatient) agents consume one unit in period 1, and type 2 (patient) agents consume R units with probability $p(\theta)$. One detail to note is the importance of risk aversion. Since agents are assumed to be highly risk averse, they gain higher utility by having some insurance in case they become type 1. This insurance takes the form of higher consumption by type 1 agents and lower consumption by type 2 agents, which is stated in Wallace's camping analogy and by Diamond and Dybvig to be preferable. Goldstein and Pauzner also determine this by equating the benefit of type 1 agents to the cost of type 2 agents using the following first-order condition:

$$u'(c_1^{FB}) = Ru'\left(\frac{1-\lambda c_1^{FB}}{1-\lambda}R\right)\mathbf{E}_\theta[p(\theta)].$$

This is very similar in form to equation (2), the second constraint from the original model. On the left hand side, one can see the marginal utility benefit gained by type 1 agents where c_1^{FB} is the optimal or first-best c_1. On the right hand side, one can see the marginal utility cost that type 2 agents must bear where λ is the fraction of type 1 agents (t and π in Diamond and Dybvig and Cooper and Ross respectively). By first observing $c_1 = 1$, it can be seen that the marginal utility benefit is greater than the marginal utility cost. Since $cu'(c)$ is a decreasing function of c due to the coefficient of relative risk aversion being greater than 1, $R > 1$, and $\mathbf{E}_\theta[p(\theta)] \leq 1$, we have $1 \cdot u'(1) > R \cdot u'(R) \cdot \mathbf{E}_\theta[p(\theta)]$. However, we desire the equality, and since the marginal utility benefit is decreasing in c_1 and the marginal utility cost increasing again due to $cu'(c)$ being a decreasing function, c_1^{FB} must be greater than 1. Thus, Goldstein and Pauzner also conclude that risk sharing is optimal.

4.2.3 Banks and Payments

Again in concurrence with Diamond and Dybvig, Goldstein and Pauzner observe demand-deposit contracts used by banks to enable risk sharing when agents' types are not observable. At period 1, an agent obtains a fixed payment of $r_1 > 1$, but at period 2, a patient agent receives a *stochastic* payoff of $\tilde{r_2}$. This is distinct from past models due to the usage of $p(\theta)$. $\tilde{r_2}$ is the remaining non-liquidated investments divided by the number of depositors left after the first period. As usual, the sequential service constraint is assumed. For a simple summary, one can look at the following table based on the one found in the Goldstein-Pauzner paper:

Withdrawal in Period	$f < 1/r_1$	$f \geq 1/r_1$
1	r_1	$\begin{cases} r_1 & \text{with probability } \frac{1}{fr_1} \\ 0 & \text{with probability } 1-\frac{1}{fr_1} \end{cases}$
2	$\begin{cases} \frac{(1-fr_1)}{1-f}R & \text{with probability } p(\theta) \\ 0 & \text{with probability } 1-p(\theta) \end{cases}$	0

Table 1: Subsequent Payments to Agents

Here, f is equivalent to Diamond and Dybvig's f, or the fraction of agents wanting to withdraw early. As one can see, the table has payment values almost the same as the V_T payoff equations shown above. In fact, the original Diamond-Dybvig model is a speical case of the Goldstein-Pauzner model with $\mathbf{E}_\theta[p(\theta)] = 1$. Thus Goldstein and Pauzner have actually generalized the previous model by the inclusion of a stochastic variable.

One criticism of the Diamond-Dybvig model is their assumption that r_1 is the optimal payment from a bank to impatient agents. They make this assumption that the good equilibrium of efficient risk sharing is always selected, rather than using any probabilistic argument. Thus they set c_1^{FB} equal to r_1. Goldstein and Pauzner propose the counter-argument that such a contract is non-optimal if there is a large enough probability of bank runs. This is also supported by Cooper and Ross with their runs-preventing contract. Additionally, using such a contract ignores the possible relation between the amount of liquidity a contract provides and the probability of a bank run. If a high r_1 leads to a higher probability of a bank run due to liquidity issues, the optimal r_1 could fail to be c_1^{FB}. The Goldstein-Pauzner model resolves these problems using their largest model extension: private signals received by agents.

4.3 Private Signals

Assume now that at the start of the first period, each agent receives a private signal about the economic fundamentals. Now, each agent's action depends on their signal. First, assume that state θ is fixed at the start of the first period. However, it is not yet publicly revealed. Each agent i receives a signal of $\theta_i = \theta + \epsilon_i$. Each ϵ_i is a small error term corresponding to agent i and the terms are independently and uniformly diestributed over $[-\epsilon, \epsilon]$. As stated above, one can think of each ϵ_i as private information received about the state of the economy, and θ_i as their overall opinion about the economy and the potential for a return from the bank investment project. Even though agents all have different information and different opinions, the quality of their signal gives them no added advantage.

Starting from period 1, the payoff of a patient agent is contingent on not only θ, but also the fraction of f agents seeking early withdrawal as one can see in Table 1. In this way, with information about θ and f given to an agent through their signal, their choice to wait or to run depends on their personal signal. Using this model, Goldstein and Pauzner make the assumption that there exist ranges of extremely positive or extremely negative economy fundamentals. In these ranges, a type 2 agent's best course of action is not affected by the actions of other agents; for example, the economy is doing so terribly that no matter what other agents are doing, a patient agent should always run.

4.3.1 Lower Dominance Region

In the lower range of θ, the probability of the bank defaulting and type 2 agents receiving a payoff of 0 is very high. Therefore, the expected utility of choosing to wait is lower than the utility gained from withdrawing early. This is true even if all other patient agents wait. Goldstein and Pauzner denote $\underline{\theta}(r_1)$ as the value of θ which gives $u(r_1) = p(\theta)u(\frac{1-\lambda r_1}{1-\lambda}R)$. The interval $[0, \underline{\theta}(r_1))$ is known as the lower dominance region (LDR). Thus, a type 2 agent will always withdraw early if they observe a signal $\theta_i < \underline{\theta}(r_1) - \epsilon$ as the true state of the economy (θ) and their private signal differs no more than ϵ. Another important assumption here is that there are values of θ for which all agents obtain signals which assure them that θ is in the LDR. This can be thought of as the critical point where actions no longer matter and it is always better for an agent to run than to wait. The critical point that guarantees this for any $r_1 \geq 1$ is shown to be $\underline{\theta}(1) > 2\epsilon$. Conclusively, when $\theta = 0$, all type 2 agents receive a signal below ϵ and must run.

4.3.2 Upper Dominance Region

Symmetrically, Goldstein and Pauzner also assume the existence of an upper dominance region (UDR) where no matter the actions of others, a patient agent will always wait to withdraw. This region has a range of $(\bar{\theta}, 1]$. One modification of the investment technology is needed to produce this region. The investment return, or production, normally produces a return of 1 at period 1. However, the return for the bank is modified to return 1 in the range $[0, \bar{\theta}]$, and R in the range $(\bar{\theta}, 1]$. One can interpret this modification as an extremely positive economy produces improved short-term returns when long-term returns are nearly certain. In this way, the return gained by the bank is higher than the maximal possible value of r_1, and there is no point in liquidating more than a single unit of investment to pay r_1 to an early agent. Therefore, the payments to agents withdrawing in period 2 are guaranteed. Like the LDR, Goldstein and Pauzner assume $\bar{\theta} < 1 - 2\epsilon$.

4.3.3 Analysis of Regions

To summarize the regions and provide some ease to visual learners, we can look at the following figure:

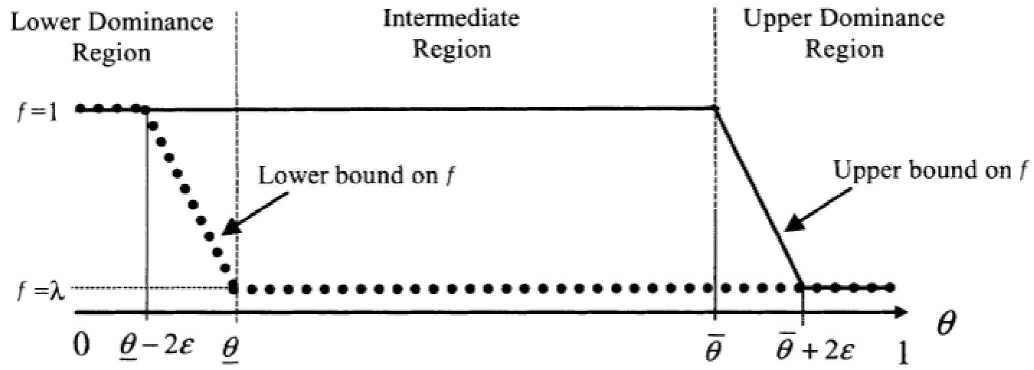

Figure 2: Dominance Regions and Agents' Behavior.
Taken from Goldstein and Pauzner (2005) with some modifications

To explain the figure further: the dotted line is a lower bound on f, and is constructed as the proportion of type 1 agents (λ) plus the type 2 agents who receive a signal of $\theta_i < \underline{\theta}(r_1) - \epsilon$ (the threshold level). As in section 4.3.1, $\theta < \underline{\theta}(r_1) - 2\epsilon$ implies that all type 2 agents get signals below $\underline{\theta}(r_1) - \epsilon$ and they must all run so f is 1. If $\theta > \underline{\theta}(r_1)$, then no type 2 agent gets a signal below $\underline{\theta}(r_1) - \epsilon$ and none of these agents *must* run. Note that they can still run depending on the actions of others, but it is not a given. Therefore, the lower bound is λ, the original proportion of type 1 agents. Finally, the distribution of the ϵ_i's is uniform and thus as θ increases from $\underline{\theta}(r_1) - 2\epsilon$ to $\underline{\theta}(r_1)$, the fraction of type 2 agents with signals below $\underline{\theta}(r_1) - \epsilon$ decreases linearly with a rate of $\frac{1-\lambda}{2\epsilon}$.

On the other hand, the solid line is the upper bound of f, and is constructed similarly with type 2 agents waiting if their signal is above $\bar{\theta} + \epsilon$.

Goldstein and Pauzner admit that little direct information about agents' behavior can be determined from the LDR and UDR. The intermediate region is far more interesting where the optimal strategy for an agent depends on their beliefs towards the actions of other agents. Here, agents do not know exactly what signals the other agents received; therefore, to choose the optimal action (or equilibrium action) at a certain signal, an agent takes into account the equilibrium actions of other agents at nearby signals. These actions depend on equilibrium actions at other, more distant signals. This process repeats until the equilibrium becomes consistent with the behavior at the dominance regions, but luckily this behavior is known! Goldstein and Pauzner are able to use this fact to determine the actions of agents in the intermediate region and with it, an optimal banking contract.

4.4 Findings

The first finding of Goldstein and Pauzner is that their model has a unique equilibrium in which type 2 agents run if they receive a signal below a critical threshold $\theta^*(r_1)$ and do not run if they receive a signal above. This is a distinct finding from the other models, where there are always two equilibria. In other words, they find that the behavior of a patient agent is in fact uniquely determined by their signal, and they will rush on the bank if and only if the signal received is below some critical value. Goldstein and Pauzner also define a deterministic function $f(\theta, \theta^*(r_1))$ which specifies the fraction of agents who run when the economic fundamentals are θ with $\theta^*(r_1)$ being the threshold level. It is defined as follows:

$$f(\theta, \theta^*(r_1)) = \begin{cases} 1 & \text{if } \theta \leq \theta^*(r_1) - \epsilon \\ \lambda + (1-\lambda)\left(\frac{\theta^*(r_1)+\epsilon-\theta}{2\epsilon}\right) & \text{if } \theta^*(r_1) - \epsilon \leq \theta \leq \theta^*(r_1) + \epsilon \\ \lambda & \text{if } \theta \geq \theta^*(r_1) + \epsilon. \end{cases}$$

Notice that the fraction of agents who run is 1 when θ is below $\theta^*(r_1) - \epsilon$ since there all type 2 agents receive signals below θ^*, and λ when θ is above $\theta^*(r_1) + \epsilon$ since the signals are above θ^*. Again, the economic fundamentals and the noise added to signals are uniformly distributed as we originally assumed, so $f(\theta, \theta^*)$ decreases linearly between $\theta^* - \epsilon$ and $\theta^* + \epsilon$.

Using this, Goldstein and Pauzner are able to study how the likelihood of runs is dependent on r_1 and arrive at their second finding. Namely that a larger r_1 implies that the set of signals which leads to type 2 agents running are also larger. Therefore, increased risk sharing between agents allows the banking system to become more vulnerable to bank runs. Intuitively, the utility type 2 agents gain from withdrawing in period 1 is larger since the payment in period 1 is increased and the payment in period 2 decreased. With this information, Goldstein and Pauzner understand how r_1 affects the actions of agents in period 1. Thus, they are able to compute the optimal r_1 by looking back at period 0. They find that if the LDR at $r_1 = 1$, $[0, \underline{\theta}(1))$ in mathematical notation, is not too large, then the optimal r_1 is larger than 1 and risk sharing benefits agents more than hinders them. In other words, a bank is a viable institution if the range where it is better to withdraw assets earlier is not too large. This of course makes sense in the real world since if it is likely that withdrawing one's cash early is better in the long run, there is no reason to put money in a bank in the first place. Finally, Goldstein and Pauzner are also able to determine that the optimal r_1 is actually lower than c_1^{FB}. Unlike Diamond and Dybvig, who calculated c_1^{FB} to maximize the increased utility from risk sharing without considering the consequences of a bank run, here bank runs are taken into consideration. Therefore, a bank must trade off the benefit of risk sharing with the cost of a bank run.

4.5 Conclusions from Goldstein and Pauzner

In contrast to Cooper and Ross, agents' ability to coordinate their actions from the economic fundamentals is not just a sunspot like the variable q. Instead, it is a *payoff-relevant variable*.

There is a unique outcome where agents must rely on their signals and cannot ignore them. This is in contrast with how the previous model depends on a critical q^* to determine the more efficient contract. The main takeaway from this final model is that there is one unique equilibrium, and here a run only occurs if the fundamentals are below some critical level. The probability of a run occurring depends on the banking contract itself, and specifically how small or large the payment to type 1 agents is. The optimal r_1 is found to not have the maximal utility gain from risk sharing as otherwise bank runs would be too likely. The overall decrease in welfare from the higher probability of runs outweighs the utility benefit gained by agents through risk sharing. Goldstein and Pauzner end with questioning how banking policies such as suspension of convertibility and deposit insurance can affect agent utility given the probability of runs. They argue that their model is much more suitable for this analysis since expected welfare cannot be calculated if the likelihood of a run is not known, but the exact details are left for future research.

5 Silicon Valley Bank

5.1 Background

On March 10, 2023, Silicon Valley Bank (SVB) was struck by, and failed due to, a bank run. This marked the third-largest bank failure in United States history and the most significant since the 2007–2008 financial crisis [10]. This event was part of a series of three bank failures that occurred in March 2023 in the United States, additionally involving Silvergate Bank and Signature Bank.

To decide how we may be able to apply the Diamond-Dybvig model, we must first understand what exactly went wrong. Firstly, from 2021, SVB had a significant increase in depositors, mostly resulting from entrepreneurs and their start-up companies. In an attempt to secure higher investment returns from the growing number of depositors, the bank significantly increased its holdings of long-term securities, recording them as hold-to-maturity assets. Hold-to-maturity is exactly what it sounds like: the bank plans to hold these assets until they mature, otherwise they will take a loss if they attempt to sell them before the maturity date. Therefore, these assets are rather illiquid and must be held until a future period, which is exactly the main scenario covered by the Diamond-Dybvig model. However, due to the Federal Reserve hiking interest rates to address rising inflation, the market value of these bonds depreciated considerably throughout 2022 and into 2023 since new bonds with higher interest rates were worth much more than the old ones. This resulted in unrealized losses on the portfolio of Silicon Valley Bank. The surge in interest rates also escalated borrowing costs across the economy, prompting some of SVB's clients in Silicon Valley to withdraw funds to meet their liquidity requirements including paying salaries. These customers would be thought of as Type 1 in the model.

To address the situation and meet withdrawal demands, SVB revealed on March 8th that it had liquidated over $21 billion USD worth of securities and planned an emergency sale to generate $2.25 billion from its treasury stock [11]. Unfortunately, this announcement combined with additional warnings from Silicon Valley investors triggered a panic-based bank run. The very next day, customers withdrew a total of $42 billion.

The California Department of Financial Protection and Innovation took control of SVB on March 10th and placed it under the control of the Federal Deposit Insurance Corporation (FDIC). Another unfortunate fact was that approximately 89 percent of the bank's $175 billion in depositor funds exceeded the maximum insured by the FDIC due to most customers being start-ups [12]. This was of course another factor in the bank run as the entrepreneurs did not know if the government would refund the entire deposit. Luckily, the FDIC quickly ensured full access to funds for all depositors without utilizing taxpayer money with SVB reopening as Silicon Valley Bridge Bank and offering its assets for auction.

The result of this bank run had serious consequences for start-ups both within the U.S. and foreign, with many unable to meet their monetary obligations. Technology, media, and winery companies, along with venture capital-backed founders, also faced disruptions. With SVB closed, many start-ups and founders were forced to take their business elsewhere to banks with less favorable conditions and entrepreneurship as a whole took a massive hit [13].

5.2 Applying the Model

There are several similarities between the crisis and the original Diamond-Dybvig model, though the main question is, can such a model be applied in this situation? As seen in the previous section, there are several variables which may have been responsible. The first and most obvious reason was that there was a large amount of unexpected type 1 consumers due to a sudden panic.

This was the ultimate reason for the bank's closing as they did not have the liquidity to pay back depositors. However, in order to get to such a place, there must be a larger reason why SVB failed. The second reason was where their investments were concentrated: in Hold-to-Maturity assets. Therefore, in this section, we will be looking at both the Diamond and Dybvig model and the Cooper and Ross model to explore how these aspects of bank runs can be simulated in MATLAB and if these models are a good fit for describing the Silicon Valley Bank run. The exact code can be found below in the appendix.

5.2.1 Diamond and Dybvig

Using the Diamond and Dybvig model, we first looked at how the amount given to type 1 consumers (r_1) would affect the amount of agents who can withdraw from a bank before there is a bank run. Here, it was assumed that if agent 2 utility gained from withdrawing in period 2 was less than agent 1 utility gained by withdrawing in period 1, there would be a run since type 2 agents would have no reason to wait as the bank is running out of assets. For the utility function, we used $\frac{-1}{x}$ as this met the desired conditions given in the model, especially the Inada conditions of $u'(0) = \infty$ and $u'(\infty) = 0$ as well as the relative risk aversion coefficient being greater than 1. One issue with the utility assumption is that this is not quite a panic-based run. In the real world, agents do not compare their "utility" with other agents, but instead make decisions based on what they hear, which was the case with SVB and social media. Therefore, the model may have been more realistic with the inclusion of private signals as seen in the Goldstein and Pauzner model, however due to time constraints these are not included in the code. Nevertheless, one can imagine that agents would talk to each other or read the news and learn that a large percentage has already withdrawn. This will of course lead to a run since if the percentage of running agents is large enough, it's in everyone's best interest to also run. Thus, the model can be assessed in this manner, and either way can still be used to explore how Diamond and Dybvig would assess the SVB situation.

Below, one can see in Figure 3 the graph created by recording varying values of r_1 given a constant R and simulating how many agents can withdraw early before a run.

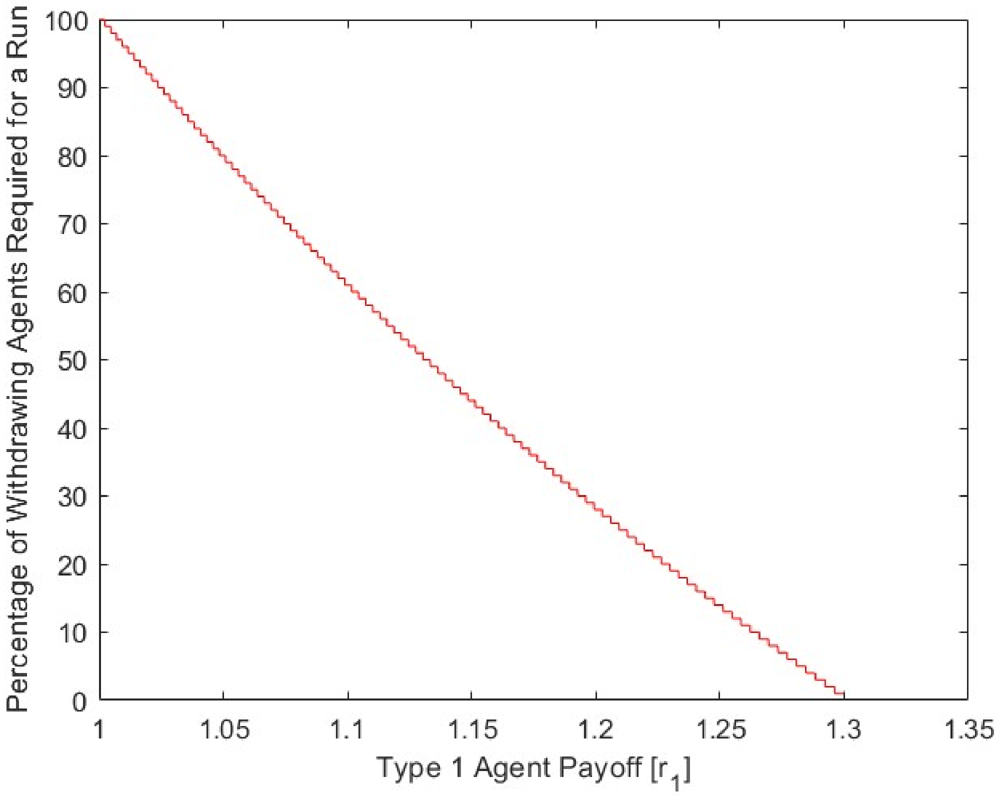

Figure 3

In this first graph, R is 1.3, which reflects the situation where SVB held a large part of its portfolio in 10-year treasury bonds which had an interest rate of approximately 3% in 2022 [15]. Remember that according to Diamond and Dybvig, R is the amount the bank receives after the completion of an investment project, and it must be greater than one. Technically, 10-year treasury bonds will pay an average coupon of 3% per year and the principle at maturity rather than 30% plus principle at the end of the 10 years. For simplicity and to keep in line with the original model, we assume that the latter is the case. This implies that SVB would have had more cash on hand in reality due to the frequent coupon payments (depending on if and where they invested this cash), and thus the percentage of withdrawing agents required for a run would be higher than what has been simulated.

The points on the red line indicate the highest percentage of withdrawals by both agent types in period 1 that can be serviced before the bank cannot afford to pay type 2 agents enough, given an r_1. By 'enough', we mean the critical amount paid where agent utility is equal and any less to type 2 agents will mean that their utility is lower than type 1 agents, triggering a run. For example, if r_1 is 1.05, then approximately 80% of depositors must withdraw before the bank is in danger of a run. The graph itself has step-like behavior due to some values of r_1 having runs at the same number of agents since we cannot have fractions of agents. In the next simulations we will also use a smoothing average for readability, though both types of graphs will be shown.

The above model could serve as a type of simplified stress test for determining the soundness of a certain amount of deposit interest. If the interest is too high, the bank can easily be in danger of a run. However, SVB had quite a low deposit interest rate, and faced a run before reaching the red line. This may be partly due to not only the nature of SVB's depositor base but also the costs of asset liquidation. Since their depositors consisted mostly of tech start-ups and large investors, a single full withdrawal could already reach several million dollars, and because of an increased cost of borrowing, many start-ups had to start funding their ventures with their own funds, leading to additional large withdrawals. This most likely led to a much bigger shock than expected and SVB showed signs of trouble sooner than if withdrawals had been more regular when compared to other large banks.

To discuss the costs due to asset liquidation, we added our own "liquidation cost variable" to the original model. Mathematically, we have:

$$V_2(f, r_1) = \max\left\{\frac{L * R(1 - r_1 f)}{1 - f}, 0\right\}, \text{ where } 0 \leq L \leq 1.$$

This L represents the costs of selling a hold-to-maturity asset sooner than it's maturity. For SVB, due to rising interest rates, their HTM assets were worth far less than the principle value, and thus they were forced to take a loss on them when liquidating in order to pay back depositors.

In figures 4 and 5 on the following page, we include the original simulation where $R = 1.3$ and liquidation costs are not taken into account, and additional simulations where a bank takes a 16.4% loss when selling their assets early ($L = 0.836$). 16.4 percent here is taken from the loss SVB would have incurred had it sold its HTM investments at the end of 2022 to meet depositor demands [16]. Additional simulations were also made to see how the amount of agents necessary for a run would be affected by a larger R with the same L.

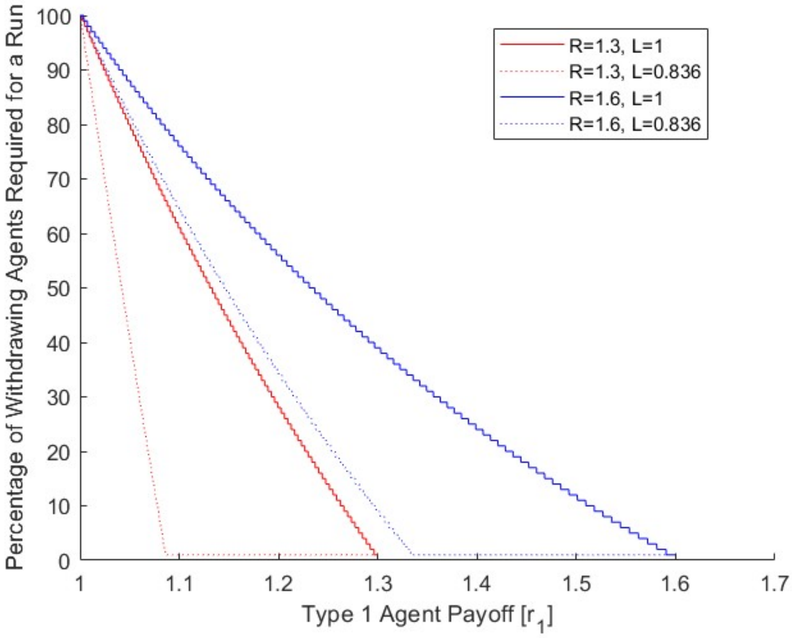

Figure 4

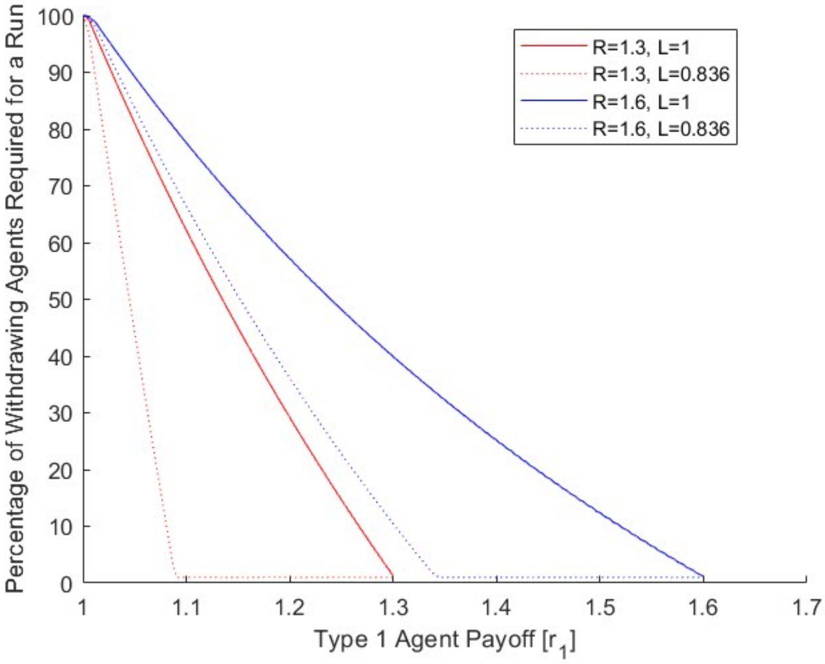

Figure 5: Smoothed Averages

With R at 1.3 and L at 0.836, one can see that the number of withdrawals required before a run is severely impacted and much lower than before. For instance, with the same r_1 of 1.05, the percentage is now below 41%. This model of course assumes that a bank's portfolio is entirely made of HTM assets which is rarely, if ever, the case. However, it still demonstrates how severely HTM assets can affect liquidity and withdrawals if they must be sold early. This is why SVB's portfolio, nearly half of which was comprised of HTM assets, worried analysts and led to a panic once large withdrawals started coming in. The blue line shows a different simulation with R at 1.6 and L at 0.836. A high liquidation cost can be seen to have a similar affect even with a higher R. Interestingly, a larger R makes it apparent that the results are not linear in nature, but appear more hyperbolic. This may stem from how the utility function is chosen. Finally, figure 5 shows the same data as figure 4, but with a weighted moving average to smooth out the lines for legibility.

5.2.2 Cooper and Ross

Using the Cooper and Ross model, we attempted to reproduce their results and explored how different combinations of λ and ν affected $\kappa*$, and if these could say anything about the risks Silicon Valley Bank took.

In appendix section 6.2, one can find the code used to recreate the model described in Cooper and Ross using MATLAB's optimization toolbox. Using this, we found the optimal early and late consumption for agents for a demand deposit contract given a random t and q chosen from a uniform distribution as well as the optimal amount for a bank to put in the risky investment, i. Most results were successfully recreated and aligned with what one would expect. With risk sharing and $\frac{-1}{x}$ as the utility function, early consumption was slightly higher than 1 and late consumption was lower than R. However, when including q in the simulation, MATLAB often gave very similar early and late consumption values. This was not expected as Cooper and Ross described the optimal consumption as 1 and R when q is high enough. Finally, without a capital requirement, the optimal i was 1 with various λ and ν tested.

Next, we looked at how various values of λ and ν would change the capital requirement that Cooper and Ross describe in their paper. These results were graphed as below:

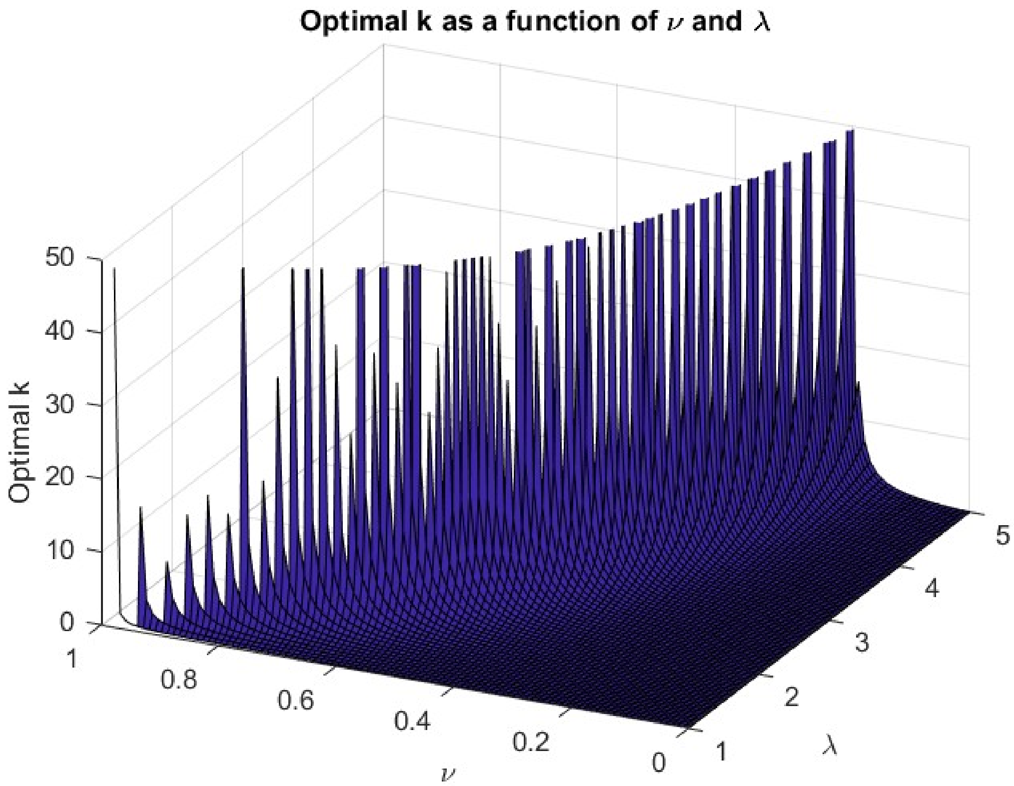

Figure 6

The surface shown in Figure 6 demonstrates all κ which fulfill the capital requirement as per Cooper and Ross' Proposition 1. Any values which did not fit the specifications of a "risky" investment ($\lambda > 1$ and $\nu\lambda \leq 1$) are excluded, which one can more easily see in Figure 7 with the top-down view of the surface. While most optimal capital requirements are within a fairly low range, near the edges of the surface lie large values of κ. These points can be interpreted as investments where the combination of risk and payoff make them particularly attractive to portfolio managers, and if the government does not wish for managers to invest in these assets, extra care and high capital equity requirements are needed.

However, the hold-to-maturity assets which Silicon Valley Bank had heavily invested in consisted largely of treasury bonds, as described above. Treasury bonds are not considered risky assets as they are in theory backed by the US government, who are extremely unlikely to default. Therefore, it would be very counter-intuitive for the government to impose any sort of base capital requirement on T-bonds. Additionally, Figure 8 shows a red dot where the optimal κ would approximately lie for these assets. With λ barely above 1 and ν nearly 1, $\kappa*$ skyrockets, meaning that in order for a bank manager with a risk-neutral outlook to be dissuaded from investing in this, the minimum capital requirement would be unfeasible.

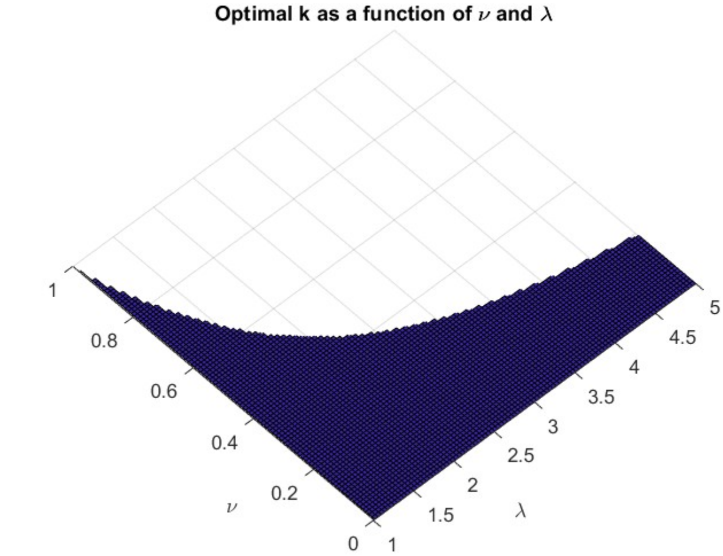

Figure 7: Aerial view of Figure 6

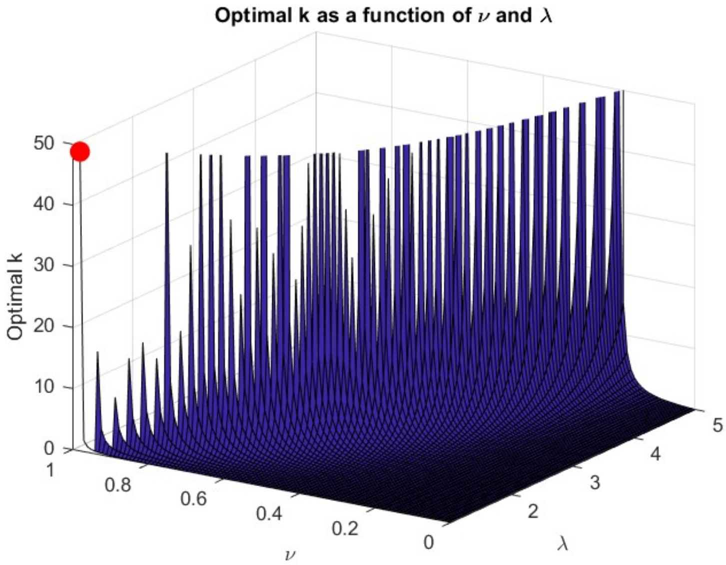

Figure 8

5.3 Conclusion and Final Thoughts

The results of MATLAB simulations indicate that the Diamond and Dybvig model and Cooper and Ross model are not suitable for the Silicon Valley Bank situation, although they do provide some information on other parts of banking. We can see that the models have a heavy focus on liquidity risk and the mindsets of both depositors and bank managers. However, SVB's problems arose largely due to a disregard of interest rate risk on HTM assets instead of pure liquidity risk, which played a far smaller role. The models studied were unfortunately not built with interest rate risk in mind, though they could perhaps be modified to include this type of risk.

Conclusively, the Diamond and Dybvig model remains a vital piece in understanding the mechanics and implications of bank runs. It highlights the importance of liquidity requirements and other potential risks in the banking industry. Extensions by Cooper and Ross and Goldstein and Pauzner have improved the framework by adding elements such as moral hazard, capital requirements, and information asymmetries. The fall of Silicon Valley Bank many months ago serves as a cautionary tale to larger institutions and shows the relevance of these kinds of models, even if the ones focused on in this paper did not completely reflect the situation at hand. This thesis not only validates some of the extensions in the literature, but also describes the importance of proper risk management and the necessity for more robust and realistic models. This will hopefully encourage more research into designing a model which can help institutions manage bank runs in the worst-case scenario of a depositor panic. Such a model will help to add more stability to the system for both bank managers and customers in the modern world.

6 Appendix

6.1 Proofs

Proof of Diamond and Dybvig Proposition 1

The following proof has been paraphrased, the original can be found in Diamond and Dybvig (1983).

Proof. Any run equilibrium does not achieve optimal risk sharing with $c_1^{1*} < c_2^{2*}$, since both types of agents receive the same consumption in the event of a run. Thus, as it is clear that uninsured demand deposit contracts are subject to runs, proposition 1 holds for all equilibria of uninsured bank contracts of the general form $V_1(f_j)$ and $V_2(f)$, where these can be any function.

Now consider the good equilibrium for any feasible contract. We will do this by contradiction. Recall that the "place in line" f_j is uniformly distributed over $[0, t]$ if only type 1 agents withdraw at $T = 1$. Suppose that the payments to those who withdraw at $T = 1$ is a nonconstant function of f_j over feasible values of t: for two possible values of $\tilde{t}$, t_1 and t_2, the value of a period 1 withdrawal varies, that is, $V_1(t_1) \neq V_1(t_2)$. Therefore there is a positive probability of different consumption levels by two type 1 agents who will withdraw at $T = 1$, which contradicts an unconstrained optimum where agents should receive the same consumption.

Now suppose that for all possible realizations of $\tilde{t} = t$, $V_1(f_j)$ is constant for all $f_j \in [0, t]$. This implies that $c_1^1(t)$ is a constant independent of the realization of $\tilde{t}$ since constant payoff implies constant consumption. On the other hand, $c_2^2(t)$ must vary with t, otherwise the resource constraint from (3) will not be met. Note that this will not be the case with $r_1 = 1$, but this is itself incompatible with optimal risk sharing. Constant $c_1^1(t)$ and varying $c_2^2(t)$ contradict (2), and thus also contradict optimal risk sharing. Therefore, optimal risk sharing is inconsistent with sequential service. □

Proof of Diamond and Dybvig Proposition 2

The following proof has been paraphrased, the original can be found in Diamond and Dybvig (1983).

Proof. Proposition 2 follows from the ability of tax-financed deposit insurance to duplicate the optimal consumptions $c_1^1(t) = c_1^{1*}(t)$, $c_2^2(t) = c_2^{2*}(t)$, $c_1^2(t) = c_2^1(t) = 0$ from the optimal risk sharing constraints characterized in equations 1, 2, and 3. Let the government impose a tax on all wealth held at the beginning of period $T = 1$, which is payable either in goods or in deposits. Let deposits be accepted for taxes at the pre-tax amount of goods which could be obtained if withdrawn at $T = 1$. The amount of tax that must be raised at $T = 1$ depends on the number of withdrawals then and the asset liquidation policy. Consider the proportionate tax as it is described in equation 4.

The after-tax proceeds, per unit of initial deposit, of a withdrawal at $T = 1$ depend on f through the tax payment and are identical for all $f_j \leq f$. Denote these after-tax proceeds by $\hat{V}_1(f)$, as given by equation 5.

The net payments to those who withdraw at $T = 1$ determine the asset liquidation policy and the after-tax value of a withdrawal at $T = 2$. Any excess tax collected to pay withdrawals at $T = 1$ is moved back into the bank in order to minimize the fraction of assets liquidated. This implies that the after-tax proceeds, per unit of initial deposit, of a withdrawal at $T = 2$ is given by $\hat{V}_2(f)$ as in equation 6.

As discussed in section 2.6, notice that $\hat{V}_1(f) < \hat{V}_2(f)$ for all $f \in [0,1]$, implying that no type 2 agents will withdraw in the first period no matter what they think the others will do. For all $f \in [0,1]$, $\hat{V}_1(f) > 0$, which implies that all type 1 agents *will* withdraw in the first period. Therefore, the unique dominant strategy equilibrium is $f = t$, the realization of $\tilde{t}$. Evaluated at a realization t:

$$\hat{V}_1(f = t) = c_1^{1*}(t)$$

and

$$\hat{V}_2(f = t) = \frac{[1 - tc_1^{1*}(t)]R}{1 - t} = c_2^{2*}(t)$$

and the optimum is achieved. □

Proof of Cooper and Ross Proposition 1

The following proof has been paraphrased, the original can be found in Cooper and Ross (2002).

Proof. Deposit insurance is complete up to the first-best contract (c_E^*, c_L^*), thus bank runs are eliminated.

Using the first-best contract, (8) becomes

$$\max_i[\nu(i\lambda R + (\kappa + 1 - i - \pi c_E^*)R - (1-\pi)c_L^*) + (1-\nu)\max((\kappa + 1 - i - \pi c_E^*)R - (1-\pi)c_L^*, 0)] \quad (9)$$

By the resource constraint of (7), $R = (1-\pi)c_L^* + R\pi c_E^*$ which in turn reduces (9) to:

$$\max_i[\nu(i\lambda R + (\kappa - i)R) + (1-\nu)\max((\kappa - i)R, 0)]$$

With this, one can see that i will be set to 0 or to its maximal value of $(1+\kappa)$ since any other choice of i is dominated by one of these extremes. The bank's profits are higher at $i = 0$ than at $i = 1 + \kappa$ if and only if:

$$R\kappa \geq \nu(1+\kappa)\lambda R - R\nu$$

which can be reduced to κ^* as given in the proposition.
Finally, from the definition of first-best, no other contract will increase the expected utility of the depositor. Therefore, if capital requirements meet the bound in the proposition, banks will offer the first-best contract and will not have any reason to invest in the risky fund. Then, there will be no incentive for the depositors to monitor and no bank runs. □

6.2 MATLAB Code

Figure 3

```
% Parameters
num_agents = 100; % Number of depositors
R = 1.3; % R gained by bank
Steps = 5000;
r_1 = linspace(1,R,Steps); % optimal consumption type 1

% Define utility function
utility_function = @(x) -1/(x); % utility function type 1

% Initialize variables
V = zeros(Steps, num_agents);
V_2 = zeros(Steps, num_agents);
```

```
agent1_utility = zeros(Steps, num_agents);
agent2_utility = zeros(Steps, num_agents);
f = zeros(1, num_agents);
f(1) = 0;

% Simulate withdrawals for each depositor period 1
for i = 1:Steps
for j = 1:num_agents

    % payoff type 1
    if f(j) < 1/r_1(i)
        V(i,j) = r_1(i);
    else
        V(i,j) = 0;
    end

    f(j+1) = f(j) + 1 / num_agents;

    % payoff type 2
    V_2(i,j) = max(R*(1-r_1(i)*(j/num_agents))/(1-(j/num_agents)),0);

    % Calculate agent utility
    agent1_utility(i,j) = utility_function(V(i,j));
    agent2_utility(i,j) = utility_function(V_2(i,j)); % if there have been j
    withdrawals with r_1, then this is type 2 utility

    if agent2_utility(i,j) < agent1_utility(i,j) % if type 2 utility is too low,
    type 2 would be better off withdrawing early and there will be a run
        if j == num_agents
            fprintf('Contract is runs preventing!\n')
            break
        end
            fprintf('Bank run occurs at agent %d!\n', j);
        break
    end
end
A(i) = j/num_agents * 100; % j/num_agents is f in D-D.
end

figure(1)
plot(r_1,A,'r')
xlabel('Type 1 Agent Payoff [r_1]');
ylabel('Percentage of Withdrawing Agents Required for a Run')
```

Figure 4

```
% Parameters
num_agents = 100; % Number of depositors
Steps = 5000;

% Function for simulation
function A = run_simulation(R, r_1, L)

    % Parameters
    num_agents = 100; % Number of depositors
    Steps = 5000;

    % Define utility function
    utility_function = @(x) -1/(x);

    % Initialize variables
    V = zeros(Steps, num_agents);
```

```
    V_2 = zeros(Steps, num_agents);
    agent1_utility = zeros(Steps, num_agents);
    agent2_utility = zeros(Steps, num_agents);
    f = zeros(1, num_agents);
    f(1) = 0;

    % Simulate withdrawals for each depositor
    for i = 1:Steps
        for j = 1:num_agents
            % payoff type 1
            if f(j) < 1/r_1(i)
                V(i,j) = r_1(i);
            else
                V(i,j) = 0;
            end
            f(j+1) = f(j) + 1 / num_agents;
            V_2(i,j) = max(L*R*(1-r_1(i)*(j/num_agents))/(1-(j/num_agents)),0); % payoff type 2

            % Calculate agent utility
            agent1_utility(i,j) = utility_function(V(i,j));
            agent2_utility(i,j) = utility_function(V_2(i,j)); % if there have been j withdrawals with r_1, then this is type 2 utility
            if agent2_utility(i,j) < agent1_utility(i,j) % if type 2 utility is too low, type 2 would be better off withdrawing early and there will be a run
                if j == num_agents
                    fprintf('Contract is runs preventing!\n')
                    break
                end
                fprintf('Bank run occurs at agent %d!\n', j);
                break
            end
        end
        A(i) = j/num_agents * 100;
    end
end

% Run simulations
R_values = [1.3, 1.3, 1.6,1.6];
L_values = [1, 0.836, 1, 0.836];
colors = ['r', 'r', 'b', 'b'];
linestyles = ['-', ':', '-', ':'];
legends = ["R=1.3, L=1", "R=1.3, L=0.836", "R=1.6, L=1", "R=1.6, L=0.836"];

figure(1)
hold on
for i = 1:4
    r_1 = linspace(1,R_values(i),Steps);
    A = run_simulation(R_values(i), r_1, L_values(i));
    plot(r_1,A,colors(i),'LineStyle',linestyles(i))
end

xlabel('Type 1 Agent Payoff [r_1]');
ylabel('Percentage of Withdrawing Agents Required for a Run');
legend(legends, 'Location', 'best');
```

Figure 5

```
% Parameters
num_agents = 100; % Number of depositors
Steps = 5000;

```

```
% Function to compute weighted moving average
function smoothed_data = weighted_moving_average(data, window_size)
    smoothed_data = zeros(size(data));
    for i = 1:length(data)
        if i < window_size
            smoothed_data(i) = mean(data(1:i));
        else
            smoothed_data(i) = mean(data(i-window_size+1:i));
        end
    end
end

% Function for simulation
function A = run_simulation(R, r_1, L)

    % Parameters
    num_agents = 100; % Number of depositors
    Steps = 5000;

    % Define utility function
    utility_function = @(x) -1/(x); % utility function type 1

    % Initialize variables
    V = zeros(Steps, num_agents);
    V_2 = zeros(Steps, num_agents);
    agent1_utility = zeros(Steps, num_agents);
    agent2_utility = zeros(Steps, num_agents);
    f = zeros(1, num_agents);
    f(1) = 0;

    % Simulate withdrawals for each depositor
    for i = 1:Steps
        for j = 1:num_agents
            % payoff type 1
            if f(j) < 1 / r_1(i)
                V(i, j) = r_1(i);
            else
                V(i, j) = 0;
            end
            f(j + 1) = f(j) + 1 / num_agents;
            V_2(i, j) = max(L * R * (1 - r_1(i) * (j / num_agents)) / (1 - (j / num_agents)), 0); % payoff type 2

            % Calculate agent utility
            agent1_utility(i, j) = utility_function(V(i, j));
            agent2_utility(i, j) = utility_function(V_2(i, j)); % if there have been j withdrawals with r_1, then this is type 2 utility
            if agent2_utility(i, j) < agent1_utility(i, j) % if type 2 utility is too low, type 2 would be better off withdrawing early and there will be a run
                if j == num_agents
                    fprintf('Contract is runs preventing!\n')
                    break
                end
                fprintf('Bank run occurs at agent %d!\n', j);
                break
            end
        end
        A(i) = j / num_agents * 100; % j/num_agents is f in D-D.
    end
end

% Run simulations
```

```
R_values = [1.3, 1.3, 1.6, 1.6];
L_values = [1, 0.836, 1, 0.836];
colors = ['r', 'r', 'b', 'b'];
linestyles = ['-', ':', '-', ':'];
legends = ["R=1.3, L=1", "R=1.3, L=0.836", "R=1.6, L=1", "R=1.6, L=0.836"];

figure(1)
hold on
window_size = 100; % Size of the moving average window
for i = 1:4
    r_1 = linspace(1, R_values(i), Steps);
    A = run_simulation(R_values(i), r_1, L_values(i));
    A_smoothed = weighted_moving_average(A, window_size); % Apply weighted moving average
    plot(r_1, A_smoothed, colors(i), 'LineStyle', linestyles(i))
end
xlabel('Type 1 Agent Payoff [r_1]');
ylabel('Percentage of Withdrawing Agents Required for a Run');
legend(legends, 'Location', 'best');
```

Cooper & Ross Basic Contract Model

```
% Define Variables
t = rand(); % type 1 vs type 2 ratio
utility_function = @(x) -1/(x); % utility function
R = 1.3;
q = rand(); % prob of economy wide pessimism

c_e = optimvar('c_e');
c_l = optimvar('c_l');
prob = optimproblem();
prob.ObjectiveSense = 'max';
prob.Objective = t*utility_function(c_e)+(1-t)*utility_function(c_l);
prob.Constraints.cons1 = t*c_e + (1-t)*c_l/R == 1;
prob.Constraints.cons2 = utility_function(c_l) >= utility_function(c_e);
prob.Constraints.cons3 = c_l <= R;

x0.c_e = 1.1;
x0.c_l = R;
sol = solve(prob,x0) % solve for optimal early and late consumption

% Define Variables

prob2 = optimproblem();
prob2.ObjectiveSense = 'max';
prob2.Objective = (1-q)*(t*utility_function(c_e)+(1-t)*utility_function(c_l))+q*utility_function(c_e)*(1/c_e);
prob2.Constraints.cons1 = t*c_e + (1-t)*c_l/R == 1;
prob2.Constraints.cons2 = utility_function(c_l) >= utility_function(c_e);
prob2.Constraints.con3 = c_l <= R;

x0.c_e = 1.1;
x0.c_l = R;
sol2 = solve(prob2,x0) % solve for optimum early and late consumption with q

% Define Variables
lambda = 4; % risky investment
nu = 0.2; % probability of risky investment paying off
c_e = sol.c_e;
c_l = sol.c_l;

i = optimvar('i','LowerBound', 0, 'UpperBound', 1);
```

```
fun = @(i)nu*(i*lambda*R+(1-i-t*c_e)*R-(1-t)*c_l)+(1-nu)*max((1-i-t*c_e)*R-(1-t)*c_l,0);
obj = fcn2optimexpr(fun,i);
prob3 = optimproblem('Objective',obj);
prob3.ObjectiveSense = 'max';

x1.i = 0.5;
sol3 = solve(prob3,x1) % Solve for optimal i
```

Figures 6-8

```
% Define range for nu and lambda
nu_values = linspace(0, 0.99, 100);
lambda_values = linspace(1.02, 5, 100);

% Initialize matrix to store optimal k values
optimal_k = NaN(length(nu_values), length(lambda_values)); % NaN to mark invalid combinations

% Loop over values of nu and lambda
for i = 1:length(nu_values)
    for j = 1:length(lambda_values)
        nu = nu_values(i);
        lambda = lambda_values(j);

        % Constraints
        if lambda > 1 && nu * lambda <= 1
            % Compute k_star
            k_star = nu * (lambda - 1) / (1 - nu * lambda);

            % Store the optimal k value
            optimal_k(i, j) = k_star;
        end
    end
end

% Plot
[X, Y] = meshgrid(lambda_values, nu_values);
surf(X, Y, optimal_k);
xlabel('\lambda');
ylabel('\nu');
zlabel('Optimal k');
title('Optimal k as a function of \nu and \lambda');

zlim([0, 50]);
% Specified point (lambda = 1.02, nu = 0.98)
hold on;
lambda_dot = 1.02;
nu_dot = 0.98;

k_star_dot = nu_dot * (lambda_dot - 1) / (1 - nu_dot * lambda_dot);

plot3(lambda_dot, nu_dot, k_star_dot, 'ro', 'MarkerSize', 10, 'MarkerFaceColor', 'r');
```

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